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論 文

분할법에 의한 대형회로망의 고유치 해석

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The Diakoptics Solution of Eigenvalue Problems in Large Scale Network

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要 約 분할법은 대규모회로망을 부분회로망으로 나누어서 해석하는 것으로 그 개념은 개회로와 폐회로에 대한 그래프 이론에 근거를 두고 있다. 본 논문에서는 대형회로망 고유값의 특성방정식을 분할법에 의하여 수식화 하고 반복법을 적용하여 간편하게 특성방정식의 해를 구함으로서 분할하지 않고 해석하는 방법에 비하여 효율적으로 처리할 수 있었다.

ABSTRACT The concept of diakoptics is to analyze a large scale network by partitioning it in to a number of smaller subnetworks. The theory has been developed from the concepts of open path and closed path through the conventional graph theoretic approach. In this paper, the formulation of characteristic equation of the eigenvalues of the network is represented by the aplication of diakoptics to the simulated network model of any linear large scale network. Furthermore, diakoptics coupled with appropriately proposed algorithm for the iterative solution of the characteristic equation results in considerable computational efficiency as compared with nondiakoptical methods.

I. Introduction

In linear network analysis, determination of complete solution comprising the general solution and steady state solution is often desirable. Determination of general solution is identifiable with the solution of generalized eigenvalue problem, the eigenvalues being poles of the transfer function[1].

Solution to the generalized eigenvalue problem of a large scale nature involves, inversion of matrix. The complexity and magnitude of the problem is directly function of matrix order and its sparsity[2]. Computer storage, number of operations needed and computation time increase with the order of matrix. The smaller the matrix is, the lesser effort would be.

Diakoptics is a method for the analysis of large scale system[3-7] The method involves conversion of eigenvalue problem in to equivalent

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electrical network, divides the large scale simulated network in to smaller networks, finds the solution models for the subnetworks, and finally obtains the solution to the original network, through a interconnected model. For solving large scale simulated equivalent networks, diakoptics substitutes the inversion of one large matrix by inversions of 'S' smaller subnetwork matrices plus one matrix of the intersection network for the tie line's parameter determination.

Another important technique employed by Brameller, for solution of the eigenvalue problem, is hybrid combination of diakoptics and escalator method[8]. The escalator method is a well established one, which consists of systematic way of escalating a 2×2 matrix up to any desired order in steps of one row and a column at a time.

The results of the diakoptics method is identical to one that would have been obtained if the network had been solved as an interconnected one. The objective of tearing method is to reduce the storage requirements and computation time.

To accomplish this objectives, this paper considered as following terms:

1) Based on an open path and a seg theory we shall establish the new mathematical foundation of Kron's orthogonal networks and diakoptics through conventional linear graph theory.

2) A new rigorous mathematical foundation for Kron's piecewise solution of large scale eigenvalue problems will be established, the derivation is also based on Kron's orthogonal network theory.

3) Derive the frequency equation (characteristic equation). The roots of the equation are the eigenvalues of the original large-scale system and determine all of the exact eigenvalues and eigenvectors. The escalator method related to Kron's piecewise solution of large scale eigenvalue problems will be discussed.

Thus the method of tearing reduces the computer storage, calculation time and improves accuracy, result in the more economical solution of large and complicated problems which can not be conveniently solved by any standard method. The advantages of tearing become apparent when the order of system to be simulated greatly exceeds that defining the capacity of the largest digital computer available.

II. Orthogonal Network

The topological properties of the linear graphs can be explored with the concepts of open path and closed path. For a connected graph of b edges and n nodes, there are (n-1) independent open paths and (b-n+1) closed paths. In electrical networks, the constitutive equations are Ohm's law and various equations for the components, and the equilibrium equations are obtained from Kirchhoff's current law and voltage law.

Using the standard edge symbols are shown in Fig. 1.

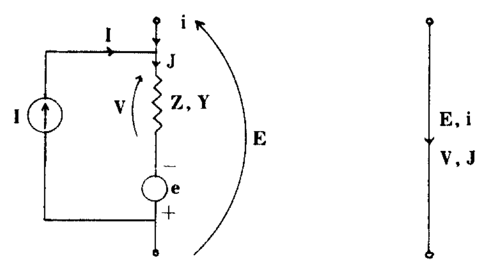

Fig 1 Standard edge and its linear graph representation.

Each edge is associated with E and i, and V and J. The Ohm's law equation can be written in the impedance form as

$$V = E + e \doteq Z\ (I + i) \tag{1}$$

and in the admittance form as

$$J = I + i = Y(E + e) \tag{2}$$

The vectors J and V are currents through and voltages across the edges of a graph. The vector I is composed of currents through the open paths due to injected currents; they are called the open path currents. The vector E contains the sums of voltage drops across the edges in the open paths. The vector i contains the closed path, or loop currents, the vector e is composed of the sums of the voltage drops in the closed paths.

It is well known that for a connected graph K of n nodes and b edges, KCL is given by

$$A_0^t = 0 \tag{3}$$

and KVL by

$$C_c^t E = 0 \tag{4}$$

where A_0 is the basic seg matrix and C_c is a basic circuit matrix. The superscript t denotes the transpose of a matrix. Since the nonsingular transformation C relates the edges to the open paths and closed paths, we can use C to define a new set of currents $[I_0 \vdots i_c]$ which is in one-to-one correspondence to the currents through the immittance J by

$$[J] = \overset{\quad o \qquad c}{[c_0 \vdots c_c]} \begin{bmatrix} I_0 \\ \hdashline i_c \end{bmatrix} \tag{5}$$

or

$$\begin{bmatrix} I_0 \\ \hdashline i_c \end{bmatrix} = \begin{matrix} o \\ c \end{matrix} \begin{bmatrix} A_0 \\ \hdashline A_c \end{bmatrix} [J]$$

where Io and ic are called the open path current vector and the closed path current vector respectively. Normally, the vector I is given for a network so that Io is known. The closed path current vector ic is unknown. Also, since each row of C_0^t and C_c^t represents an open path or closed path, the premultiplication of the V vector by C^t will give the algebraic sums of immittance voltages in this paths.

$$\begin{matrix} o \\ c \end{matrix} \begin{bmatrix} C^t_0 \\ \hdashline C^t_c \end{bmatrix} [V] = \begin{bmatrix} V_0 \\ \hdashline e_c \end{bmatrix} \tag{6}$$

or

$$V = \overset{\quad o \qquad c}{[A^t_0 \vdots A^t_c]} \begin{bmatrix} V_0 \\ \hdashline e_c \end{bmatrix}$$

where V is the open path vector and e_c is the closed path voltage vector.
when the constitutive equation is given as

$$J = YV \tag{7}$$

we have the orthogonal Y network equation, as

$$\begin{bmatrix} I_0 \\ \hdashline i_c \end{bmatrix} = \begin{bmatrix} A_0 \\ \hdashline A_c \end{bmatrix} [Y] \; [A^{t_0} \vdots A^{t_c}] \begin{bmatrix} E_0 + e_0 \\ \hdashline e_c \end{bmatrix}$$

$$= \begin{matrix} o \\ c \end{matrix} \begin{bmatrix} Y_{00} & \vdots & Y_{0c} \\ \hdashline Y_{c0} & \vdots & Y_{c0} \end{bmatrix} \begin{bmatrix} E_0 + e_0 \\ \hdashline e_c \end{bmatrix} \tag{8}$$

where

$$Y_{00} = A_0 Y A_0^t$$
$$Y_{0c} = A_0 Y A_c^t$$
$$Y_{c0} = A_c Y A_0^t$$
$$Y_{cc} = A_c Y A_c^t$$

Using the C and A transformations, the nonsingular transformations between the different orthogonal networks composed of the same set of standard branches can be constructed.

Let K_ℓ and K_J denote the graphs of two orthogonal networks. The transformations can be written as

$$[J^{(\ell)}]=[C_\ell]\begin{bmatrix} I_o^{(\ell)} \\ i_c^{(\ell)} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} E_o^{(\ell)}+e_o^{(\ell)} \\ e_c^{(\ell)} \end{bmatrix}=[C_\ell^t][V^{(\ell)}] \quad (10)$$

Then $T_{J\ell}=A_J C_\ell$

$$\begin{bmatrix} I_o^{(J)} \\ i_c^{(J)} \end{bmatrix}=[J_{J\ell}]\begin{bmatrix} I_o^{(\ell)} \\ i_c^{(\ell)} \end{bmatrix} \quad (11)$$

and $T_{\ell J}^t=C_\ell^t A_J^t$

$$\begin{bmatrix} E_o^{(\ell)}+e_o^{(\ell)} \\ e_c^{(\ell)} \end{bmatrix}=[T_{\ell J}^t]\begin{bmatrix} E_o^{(J)}+e_o^{(J)} \\ e_c^{(J)} \end{bmatrix} \quad (12)$$

Ⅲ. Eigenvalue problems by Diakoptics

A eigenvalue of a $n\times n$ square matrix A is a number, λ, that satisfies the equation,

$$(A-\lambda I)X=0 \quad (13)$$

where X is a column vector and is known as a eigenvector of matrix A. We shall derive the eigenvalue problems by means of diakoptics through the viewpoint of the orthogonal networks.

The orthogonal Y equations for the subdivisions from Eq. (8) is

$$\begin{bmatrix} I_o^{(S)} \\ i_c^{(S)} \end{bmatrix}=\begin{matrix} o \\ c \end{matrix}\overset{\quad o \qquad c}{\begin{bmatrix} Y_{oo} & Y_{oc} \\ Y_{co} & Y_{cc} \end{bmatrix}}\ \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} E_o^{(S)} \\ o \end{bmatrix} \quad (14)$$

where the subscript a represents subdivision, and the $e_o^{(S)}$ and $e_c^{(S)}$ are zero due to the nonexistance of independent voltage sources in the electric circuit model. From Eq. (14), the open path currents $I_o^{(S)}$ are found to be

$$I_o^{(S)}=Y_{oo}^{(S)}E_o^{(S)} \quad (15)$$

Where $I_o^{(S)}=C_K i^{(t)}$

and $Y_{oo}^{(S)}=(A^{(S)}-I)$

Notice that $I_o^{(S)}$ contains unknown tie line currents $i^{(t)}$ only, where $C_K i^{(t)}$ gives the new injected currents in the subdivisions due to tearing.

When there is no conpling between the subdivisions, $Y_{oo}^{(S)}$ has the form of

$$Y_{oo}^{(S)}=\text{diag}(A^{(1)}-\lambda I,\ A^{(2)}-\lambda I,\ \cdots,\ A^{(n_s)}-\lambda I)$$

where diag[·] means diagonal square matrix and

$$(A^{(1)}-\lambda I),\ (A^{(2)}-\lambda I),\ \cdots,\ (A^{(n_s)}-\lambda I)$$

are the admittance matrices of the subdivisions. Eq. (15) is rewritten as

$$(\lambda^{(S)}-\lambda I)E_o^{(S)}=C_K i^{(t)} \quad (16)$$

Using the similar transformation on the matrix $A^{(S)}$ which diagonalized $A^{(m)}$, we have

$$\Lambda^{(S)}=X^{-1}A^{(S)}X \quad (17)$$

where $\Lambda^{(S)}$ is the diagonal matrix of eigenvalues of the subdivisions, $\lambda_i^{(S)}$ of the form

$$\Lambda^{(S)}=\text{diag}(\lambda_1^{(S)},\ \lambda_2^{(S)}\cdots\lambda_n^{(S)})$$

X is the modal matrix of the subdivisions, $X_i^{(S)}$, of the form

$$X=\begin{bmatrix} X_1^{(S)} & & & \\ & X_2^{(S)} & & \\ & & \ddots & \\ & & & X_n^{(S)} \end{bmatrix}$$

Applying this transformation to the matrix $(A^{(S)}-I)$ results in the expression

$$\mathbf{X}^{-1}(\mathbf{A}^{(S)}-\lambda\mathbf{I})\mathbf{x}=(\Lambda^{(S)}-\lambda\mathbf{I}) \tag{18}$$

The inverse of $(\mathbf{A}^{(S)}-\mathbf{I})$ is found to be

$$(\mathbf{A}^{(S)}-\lambda\mathbf{I})^{-1}=\mathbf{X}(\Lambda^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{X}^{-1} \tag{19}$$

where

$$(\Lambda^{(S)}-\lambda\mathbf{I})^{-1} = \mathrm{diag}\left(\frac{1}{\lambda_1^{(S)}-\lambda}, \frac{1}{\lambda_2^{(S)}-\lambda}, \cdots, \frac{1}{\lambda_n^{(S)}-\lambda}\right)$$

The solution of Eq. (16) may be written as follows, using Eq. (19) for the inverse of $(\mathbf{A}^{(S)}-\lambda\mathbf{I})$

$$\mathbf{E}_0^{(S)}=(\mathbf{A}^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{C}_K\mathbf{i}^{(t)} = \mathbf{Z}_{00}^{(S)}\mathbf{I}_0^{(S)} \tag{20}$$

where $\mathbf{Z}_{00}^{(S)}=\mathbf{X}(\lambda^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{X}^{-1}$

and $\mathbf{I}_0^{(S)}=\mathbf{C}_K\mathbf{i}^{(t)}$

The network equation for the set of tie line is

$$\mathbf{i}^{(t)}=\mathbf{Y}^{(t)}\mathbf{E}^{(t)} \tag{21}$$

when $\mathbf{Y}^{(t)}$ is the admittance matrix of the tie lines.

From Eqs. (20) and (21), the orthogonal $\mathbf{Y}$ equation is

$$\begin{bmatrix}\mathbf{E}_0^{(S)}\\ \mathbf{E}^{(t)}\end{bmatrix}=\begin{matrix} & s & t\\ s & \mathbf{Z}_{00} & \\ t & & \mathbf{Z}^{(t)}\end{matrix}\quad \begin{matrix} s & \mathbf{C}_K\mathbf{i}^{(t)}\\ t & \mathbf{i}^{(t)}\end{matrix} \tag{22}$$

where $C\,\mathbf{Z}^{(t)}=[\mathbf{Y}^{(t)}]^{-1}$ is the impedance matrix of the tie lines.

Let the all open network and the tie lines be considered as a disconnected network, then graph K_s and K_t form the graph K_J of this disconnected network.

Since K_s is all open network and Kt is a primitive network, then the transformation matrix $\mathbf{A}_J$ of Eq. (11) is a unit matrix. Let the tie lines and the all open network be interconnected.

In the process of interconnection, each edge of K_t creates a fundamental closed path, which consists of exactly three edges, one from K_t and two from K_s. Let the linear graph of the interconnected network be denoted by K_ι. The open paths of K_ι and K_s are identicial and the closed paths are the fundamental closed paths formed by restoring tie lines. Thus

$$\mathbf{E}_0^{(\iota)}=\mathbf{E}_0^{(S)} \tag{23}$$

where $\mathbf{E}_0^{(\iota)}$ is the open path voltage vector of K_ι.

Let the open path and closed path currents of K_ι are denoted by $\mathbf{I}_0^{(\iota)}$ and $\mathbf{i}_C^{(\iota)}$ respectively, the tie line currents $\mathbf{i}^{(t)}$ are no longer included in the open path currents $\mathbf{i}_0^{(\iota)}$ of K_ι and the tie line currents $\mathbf{i}^{(t)}$ becomes the link currents $\mathbf{i}_C^{(\iota)}$ of K_ι. We have

$$\text{and}\quad \left.\begin{matrix}\mathbf{I}_0^{(\iota)}=0\\ \mathbf{i}_C^{(\iota)}=\mathbf{i}^{(t)}\end{matrix}\right| \tag{24}$$

From Eq. (13), the matrix of linear transformation $\mathbf{T}_{J\iota}$ is given by

$$\mathbf{T}_{J\iota}=\mathbf{A}_J\mathbf{C}_\iota=\mathbf{C}_\iota \tag{25}$$

where

$$\mathbf{C}_\iota=\begin{matrix} & o & c\\ T & \mathbf{U} & \mathbf{C}_{TC}\\ L & \mathbf{O} & \mathbf{U}\end{matrix}$$

here T and L stand for the K_s tree branches and the K_t links respectively, and $\mathbf{U}$ is a unit matrix.

The path voltage of Eq. (12) is

$$\begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{E}_0^{(t)} \\ \hdashline \mathbf{e}_c^{(t)}=0 \end{bmatrix} = \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \overset{s}{\mathbf{U}} & \overset{t}{\mathbf{C}} \\ \hdashline \mathbf{C}_{TC}^{t} & \mathbf{U} \end{bmatrix} \begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \mathbf{E}_0^{(S)} \\ \hdashline \mathbf{E}^{(t)} \end{bmatrix} \tag{26}$$

The inverse of Eq. (26) is

$$\begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \mathbf{E}_0^{(S)} \\ \hdashline \mathbf{E}^{(t)} \end{bmatrix} = \begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \overset{o}{\mathbf{U}} & \overset{c}{\mathbf{O}} \\ \hdashline -\mathbf{C}_{TC}^{t} & \mathbf{U} \end{bmatrix} \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{E}_0^{(t)} \\ \hdashline \mathbf{O} \end{bmatrix} \tag{27}$$

where $\mathbf{E}^{(t)}$ is the voltage vector across the tie lines. The path current vector of Eq. (11) is

$$\begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \mathbf{I}_0^{(S)} \\ \hdashline \mathbf{i}^{(t)} \end{bmatrix} = \begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \overset{o}{\mathbf{U}} & \overset{c}{\mathbf{C}_{TC}} \\ \hdashline \mathbf{O} & \mathbf{U} \end{bmatrix} \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{I}_0^{(t)} \\ \hdashline \mathbf{i}_c^{(t)} \end{bmatrix} \tag{28}$$

comparing $\mathbf{I}_0^{(S)}$ obtained from Eqs. (28) and (20), since the tie line currents $\mathbf{i}^{(t)}$ are equal to the link currentd $\mathbf{i}_c^{(t)}$ and the open path currents $\mathbf{I}_0^{(t)}=\mathbf{O}$, it is seen that the matrix $\mathbf{C}_K=\mathbf{C}_{TC}$.
Substituting Eqs. (27) and (28) into Eq. (22), we obtain

$$\begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \overset{o}{\mathbf{U}} & \overset{c}{\mathbf{O}} \\ \hdashline -\mathbf{C}_K^{t} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{E}_0^{(t)} \\ \hdashline \mathbf{O} \end{bmatrix}$$

$$= \begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \overset{s}{\mathbf{Z}_{00}^{(S)}} & \overset{t}{} \\ \hdashline & \mathbf{Z}^{(t)} \end{bmatrix} \begin{matrix} s \\ t \end{matrix}\begin{bmatrix} \overset{o}{\mathbf{U}} & \overset{c}{\mathbf{C}_K} \\ \hdashline \mathbf{O} & \mathbf{U} \end{bmatrix} \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{O} \\ \hdashline \mathbf{i}^{(t)} \end{bmatrix} \tag{29}$$

From Eq. (29), the equation of solution for the interconnected network K_t is

$$\begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{E}_0^{(t)} \\ \hdashline \mathbf{O} \end{bmatrix}$$

$$= \begin{bmatrix} \overset{s}{\mathbf{U}} & \overset{t}{\mathbf{O}} \\ \hdashline \mathbf{C}_K^{t} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \overset{s}{\mathbf{Z}_{00}^{(S)}} & \overset{t}{} \\ \hdashline & \mathbf{Z}^{(t)} \end{bmatrix} \begin{bmatrix} \overset{o}{\mathbf{U}} & \overset{c}{\mathbf{C}_K} \\ \hdashline \mathbf{O} & \mathbf{U} \end{bmatrix} \begin{bmatrix} \mathbf{O} \\ \hdashline \mathbf{i}^{(t)} \end{bmatrix}$$

$$= \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \overset{o}{\mathbf{Z}_{00}^{(S)}} & \overset{c}{\mathbf{Z}_{00}^{(S)}\mathbf{C}_K} \\ \hdashline \mathbf{C}_K^{t}\mathbf{Z}_{00}^{(S)} & \mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\mathbf{Z}_{00}^{(S)}\mathbf{C}_K \end{bmatrix} \begin{matrix} o \\ c \end{matrix}\begin{bmatrix} \mathbf{O} \\ \hdashline \mathbf{i}^{(t)} \end{bmatrix} \tag{30}$$

From Eq. (30), we obtain

$$(\mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\mathbf{Z}_{00}^{(S)}\mathbf{C}_K)\,\mathbf{i}^{(t)}=0 \tag{31}$$

Substituting Eq. (20) into Eq. (31), we obtain

$$(\mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\cdot\mathbf{X}(\Lambda^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{X}^{-1}\mathbf{C}_K)\,\mathbf{i}^{(t)}=0 \tag{32}$$

and

$$\mathbf{E}_0^{(t)}=\mathbf{Z}_{00}^{(S)}\mathbf{C}_K\,\mathbf{i}^{(t)} \tag{33}$$

which is the same as Eq. (20) since $\mathbf{E}_0^{(t)}=\mathbf{E}_0^{(S)}$
This is the basic relation of the present method of calculating the open path voltages (eigenvectors). Notice that, substitution of $\mathbf{i}^{(t)}$ vector into Eq. (33) gives the open path voltages (eigenvectors) of the original system.
The general theory of simultaneous algebraic equations show that there is a non-trivial solution if and only if, the matrix $(\mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\mathbf{X}(\Lambda^{(S)}-\mathbf{I})^{-1}\mathbf{X}^{-1}\mathbf{C}_K)$ of Eq. (32) is singular, that is

$$\det\ (\mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\mathbf{X}(\Lambda^{(S)}-\mathbf{I})^{-1}\mathbf{X}^{-1}\mathbf{C}_K)=0 \tag{34}$$

The roots of Eq. (34) are the eigenvalues of the original system. The above equation is the frequency equation (characteristic equation) of Eq. (32). The important point to observe is that the frequency equation involves the solution of only the $n_t \times n_t$ matrix, where n_t represents the total number of tie lines.

But the determinant of Eq. (34) is not wholly arithmetic, its evaluation will involve of the order of n_{tt} calculations. The general solution of the problem in this form is clearly impracticable.

Ⅲ－1 Solution of Frequency Equation

There are a number of ways of solving Eq. (34). One convenient way is by interconnecting one tie line at a time, that is, $\mathbf{C}_K^{t}\mathbf{X}$ and $\mathbf{X}^{-1}\mathbf{C}_K$ are single row and column vectors and $\mathbf{Z}^{(t)}$ is a scalar. By connecting one tie line at a time only the inversion of a 1×1 matrix is required. The frequency equation (34) for the interconnection of the K_{th} tie line can be expanded and conveniently solved by the same technique used in the escalator method as described in reference[8].

From the solution of eigenvalues, the corresponding eigenvectors of the interconnected system may be evaluated as follows. From Eq. (32) and (33), substituting $Z_{oo}^{(S)}$ from Eq. (23), we have

$$E_o^{(t)} = X(\Lambda^{(S)} - \lambda I)^{-1} X^{-1} C_K i^{(t)} \qquad (35)$$

From Eq. (34), the coefficient of $i^{(t)}$ in Eq. (32) is zero. Thus $i^{(t)}$ can be any scalar, it may be considered as a common scaling factor of the eigenvector $E^{(t)}$ and can be chosen to be unity. Therefore

$$E_o^{(t)} = X(\Lambda^{(S)} - \lambda I)^{-1} X^{-1} C_K \qquad (36)$$

For orthonormal eigenvectors

$$X^{-1} = X^t$$

Therefore

$$E_o^{(t)} = X(\Lambda^{(S)} - \lambda I)^{-1} X^t C_K \qquad (37)$$

Based on the escalator method, it is then necessary to invert the modal matrix X of the subdivisions for each interconnection. This technique is inqppropriate because the inversion of a large matrix requires an excessive computation time and may induce numerical inaccuracy[2].

To overcome the problems cited above for the real matrix A, we shall apply the theory to a square matrix.

The following steps taken in order facilitate the solution of eigenvalues in a systematic manner.

(1) Tear the given system into a number of subdivisions and the lines.

(2) Establish the connection matrix Ck.

(3) Calculate eigenvalues and corresponding eigenvectors for each subdivisions by any method and establish the corresponding modal matrix X from the eigenvectors of subdivisions.

(4) Interconnect one tie line at a time to the models to form ultimately an interconnected network which has exactly the same eigenvalues as the original system.

(5) For the interconnection of the K_{th} tie line:

(a) Calculate $D = X^t C_K = C_K^t X$

(b) Solve by Newton's method the escalator form of the frequency equation, Eq. (34):

$$f(\lambda) = \sum_{i=1}^{n} \frac{d_i^2}{(\lambda_i^{(S)} - \lambda)} + Z_K^{(t)} = 0$$

and

$$f'(\lambda) = \sum_{i=1}^{n} \frac{d_i^2}{(\lambda_i^{(S)} - \lambda)^2}$$

(c) Evaluate the eigenvectors from Eq. (37)

$$E_o^{(t)} = X(\Lambda^{(S)} - \lambda I)^{-1} X^t C_K$$

(d) Establish a new modal matrix from (c):

$$X = X_{new}$$

and

$$\lambda^{(S)} = \lambda_{new}$$

(e) Repeat step (a) for the next interconnection.

Ⅳ. The result of numerical study

The sample network has been used to analysis a large scale eigenvalue problem by diakoptics.

$$\begin{bmatrix} 3-\lambda & -1 & 0 & 0 & -1 \\ -1 & 3-\lambda & -1 & 0 & 0 \\ 0 & -1 & 3-\lambda & -1 & 0 \\ 0 & 0 & -1 & 4-\lambda & -1 \\ -1 & 0 & 0 & -1 & 3-\lambda \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix} = 0$$

The equation above defines the eigenvalue problem and represented diagrammatically in Fig.2. The removal of the lines and the splitting the ground node into two nodes yield the torn network and its graph as shown in Fig.3.
Then the equation of solution, from Eq. (35), is

$$E_0^{(S)} = (A^{(S)} - I)^{-1} C_K i^{(t)}$$

or

$$\begin{bmatrix} E_1^{(S)} \\ E_2^{(S)} \\ E_3^{(S)} \\ E_4^{(S)} \\ E_5^{(S)} \end{bmatrix} = \begin{bmatrix} 2-\lambda & -1 & 0 & & \\ -1 & 3-\lambda & -1 & & \\ 0 & -1 & 2-\lambda & & \\ & & & 3-\lambda & -1 \\ & & & -1 & 2-\lambda \end{bmatrix}^{-1} \begin{bmatrix} i_2^{(t)} \\ 0 \\ i_1^{(t)} \\ -i_1^{(t)} \\ -i_2^{(t)} \end{bmatrix} \tag{38}$$

For the second torn subdivision, the eigenvalues and corresponding eigenvectors are

$$\lambda_1 = 4, \quad \lambda_2 = 2, \quad \lambda_3 = 1$$

and

$$X_1 = \begin{bmatrix} -.408248 \\ 0.816469 \\ -.408248 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -.707107 \\ -.0 \\ 0.707106 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} 0.577350 \\ 0.577350 \\ 0.577350 \end{bmatrix}$$

For the first torn subdivision, the eigenvalues and the corresponding eigenvectors are

$$\lambda_4 = 3.168033, \quad \lambda_5 = 1.381966$$

and

$$X_3 = \begin{bmatrix} 0.850651 \\ -.525731 \end{bmatrix}, \quad X_4 = \begin{bmatrix} 0.525731 \\ 0.850651 \end{bmatrix}$$

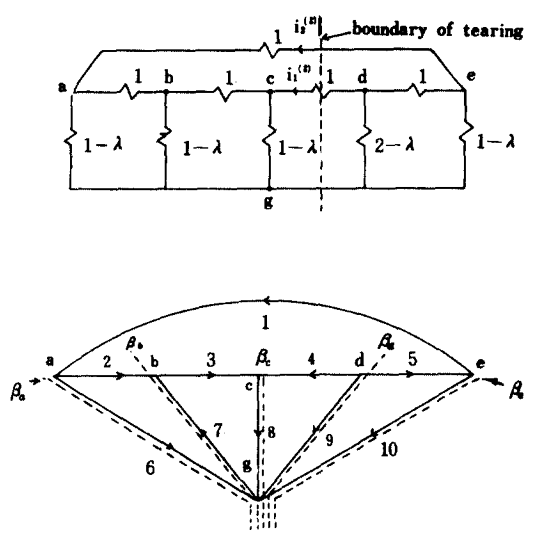

Fig 2 A network and its graph K.

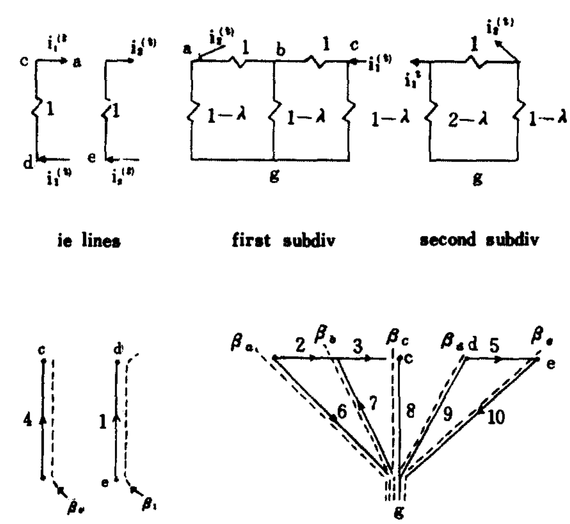

Fig 3 The torn network and its graph.

Using Eqs. (21) and (22), Eq. (23) becomes

$$
\begin{bmatrix} E_1^{(S)} \\ E_2^{(S)} \\ E_3^{(S)} \\ E_4^{(S)} \\ E_5^{(S)} \end{bmatrix}
=
\left[\begin{array}{ccc|cc}
-.408248 & -.707107 & 0.577350 & & \\
0.816496 & 0.0 & 0.577350 & & \\
-.408248 & 0.707106 & 0.577350 & & \\
\hline
 & & & 0.850651 & 0.525731 \\
 & & & -.525731 & 0.850651
\end{array}\right]
$$

$$
\times
\begin{bmatrix}
\frac{1}{4-\lambda} & & & & \\
 & \frac{1}{2-\lambda} & & & \\
 & & \frac{1}{1-\lambda} & & \\
 & & & \frac{1}{3.161803-\lambda} & \\
 & & & & \frac{1}{1.381966-\lambda}
\end{bmatrix}
\times
$$

$$
\left[\begin{array}{ccc|cc}
-.408248 & 0.816496 & -.408248 & & \\
-.707107 & 0.0 & 0.707106 & & \\
0.577350 & 0.577350 & 0.577350 & & \\
\hline
 & & & 0.850651 & -.525731 \\
 & & & 0.525731 & 0.850651
\end{array}\right]
\left[\begin{array}{c}
i_2^{(t)} \\ 0 \\ i_1^{(t)} \\ \hline -i_1^{(t)} \\ -i_2^{(t)}
\end{array}\right]
$$

The all open network of Eq. (39) and its graph K_S together with the tie lines and its graph Kt are shown in Fig.4.

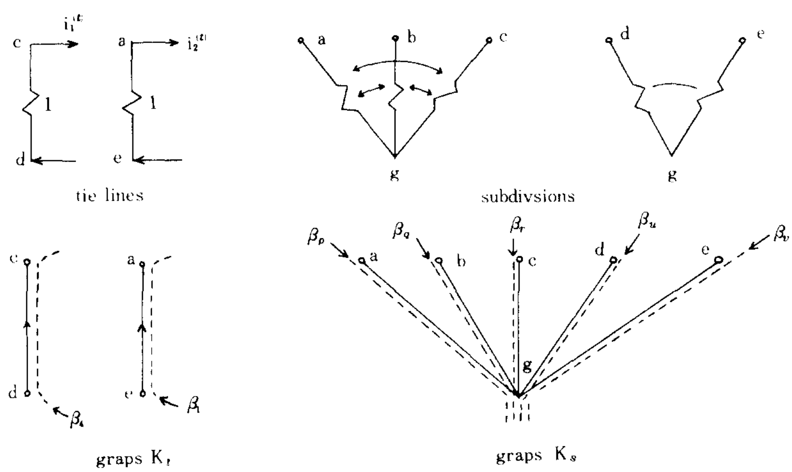

Fig 4 The all open torn network and K,

It is obvious from Fig.4 that Ci = U. The impedance matrix of the torn network, Eq. (22), is

$$\mathbf{Z}^{(J)}=\left[\begin{array}{c:cc}\mathbf{X}(\Lambda^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{X}^{t} & & \\ \hdashline & 1 & \\ & & 1\end{array}\right]$$

The interconnected network and its graph K_t are shown in Fig.5. The transformation matrix $T_{Jt}=C_t$ is obtained from Ki, and

$$\mathbf{C}_t=\begin{array}{c}\\ T\\ L\end{array}\left[\begin{array}{cc}o & c\\ \mathbf{U} & \mathbf{C}_{Tc}\\ \hdashline \mathbf{O} & \mathbf{U}\end{array}\right]$$

Thus the connection matrix C_K is

$$\mathbf{C}_{TC}=\mathbf{C}_K=\begin{array}{c}\\ \beta_p\\ \beta_q\\ \beta_r\\ \beta_u\\ \beta_v\end{array}\begin{array}{c}\begin{array}{cc}\varphi_1 & \varphi_2\end{array}\\ \left[\begin{array}{cc}0 & 1\\ 0 & 0\\ 1 & 0\\ -1 & 0\\ 0 & -1\end{array}\right]\end{array}$$

The equation of solution for the interconnected graph K_t, from Eq. (34), is

$$\det\ (\mathbf{Z}^{(t)}+\mathbf{C}_K^{t}\mathbf{X}(\Lambda^{(S)}-\lambda\mathbf{I})^{-1}\mathbf{X}^{t}\mathbf{C}_K)=0 \qquad (40)$$

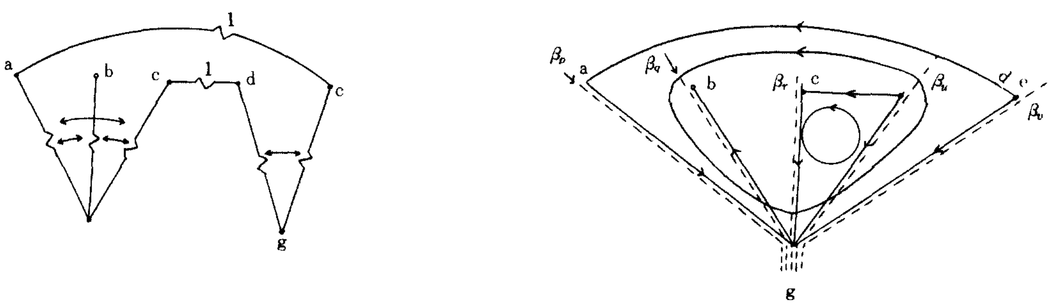

Fig 5 The interconnected network and K_1.

Where

$$\mathbf{Z}=\begin{bmatrix}1 & \\ & 1\end{bmatrix}+\begin{bmatrix}-.408248 & 0.707106 & 0.577350 & -.850651 & -.525731\\ -.408248 & -.707107 & 0.577350 & 0.525731 & -850651\end{bmatrix}\times$$

$$\begin{bmatrix}\frac{1}{4-\lambda} & & & & \\ & \frac{1}{2-\lambda} & & & \\ & & \frac{1}{1-\lambda} & & \\ & & & \frac{1}{3.161803-\lambda} & \\ & & & & \frac{1}{1,381966-\lambda}\end{bmatrix}\begin{bmatrix}-.408248 & -.408248\\ 0.707106 & 0.707107\\ 0.57735 & 0.37753\\ -.850651 & 0.525731\\ -.525731 & -.850651\end{bmatrix}$$

As mentioned, there are a number of ways of solving Eq. (40). One convenient way is by interconnecting one tie line at a time, that is, $\mathbf{C}_k^{t}\mathbf{X}$ and $\mathbf{X}^{-1}\mathbf{C}_K$ are single row and column vectors and $\mathbf{Z}^{(t)}$ is a scalar.

Our next step is to interconnect the first tie line (k=1). Therefore, we calculate

$$\mathbf{D}=\mathbf{X}^t\mathbf{C}_1$$

$$=\left[\begin{array}{ccc:cc} -.408248 & 0.816469 & -.408248 & & \\ -.707107 & 0.0 & 0.707106 & & \\ 0.577350 & 0.577350 & 0.577350 & & \\ \hdashline & & & 0.850651 & -.525731 \\ & & & 0.525731 & 0.850651 \end{array}\right]\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

Then

$$\begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \\ d_4^2 \\ d_5^2 \end{bmatrix}=\begin{bmatrix} 0.166667 \\ 0.5 \\ 0.333336 \\ 0.723607 \\ 0.276393 \end{bmatrix}$$

We may write the escalator equation for the network after the first interconnection:

$$f(\lambda)=\frac{.166667}{4-\lambda}+\frac{.5}{2-\lambda}+\frac{.333336}{1-\lambda}+\frac{.723607}{3.618033-\lambda}+\frac{.276393}{1.381966-\lambda}+1=0$$

We determine its roots by Newton's method, we obtain

Then

$$\begin{bmatrix} d_1^2 \\ d_2^2 \\ d_3^2 \\ d_4^2 \\ d_5^2 \end{bmatrix}=\begin{bmatrix} 0.125753 \\ 1.456885 \\ 0.018585 \\ 0.377456 \\ 0.021320 \end{bmatrix}$$

$$\lambda_1=1.113387,\quad \lambda_2=1.531122,\quad \lambda_3=2.415332,\quad \lambda_4=3.913728,\quad \lambda_5=5.026415$$

The corresponding eigenvectors, arranged in a new modal matrix, are

$$\mathbf{X}_{new}=\begin{bmatrix} -.623664 & -.405695 & 0.544264 & 0.374036 & -.101624 \\ -.552947 & -.190220 & -.226053 & -.715805 & 0.307555 \\ -.419531 & 0.126285 & -.676429 & 0.280018 & -.521613 \\ -.238540 & 0.375721 & -.169430 & 0.459945 & 0.749452 \\ -.269047 & 0.801320 & 0.407938 & -.240340 & -.247637 \end{bmatrix}$$

Our next step is to interconnect the second tie oine. Therefore, we calculate

$$\mathbf{D}=\mathbf{X}_{new}\cdot\mathbf{C}_2=\begin{bmatrix} -.623664 & -.552947 & -.419531 & -.238540 & -.269047 \\ -.405695 & -.190220 & 0.126285 & 0.375721 & 0.801320 \\ 0.544264 & -.226053 & -.676429 & -.169430 & 0.407938 \\ 0.374036 & -.715805 & 0.280018 & 0.459945 & -.240340 \\ -.101624 & 0.307555 & -.521613 & 0.749452 & -.247637 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ -0 \\ -1 \end{bmatrix}$$

We may write the new escalator equation for the second interconnected network as

$$f(\lambda)=\frac{0.125753}{1.113387-\lambda}+\frac{1.456885}{1.531122-\lambda}+\frac{0.018585}{2.415332-\lambda}+\frac{0.377556}{3.913728-\lambda}+\frac{0.021320}{5.026415-\lambda}+1=0$$

We determine its roots by Newton's method, we obtain

$\lambda_1=1.139191,\ \lambda_2=2.381962,\ \lambda_3=2.745893,$

$\lambda_4=4.613029,\ \lambda_5=5.114904$

These are the exact eigenvalues of the original large scale system. The corresponding eigenvectors, arranged in a modal matrix, are calculated from Eq. (37):

$$X=\begin{bmatrix} -.510041 & 0.371746 & -.469959 & -.601500 & -.137844 \\ -.510039 & -.371749 & -.469959 & 0.601501 & -.137845 \\ -.439043 & -.601501 & 0.350540 & -.371749 & 0.429374 \\ -.306932 & 0.000003 & 0.559034 & -.000001 & -.770243 \\ -.439035 & 0.601502 & 0.350543 & & 0.429374 \end{bmatrix}$$

It should be emphasized that in the described method no approximations are made. The eigenvalues and eigenvectors are exact and all of them are determined.

V. Conclusions

Mathematically rigorous theory and formulations are developed for large scale eigenvalue problems. Comprehensive derivations based on orthogonal network theory and founded on Kron's piecewise solution of large scale eigenvalue problem are presented. This method could make available the computational advantage presently accessible in diakoptical analysis of symmetric matrix ease, namely the substitution of inversion of a single large scale matrix, by inversion of (S+1) smaller matrices. Considerable computational ease and efficiency is proved through sample network for study of eigenvalue problem associated with the general solution of large scale network.

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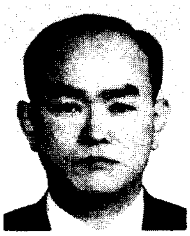

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