論 文

Seismic Wave Attenuation에 의한 Wrap-around Noise의 제거

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Wrap-around Noise Removal by Seismic Wave Attenuation

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要 約 Seismic wave가 지하 내부로 전파되어 갈때 점차적으로 에너지를 상실함으로써 attenuation현상을 일으킨다. Seismic wave의 velocity를 complex number로 표시하여 실수부는 phase velocity, 허수부는 attenuation 상수로 하여 attenuation특성을 수치적으로 modeling하는 방법을 제시하였다. 이 방법은 주파수와 독립적으로 로그특성으로 감쇄해가는 매질 속에서의 파동의 전파를 modeling한다. 본 연구는 attenuation을 위치 함수로 표시하여 순방향 및 역방향 numerical modeling에 응용하여 FFT 계산때 발생하는 wrap-arround noise를 효율적으로 제거함으로써 memory space를 절약하고 computing time을 감소시킬 수 있음을 잘 보여주고 있다.

ABSTRACT Seismic waves are attenuated by losses of energy as they propagate through the earth. One way to model this numerically is to make the velocity a complex number, the real part giving the phase velocity and the imaginary part the attenuation. This models wave propagation in a medium for which the logarithmic decrement is independent of frequency(attenuation coefficient is proportional to frequncy). The aim is to modify forward and inverse numerical modeling so that attenuation can be specified as a function of position.

I. INTRODUCTION

Seismic wave attenuation can be incorporated in numerical forward and inverse modeling by allowing the velocity to be a complex number.[1] The real part of the velocity is the phase velocity, and the imaginary part is the phase velocity multiplied by the logarithmic decrement. However, it is not enough to simply use the full acoustic wave equation and make the velocity a complex number. Some plane wave components would be attenuated, others amplified. One approach is to separate the waves into upgoing and downgoing components and propagate them so that their amplitudes decay. In this report an one-way equation with attenuation is investigated. It is based on Gazdag's method (1981) and the algorithm can be used for time-reversed depth migration.[2,3]

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An algorithm that includes attenuation in a way that is fairly realistic may have some use in modeling shot gathers or CDP profiles, or for undoing the effects in a depth migration program. Additionally, it may have some numerical applications. A border of attenuating material around a snapshot might conveniently reduce the wrap-around effects that are a part of the Fourier method.

For many earth materials the logarithmic decrement, or quality factor Q, is a constant independent of frequency, but not for all. A report on this topic has been written by Purnell (1983) in which measurements of Q for some physical modeling materials are given.[4]

For plane waves propagating in a homogeneous acoustic medium, the amplitude is given by

$$P(x, t) = P_0 e^{i(kx - wt)} = P_0 e^{ik(x - ct)} \quad (1)$$

where w is the angular frequency, k is the wavenumber, and c is the phase velocity.

Attenuation may be introduced mathematically by allowing the velocity c to be complex, $c = c_1 + ic_2$ where c_1 is the phase velocity and c_2 is c_1 multiplied by the logarithmic decrement. Then the amplitude of the plane waves propagating with attenuation can be expressed as the following,

$$\begin{aligned} P(x, t) &= P_0 e^{ik(x-(c_1+ic_2)t)} \\ &= P_0 e^{ik(x-c_1t)+kc_2t} \\ &= P_0 e^{ik(x-c_1t)} e^{c_2kt} \end{aligned} \quad (2)$$

where e^{c_2kt} is the attenuation factor provided c_2 is negative.

A snapshot of the wave described by equation (2) at a = 0 is given by $P_0 e^{ikx}$, a single frequency (wavelength = $2\pi/k$) constant amplitude disturbance. At time t the wave has travelled a distance c_1t. For material with constant Q, the amplitude is assumed to decay exponentially proportional to the number of wavelengths travelled. Hence at time t the amplitude is $P_0 e^{-\Delta(c_1t)k/2\pi}$

where ∇ is the logarithmic decrement, a constant independent of k. By comparison with equation (2), it can be seen that $c_2 = -\nabla c_1/2\pi$ Successive snapshots of the wave described by equation (2) show a constant wavelength disturbance advancing with velocity c_1 and decaying in amplitude by the factor $e^{-\Delta}$ for every wavelength of distance travelled.

The quality factor Q is related to the logarithmic decrement by $Q = \pi/\nabla$ Q is physically a measure of the peak energy in a cycle to the energy dissipated in a cycle. For rocks Q is typically in the rangee 10 to 500.

II. MATHEMATICAL BACKGROUND

Let P(x,y) be the two-dimensional pressure field (real values). The digitized values denoted by $P_{\alpha,\beta}$ ($\alpha = 0, N-1$; $\beta = 0, N-1$)can be Fourier transformed to give the complex coefficients $C_{\gamma,\sigma}$.[3,5] The relationships are,

$$C_{\gamma,\sigma} = 1/N^2 \sum_{\alpha=0}^{N-1} \sum_{\beta=0}^{N-1} P_{\alpha\beta}\, e^{-i(\alpha\gamma+\beta\sigma)2\pi/N}$$

and

$$P_{\alpha,\beta} = \sum_{\gamma=0}^{N-1} \sum_{\sigma=0}^{N-1} C_{\gamma,\sigma}\, e^{i(\alpha\gamma+\beta\sigma)2\pi/N} \quad (3)$$

A continuous complex valued signal Z(x,y) can be defined from the coefficients $C_{\gamma,\sigma}$ by the following summation

$$Z(x, y) = 2 \sum_{\gamma=-N/2}^{N/2}{}' \sum_{\sigma=0}^{N/2}{}' C_{\gamma,\sigma}\, e^{i(\gamma x+\sigma y)2\pi/L} \quad (4)$$

where L=N$\triangle$X and Σ' means Σ with the first and last terms given a weight of 1/2. The kev property of Z(x,y) is that

$$P(x, y) = \text{Real}\,(Z(z, y)).$$

The imaginary part of $Z(x, y)$ is called the Hilbert transform of $P(x,y)$.

The terms in equation (4) represent single-frequency plane waves. For example, the term $e^{i(\gamma x+\delta y)2\pi/L}$ is constant when $(\gamma x+\delta y)$ is constant. This occurs along lines in the x,y plane which make an angle θ with the x axis, where $\tan\theta = \gamma/\delta$ (Figure 1).

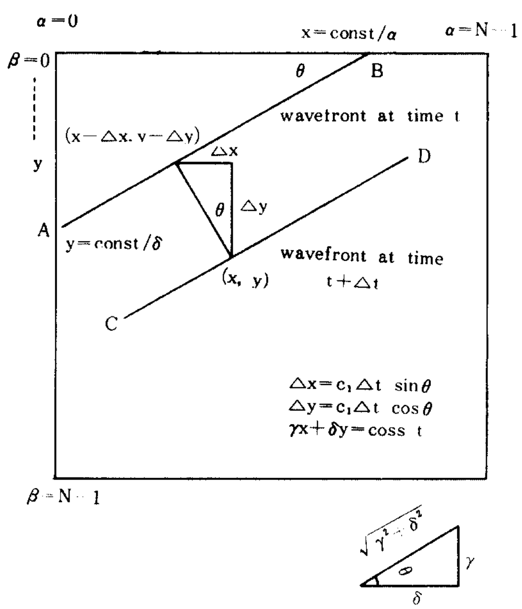


FIGURE 1 Geometric relationships for plane wave propagation. AB shows the location at time t of a plane wave component, i. e. the line along which $\gamma x+\delta y=$ const. DC shows its location at time $t+\Delta t$. The distance travelled is $c_1\Delta t$ and the direction is down if $\Delta y>0$. The pressure $P(x,y,t+\Delta t)$ is equal to the pressure $P(x-\Delta x, y-\Delta y, t)$ multiplied by the attenuation factor $\exp(-\nabla(c_1\Delta t)/\lambda)$.

The wavelength is a distance perpendicular to the lines of constant phase such that $e^{i(\gamma x+\delta y)2\pi/L}$ returns to the same value when x and y are increased by $\lambda\sin\theta$ and $\lambda\cos\theta$, respectively. That is,

$$(\gamma\lambda\sin\theta+\delta\lambda\cos\theta)\,2\pi/L = 2\pi. \tag{5}$$

Substituting for $\sin\theta$ and $\cos\theta$ in terms of γ and δ and solving for λ gives

$$\lambda = L/\sqrt{(\gamma^2+\delta^2)}. \tag{6}$$

The pressure at point (x,y) at time $t+\Delta t$ will be equal to the pressure at point $(x-\Delta x, y-\Delta y)$ at time t multiplied by an attenuation factor, where

$$\Delta x = c_1\Delta t\sin\theta = c_1\Delta t\ \gamma/\sqrt{(\gamma^2+\delta^2)}$$

$$\Delta y = c_1\Delta t\cos\theta = c_1\Delta t\ \delta/\sqrt{(\gamma^2+\delta^2)} \tag{7}$$

The attenuation factor is given by $e^{-\nabla c\Delta t/\lambda}$ where ∇ is the logarithmic decrement and λ is the wavelength. Hence

$$Z(x, y, t+\Delta t) = Z(x-\Delta x, y-\Delta y, t)\, e^{-\nabla c\Delta t/\lambda} \tag{8}$$

Substituting for Δx, Δy and λ as defined by equation (6) and (7) in equation (4), we obtain

$$Z(x, y, t+\Delta t) = \sum_{\gamma=-N/2}^{N/2}{}' \sum_{\delta=0}^{N/2}{}' C_{\gamma,\delta}\, e^{-ic(1-i\nabla/2\pi)\Delta t\sqrt{(\gamma^2+\delta^2)}\,2\pi/L}\, e^{i(\gamma x+\delta y)2\pi/L} \tag{9}$$

Since Δy is positive in this summation, each plane-wave component moves so that Δy is positive, because by equation (7)

$$\Delta y = c_1\Delta t\,\delta/\sqrt{(\gamma^2-\delta^2)} \tag{10}$$

Thus all the plane waves move downwards. Letting Δt be very small and expanding the exponential,

$$Z(x, y, t+\Delta t) = Z(x, y, t) - c_1(1-i\nabla/2\pi)\Delta t \sum_{\gamma=-N/2}^{N/2}{}' \sum_{\delta=0}^{N/2}{}' \cdot C_{\gamma,\delta}\left(i\sqrt{(\gamma^2+\delta^2)}\right)2\pi/L\cdot e^{i(\gamma x+\delta y)2\pi/L} \tag{11}$$

This can be written,

$$Z(x, y, t+\triangle t) = Z(x, y, t) + c\triangle t\, D(x, y) \quad (12)$$

where c is the complex velocity

$$c = c_1 (1 - i\nabla/2\pi)$$

and

$$D(x, y) = \sum_{\gamma=-N/2}^{N/2}{}' \sum_{\sigma=0}^{N/2}{}' C_{\gamma,\sigma} (i\sqrt{(\gamma^2+\delta^2)})2\pi/L \cdot e^{i(\gamma x+\sigma y)2\pi/L} \quad (13)$$

If $\nabla=0$ so that there is no attenuation and c is real number, then the real part of equation (12) is identical to the algorithm used by chung et al (1982) for depth migration.[6] It gives the snapshots Z(x,y)at time $t+\triangle t$ in terms of the snapshot at time t and an increment which is the product of c(x,y) and D(x,y). The function D(x,y) can be computed by Fourier transforms. The 2-D Fourier transform of P(x,y) gives the coefficients $C_{\gamma,\sigma}$; these are multiplied by filter coefficients; and the 2-D inverse Fourier transform gives D(x,y).

If $\nabla \neq 0$, the complex equation (12) can be written in the form of a real equation.

Note that the real and imaginary parts of D (x,y), from the definition in equation (13), form a Hilbert transform pair. Letting

$$D(x, y) = D_1 + iD_2$$

equation (12) can be written

$$P(x, y, t+\triangle t) = P(x, y, t) + (c_1\triangle t)D_1 + (c_1\nabla/2\pi\triangle t)D_2 \quad (14)$$

Equation (14) shows that the attenuation of the wave is caused by adding to the increment $(c_1\triangle t)D_1$ (which is the increment in the absence of attenuation) an additional increment $(c_1\nabla/2\pi\triangle t)D_2$ which is proportional to the Hilbert transform of D_1.

In the examples shown below, the algorithm used implemented equation (14).

Ⅲ. ANALYSIS OF NUMERICAL METHOD

The numerical algorithm for the propagation of a plane wave travelling forward only can be written

$$\frac{p^{\triangle t} - p^{-\triangle t}}{2\triangle t} = c\frac{dp}{dx} \quad (15)$$

where $c = c_1 + c_2 i$
$p^{\triangle t}$: pressure at time $t+\triangle t$
$p^{-\triangle t}$: pressure at time $t-\triangle t$
$\triangle t$: time interval (sampling rate)

An exact solution of equation (15) can be found in the form a single frequency wave travelling with velocity v and with an attenuation coefficient α. The problem is to find how v and α are related to c_1 and c_2. Let the exact solution be written

$$P = e^{-\alpha t}(\cos k(x-vt) + i\sin k(x-vt)) \quad (16)$$

When we substitute eqution (16) into equation (15) and solve for c_1 and c_2 as a function of v and α, we get

$$c_1 = \frac{\sin kv\triangle t}{k\triangle t}\, \mathrm{ch}\,\alpha\triangle t$$

and

$$c_2 = \frac{\cos kv\triangle t}{k\triangle t}\, \mathrm{sh}\,\alpha\triangle t \quad (17)$$

Where c_1 : phase velocity
c_2 : imaginary part of velocity
α : attenuation constant in analytic solution

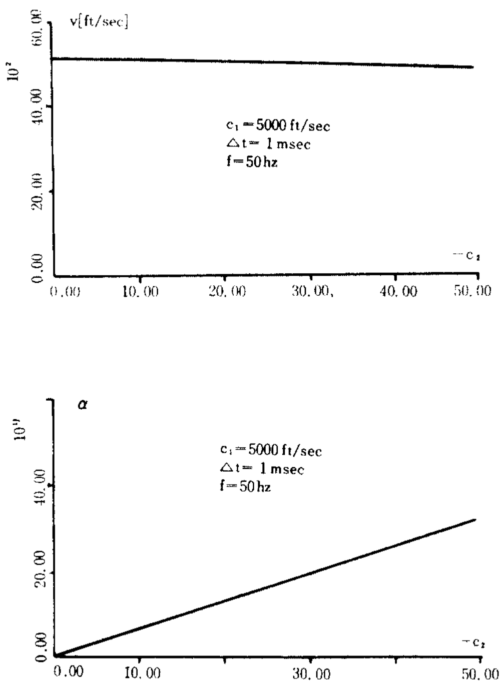


FIGURE 2 Plots for V vs. c_2 and α vs. c_2 Velocity v is almost constant and α linearly increases along the c_2 for c_1=5000 ft/sec, $\triangle t$= 1 msec, and f= 50Hz.

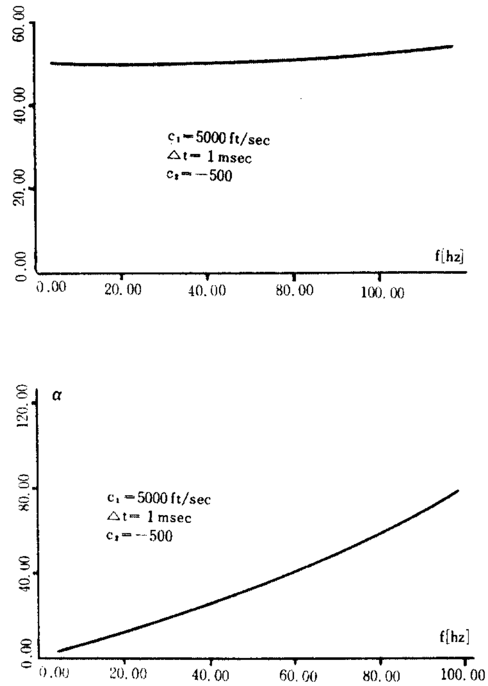


FIGURE 3 Plots for v vs. f and α vs. f Velocity v is almost constant and α linearly increases along the frequency for c_1=5000 ft/sec, $\triangle t$= 1 msec, and c_2= −500.

k: wavenumber ($= 2\pi f/c$)
f: frequency
ch: hyperbolic ocsine function
sh: hyperbolic sine function

From equation (17), through several arithmetical operations, v and α can be expressed as a function of c_1, c_2, $\triangle t$, and f. Figures 2 and 3 show examples of the relationships between c_2, f, α, and v. Figure 2 is the plot for v and α, when c_2 was varied from 0 to -5000 with c =5000 ft/sec, f= 50Hz, and $\triangle t$ = 1 msec. As can be seen in the Figure 2, 3, v is almost constant and α increases linearly along the c_2 axis. Figure 3 is the plot for v and α when f is varied from 5 to 100hz with c_1 =5000 ft/sec, c_2 =-500, and t=1 msec. α also increased linearly with f and v is almost constant.

As shown in the Figures 2 and 3, we can choose any value for c_2 and obtain corresponding a value for v and α. If we choose large value for c_2 2, the wave will be highly attenuated with the attenuation coefficient α . Dispersion is caused by v not being exactly equal to c

Ⅵ. EXPERIMENTAL RESULTS

Figure 4 shows the structure for the test. model. The velocity is 500 ft/sec at all points, and there are two different attenuation constants. In both side strips the attenuation constant is 0.1256 and in the middle strip the attenuation constant is 0.0. A horizontal plane-wave source is introduced 600 ft below the surface in the middle strip.

For comparison, the same problem was

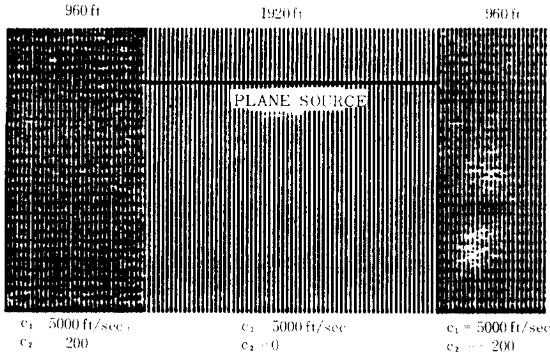


FIGURE 4 Structure diagram. A uniform velocity of 5000 ft/sec, attenuation constant for both side is 0.1256 and there is no attenuation in the middle part. A horizontal plane wave source is introduced 600 ft below the surface in the middle strip where there is no attenuation. Structure size is 128×64 and grid size is 30ft for both the x and y directions.

solved with zero attenuation at all points. The grid spacings were 30 ft in both the x and y direction and the number of grid points was 128 x 64. The disturbance introduced by the source term generated a zero phase wavelet with a frequency range 0 to 30 Hz. The time step for the computation was 1 msec.

Snapshots of the pressure distribution at times 500 and 1500 msec are shown in Figures 5 and 6. Figures 5a and 6a show the the waves when the attenuation is zero. The diffractions from the edge of the initial plane wave wrap-around because of the Fourier method of computation, as can be seen in Figure 6a. By adding attenuation within the side strips the wrap-around effect is reduced as can be seen by comparing Figures 6a and 6b.

V. DESCUSSION AND CONCLUSION

As shown by the experiments described in this paper, the inclusion of attenuation by using a complex velocity works quite well with the standard one-way numerical modeling algorithm. This algorithm would be applied to the forward

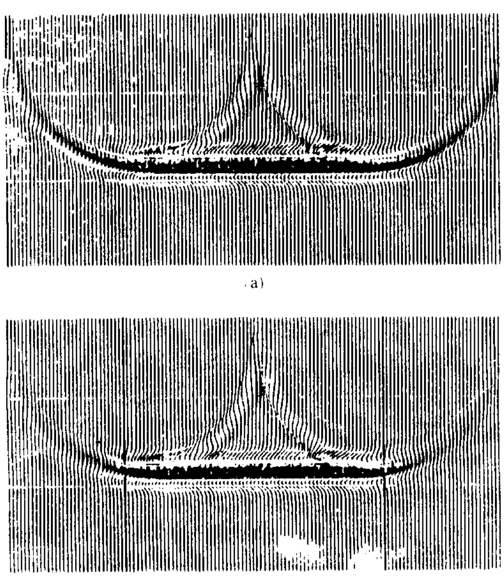


FIGURE 5 Snapshots at 500 msec. (a) Without attenuation. (b) With attenuation constant of 0.1256.

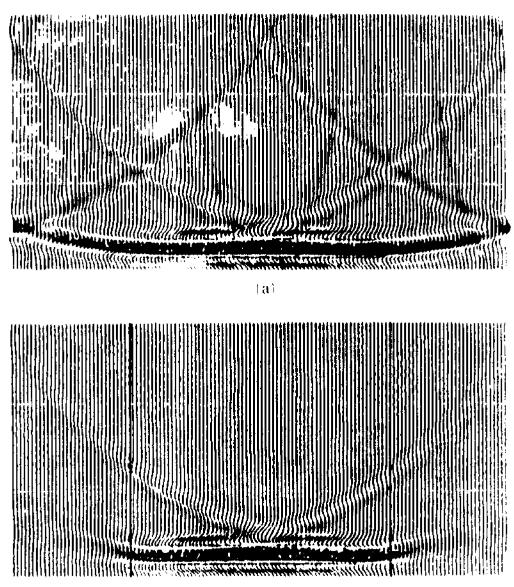


FIGURE 6 Snapshots at 1500msec. (a) Without attenuation. (b) With attenuation constant of 0.1256. This figure shows that wrap-asound noise which comes from the Fourier transform property is reduced by the attenuating medium.

and inverse modeling for more realistic models. Some advantages may be

(1) to reduce allocated storage space,

(2) to reduce computing time,

(3) to reduce the noise generated at boundaries, and

(4) to reduce the high frequency noise which is generated in the inverse modeling

Although more researches for reuction of attenuation boundaries are prospected, it is expected that these results are useful for computing wavefields by Fourier method.

REFERENCES

(1) J.E. White, "Seismic Waves: Radiation, transmission, and attenuation," New York, McGraw-Hill Book Co., 1965.

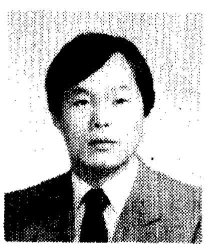
(2) J. Gazdag, "Wave Equation Migration with the Accuarate Space Derivative Method," Geophysical Prospecting, Vol.28, pp.60-70, 1980.

(3) J. Gazdang, "Modeling of the Acoustic Wave Equation with a Transform Method," Geophysics, Vol.46, pp.854-859, 1981.

(4) G.w. Purnell and G.H.F. Gardner, "Attenuation Measurement with Physical Model," SAL Sixth Year Annual Progress Review, 1983.

(5) D. Loewenthal, L. Lu, R. Roberson and J. Sherwood, "Wave Equation Applied to Migration," Geophysical Prospecting, Vol.24, pp.380-399, 1976.

(6) S.J. Chung and S.S. Lee, "Interpretation of Geological Overthrust structure by Fourier Transform and Ray Tracing," SAL Fifth Year Annual Progress Review, Vol.10, pp.473-497, 1982.



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