論 文

변형된 상승여현窓의 제안과 기존窓들과의 특성 비교

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Modified Raised-cosine Window and Comparison With Standard Windows

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要 約 새로운 窓(Window) 함수로 $w(t)=0.62-0.48|t|+0.38\cos 2\pi t$, $|t|\leqq 1/2$ 가 제시되어 기존 窓들과 성능을 비교하였다. 제안된 窓은 Bartlett, Hanning및 Hamming 窓들과 유사한 郡에 속하나 그들의 특징을 상호 조합한 특성을 나타낸다.

ABSTRACT A new window is introduced. It is shown that the window, $w(t)=0.62-0.48|t|+0.38\cos 2\pi t$, $|t|\leqq 1/2$ is of the group having similar but trade-off properties with the Bartlett, the Hanning and the Hamming windows.

I. INTRODUCTION

In the numerical evaluation of Fourier integrals or in the spectral estimation, windows are used to reduce the leakage and spectral bias(1-3). Several standard windows are used to optimize the requirements of a particular application in signal processing. The Bartlett, the Hamming and the Hanning windows, etc. are of them. These windows have good overall properties but are not optimum in any specific sense.

The common properties of these windows can be summarized as follows:

i) They are real, even, nonnegative and time-limited.

ii) Their Fourier transforms have main lobe at the origin and side lobes at both sides. These side lobes are decaying with asymptotic attenuation of f^{-n} as $f\rightarrow\infty$, where n is an integer.

In the following section, a new window,

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which can be called as modified raised-cosine window, is suggested. This window is from two partly sum of Barlett and Hanning windows. The idea is to reduce the effect of side lobes of its Fourier transform, whereas a family of windows that are a combination of rectangular and Hanning windows is considered to have sharp edges of the rectangular window be replaced by Hanning window[4]. The proposed window is compared with several standard windows in term of parameters which is generally used in the evaluatiun of the window.

II. DERIVATION OF THE WINDOW

We designate w(t) a window function and its Fourier transform be W(f). If w(t) is real, even, unity at the origin and time limited:

$$w(0) = \int_{-\infty}^{\infty} w(f)\ df = 1 \tag{1}$$

$$w(t) = 0 \quad \text{for } |t| > 1/2 \tag{2}$$

then the transform pair w(t) and w(f) form the window pair.

With the constraints of window, a new window can be derived from two partly sum of the Bartlett and the Hanning windows to reduce the side lobes of its Fourier transform. The Bartlett window is given by

$$w(t) = \begin{cases} 1 + 2|t|, & |t| \leq 1/2 \\ 0 & \text{elsewhere} \end{cases} \tag{3}$$

$$w(f) = \frac{1}{2}\,\frac{\sin^2(\pi f/2)}{(\pi f/2)^2} \tag{4}$$

and the Hanning window is given by

$$w(t) = \begin{cases} 0.5 + 0.5\cos(2\pi t), & |t| \leq 1/2 \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

$$W(f) = \frac{\sin(\pi f)}{2\pi f(1 - f^2)} \tag{6}$$

FRom their frequency spectra, they have partly opposite signs in the side lobes. This situation immediately suggests that scaled Bartlett and Hanning windows can be used to reduce the effect of side lobes, especially near side lobes. The outcome can be called as the modified Bartlett-and-Hanning window.

The proposed window function can be written by

$$w(t) = \begin{cases} \xi(1 - 2|t|) + (1 - \xi)(0.5 + 0.5\cos 2\pi t), & |t| \leq 1/2 \\ 0 & \text{elsewhere} \end{cases} \tag{7}$$

where, $0 < \xi < 1$

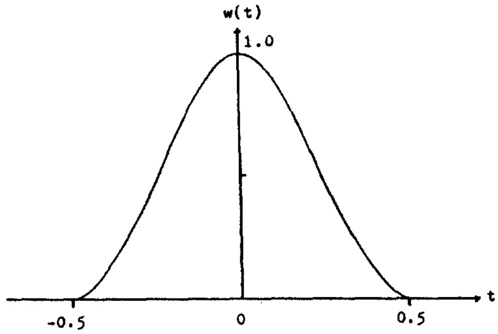
By Fourier transform, the proposed function gives

$$W(f) = \xi\,\frac{\sin^2(\pi f/2)}{2(\pi f/2)^2} + (1 - \xi)\,\frac{\sin \pi f}{2\pi f(1 - f^2)} \tag{8}$$

Wince one of the aims of this proposed window is to reduce the effect of side lobes, this requires

$$\int_{2}^{\infty} W(f)\ df = 0 \tag{9}$$

Solving Eq.(9) gives ζ approximately equal to 0.24. Therefore, the complete form of window function is given by



$$w(t) = 0.24(1-2|t|) + 0.76(0.5 + 0.5\cos 2\pi t)$$
$$= 0.62 - 0.48|t| + 0.38\cos 2\pi t, \quad |t| \leq 1/2 \tag{10}$$

and

$$W(f) = 0.12\frac{\sin^2(\pi f/2)}{(\pi f/2)^2} + 0.38\frac{\sin(\pi f)}{\pi f(1-f^2)} \tag{11}$$

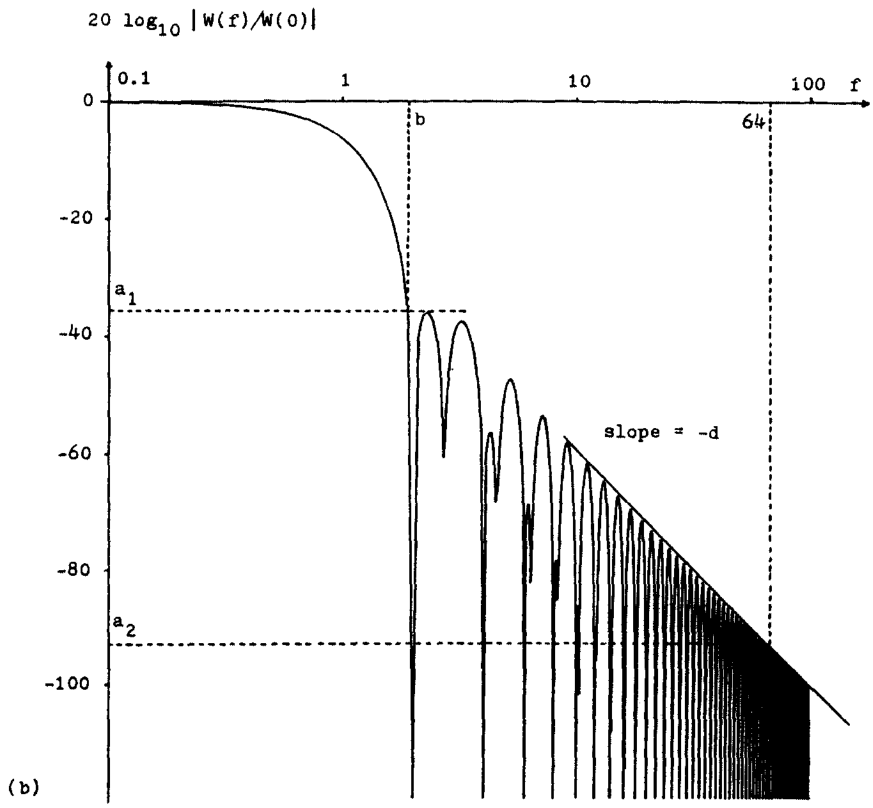


Fig 1 Plot of the modified Bartlett-and-Hanning window.
(a) Modified Bartlett-and-Hanning window.
(b) Decibel amplitude response. Parameters, b, a_1 a_2 and d, are used to compare the window with others.

Eq.(10) and (11) satisfy the conditions required to the window function and they can be the window pair. Fig.1 shows the plots of the modified Bartlett-and-Hanning and its decibel amplitude response.

III. COMPARISON OF THE WINDOWS

We shall now compare the proposed window with the existing standard windows[2] in terms of the parameters[4] that reflect the effects of resolution degradation due to the main lobe, leakage due to the near side lobes, and leakage due to far side lobes. From Fig.1-(b), the following parameters of $|W(f)/W(O)|$ are selected for comparison.

a : The frequency at which the main lobe drops to the peak ripple value of the side lobes.

a_1: The peak ripple value of the side lobes.

a_2: The ripple value of the side lobes at f=64, with f normalized according to Eq.(2), the value at which one can readily presume that asymtotic behavior has been reached.

d : Asymtotic decay rate of the sidelobe envvelope.

The smaller b is, the better the resolution of the estimates; the smaller a_1 is, the smaller the leakage though near sidelobes; the smaller a_2 and the larger d are, the smaller the leakage through far sidelobes.

To evaluate the proposed window further, the following parameters are additionally of interest.

Energy:

$$I = \int_{-\infty}^{\infty} W^2(f)\,df = \int_{-\frac{1}{2}}^{\frac{1}{2}} w^2(t)\,dt = 0.365 \qquad (12)$$

Energy moment:

$$M = \int_{-\infty}^{\infty} f^2 W^2(f)\,df = \int_{-\frac{1}{2}}^{\frac{1}{2}} [w'(t)]^2 dt = 3.975 \qquad (13)$$

Table I Comparison of various windows.

windows	b	a_1	a_2	d	I	M	m	asymptotic attenuation
Modified Bartlettand-Hanning	1.89	-36	-98	12	0.365	4.71	15.0	$\frac{0.24}{\pi^2 f^2}$
Bartlett	1.63	-26	-80	12	0.333	4.0	∞	$\frac{2}{\pi^2 f^2}$
Hanning	1.87	-32	-118	18	0.375	4.93	19.74	$\frac{0.5}{\pi f^3}$
Hamming	1.91	-43	-63	6	0.398	∞		$\frac{0.16}{2\pi f}$
Papoulis	2.70	-46	-146	24	0.268	5.58	39.48	$\frac{2}{\pi^2 f^4}$
Parzen	3.25	-53	-136	24	0.269	6	48.0	$\frac{96}{\pi^4 f^4}$

Amplitude moment:

$$m = \int_{-\infty}^{\infty} f^2 W(f)\, df = -w''(0) = 15.0 \qquad (14)$$

Asymtotic attenuation for large f:

$$W(f) = 0.12\frac{\sin^2(\pi f/2)}{(\pi f/2)^2} + 0.38\,\frac{\sin(\pi f)}{\pi f(1-f^2)}$$

$$\underset{f \to \infty}{=} \frac{0.24}{\pi^2 f^2} \qquad (15)$$

Thus W(f) goes to zero as f^{-2} for $f \to \infty$.

Table 1 shows the comparison of various windows via these parameters.

IV. CONCLUSION

The Bartlett, the Hanning, the Hamming and the modified raised - cosine windows are of the group having the similar properties. The proposed window has more effective than both the Bartlett and the Hanning in near side lobes, and more effective than both the Bartlett and the Hamming in far side lobes. No window is generally the best in all aspects, and one should select one according to the requirements of a particular application.

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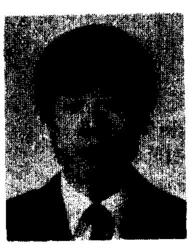

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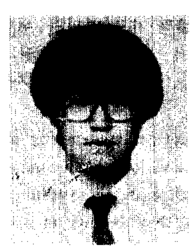

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