

# Algebraic Geometric Codes and Subfields of Hermitian Function Field

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## 대수기하부호와 Hermitian 함수체의 부분체

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### ABSTRACT

Like the Hermitian function field over  $GF(q^2)$ , those subfields defined by  $y^s + y = x^n$  where  $s$  divides  $q + 1$  are also maximal, having the maximum number of places of degree one permissible by the Hasse-Weil bound. Geometric Goppa codes(or algebraic geometric codes) arising from these subfields of the Hermitian function field are studied in this paper. Their dimension and minimum distance are explicitly and completely presented for any  $m$  with  $m < n$  where  $n$  is the length and  $m$  is a parameter that governs both dimension and minimum distance of the code.

### 요 약

$GF(q^2)$  위의 Hermitian 함수체와 마찬가지로,  $q + 1$ 을 나누는 정수  $s$ 에 대해  $y^s + y = x^n$ 로 정의되는 부분체들도 Hasse-Weil 한계식에 의해 허용되는 최대수의 1차 점들을 가지므로 최상이 된다. 본 논문에서는 Hermitian 함수체의 이러한 부분체들로부터 생성되는 기하 Goppa 부호(혹은 대수기하부호)를 연구한다.  $n$ 을 부호장,  $m$ 을 이들 부호의 차원과 최소거리를 결정하는 패러미터라 할때,  $n$ 보다 작은 임의의  $m$ 에 대해 차원과 최소거리가 명확하면서도 완전하게 주어진다.

### I. Introduction

In a series of papers ([5]-[8]), Goppa discovered an amazing connection between the theory of algebraic curves over a finite field  $GF(q)$  and the theory of error-correcting block codes over  $GF(q)$ .

His idea generalizes the well-known construction of Reed-Solomon and classical Goppa codes. Following his idea, Tsfasman, Vladut, and Zink [14] used modular and Shimura curves when  $q$  is a perfect square, and showed in a landmark result that there is a sequence of codes which exceed the Gilbert Varshamov bound for  $q \geq 49$ . Thereafter, these codes are called geometric Goppa codes or algebraic geometric codes (AGC).

A particularly interesting class of geometric

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Goppa codes are codes arising from the Hermitian function field. The large length of these codes in comparison with their alphabet size makes them attractive over conventional Reed-Solomon codes having the same alphabet. Tiersma [13] studied these codes in more detail and provided a clear description of their dual codes.

Stichtenoth [11] generalized and simplified the results of Tiersma by working with an isomorphic curve having only one point at infinity. Codes of length  $n = q^3$  and any dimension  $k$  with  $0 \leq k \leq q^3$  over  $GF(q^2)$  were considered by him. In particular, the exact minimum distance of these codes in the range that  $0 \leq m \leq q^3 - q^2$  or  $m \equiv 0 \pmod{q}$  with  $m < q^3$  (where  $m$  is a parameter that governs both dimension and minimum distance of the code) was determined in his paper. The true minimum distance for ranges of the parameter  $m$  not covered in [11] was completely determined by Yang and Kumar [15]. The generalized Hamming weights of these codes were also studied by Yang, Kumar, and Stichtenoth [16].

Like the Hermitian function field, those subfields defined by  $y^q + y = x^s$  where  $s$  divides  $q + 1$  are also maximal, having the maximum number of places of degree one permissible by the Hasse-Weil bound. Some partial results on minimum distance of codes arising from these subfields were obtained by Garcia, Kim and Lax [12] using consecutive Weierstrass gaps.

In this paper we provide complete results on the dimension and minimum distance of these codes with  $m < n$ , where  $n$  is the length and  $m$  is a parameter that governs both dimension and minimum distance of the code

## II. Geometric Goppa Codes

Originally, Goppa constructed algebraic geometric codes using differentials of a function field and the residue map, which are now well-known to be the duals of algebraic geometric codes using functions of a function field and the evaluation map. The presentation here will adopt the function/evaluation

viewpoint. See [9], [10], [12] for more details.

Let  $F/K$  be an algebraic function field of genus  $g$  over a finite constant field  $K$  (see [1], [10], [12] for example). Let  $\{P_i | i = 1, 2, \dots, n\}$  be a set of places of degree one in  $F/K$ . Let  $G$  and  $D$  be divisors of  $F/K$  such that  $D = P_1 + P_2 + \dots + P_n$  and  $\text{supp}(G) \cap \text{supp}(D) = \emptyset$  where  $\text{supp}(G)$  and  $\text{supp}(D)$  denote the supports of  $G$  and  $D$  respectively. Define the vector space  $\mathcal{L}(G)$  as follows :

$$\mathcal{L}(G) := \{f \in F \mid (f) \geq -G \text{ or } f = 0\},$$

where  $(f)$  is the principal divisor of  $f$ . For a divisor  $A$  of  $F/K$ , denote by  $\dim A$  and  $\deg A$  the dimension of  $\mathcal{L}(A)$  over  $K$  and the degree of  $A$  respectively. Now consider the  $K$ -linear evaluation map  $\varphi$  given by

$$\begin{aligned} \varphi : \mathcal{L}(G) &\rightarrow K^n \\ f &\rightarrow (f(P_1), f(P_2), \dots, f(P_n)). \end{aligned} \quad (1)$$

Then the geometric Goppa code (or algebraic geometric code) associated with two divisors  $D$  and  $G$  is defined by

$$C_{\mathcal{L}(D), G} := \text{Image of } \varphi = \varphi(\mathcal{L}(G)). \quad (2)$$

Here we assume that  $\deg G < n$  for simplicity. The following proposition is well-known [9], [12].

**Proposition 1**  $C_{\mathcal{L}(D), G}$  is an  $[n, k, d]$  code with parameters

$$k = \dim G \text{ and } d \geq n - \deg G.$$

Furthermore, if  $\deg G > 2g - 2$ , then  $k = \deg G + 1 - g$ .

## III. Some Subfields of Hermitian Function Fields

Let  $K$  be a finite field  $K = GF(q^2)$  ( $q = a$  power of some prime  $p$ ) and  $F = K(x, y)$  be the function field defined by

$$F = K(x, y) \text{ with } y^q + y = x^s \text{ and } s \mid q + 1 \quad (3)$$

(see [3], [4], [11], [12]). If  $s = q + 1$ , then  $F/K$

is called the Hermitian function field over  $K$ . If  $s < q+1$ , then  $F/K$  is isomorphic to a subfield of the Hermitian function field and we shall therefore refer to it as a subfield of the Hermitian function field. The genus  $g$  of the function field  $F/K$  is given by  $g = (q-1)(s-1)/2$  and the divisor of the differential  $dx$  can be shown to be

$$(dx) = (2g-2)Q_s, \tag{4}$$

where  $Q_s$  is the common pole of  $x$  and  $y$ .

The places of degree one of  $F/K$  are given as follows: The place  $Q_s$  is one of them. Let  $\alpha \in K$ . Note that  $T^q + T = \alpha^s$  has a root in  $K$  if and only if  $\alpha^s \in GF(q)$ . For any  $\alpha$  with  $\alpha^s \in GF(q)$ , there are exactly  $q$  distinct solutions in  $K$  of  $T^q + T = \alpha^s$ . Let  $U^*$  be the subgroup of order  $(q-1)s$  of the multiplicative group  $K^*$  and let  $U := U^* \cup \{0\}$ . Then for  $\alpha \in K$ ,  $\alpha^s \in GF(q)$  if and only if  $\alpha \in U$ . Hence the number  $N$  of places of degree one in  $F/K$  is

$$N = q \cdot |U| + 1 = q(1 + (q-1)s) + 1. \tag{5}$$

Since  $N = 1 + q^2 + 2gq = 1 + q^2 + q(q-1)(s-1)$ ,  $F/K$  achieves the Hasse-Weil bound and is therefore a maximal function field.

We define  $P_{\alpha, \beta}$  to be the common zero of  $x - \alpha$  and  $y - \beta$  whenever  $\alpha \in U$  and  $\beta \in K$  are such that  $\beta^q + \beta = \alpha^s$ . Then the divisors of  $x - \alpha$  and  $y - \beta$  are as follows:

$$(x - \alpha) = \begin{cases} \sum_{\substack{\beta \in K, \\ \beta^q + \beta = \alpha^s}} P_{\alpha, \beta} - qQ_s, & \text{if } \alpha \in U, \\ R_\alpha - qQ_s, & \text{if } \alpha \in K \setminus U \end{cases} \tag{6}$$

where  $R_\alpha$  is a divisor of degree  $q$  in  $F/K$  depending on  $\alpha$  whose support does not contain any place of degree one and

$$(y - \beta) = \begin{cases} sP_{0, \beta} - sQ_s, & \text{if } \beta^q + \beta = 0, \\ \sum_{\substack{\alpha \in K, \\ \beta^q + \beta = \alpha^s}} P_{\alpha, \beta} - sQ_s, & \text{if } \beta^q + \beta \neq 0. \end{cases} \tag{7}$$

For each integer  $m \geq 0$ , the set  $B(m)$  given by

$$B(m) := \{x^i y^j \mid 0 \leq i, 0 \leq j \leq q-1, iq + js \leq m\} \tag{8}$$

is a basis of  $\mathcal{L}(mQ_s)$  over  $K$ .

Let  $s \cdot t := q + 1$ . From here on, we will assume that

$$G := mQ_s \text{ and } D := \sum_{\alpha \in U} \sum_{\substack{\beta \in K, \\ \beta^q + \beta = \alpha^s}} P_{\alpha, \beta}. \tag{9}$$

Consider the geometric Goppa code  $C_{\mathcal{L}}(D, G)$  associated with two divisors  $D$  and  $G$ . Then  $C_{\mathcal{L}}(D, G)$  is a linear code of length  $n := q(1 + (q-1)s)$ . To simplify notation, let

$$C_m := C_{\mathcal{L}}(D, mQ_s). \tag{10}$$

Let  $d(C_m)$  denote the minimum distance of the code  $C_m$ . Note that  $C_m$  is a linear code of length  $n := q(1 + (q-1)s$  and that if  $m_1 \leq m_2$ , then  $C_{m_1} \subseteq C_{m_2}$  and therefore  $d(C_{m_1}) \geq d(C_{m_2})$ .

Consider the function  $u$  defined by  $u := \prod_{\alpha \in U} (x - \alpha)$ . Then we have

$$(u) = D - nQ_s, \tag{11}$$

and

$$u = x \cdot \prod_{\alpha \in U^*} (x - \alpha) = x(x^{(q-1)s} - 1) = x^{1 + (q-1)s} - x.$$

An integer  $l \geq 0$  is called a gap number of  $Q_s$  if there is no function  $f \in F$  such that  $f \in \mathcal{L}(lQ_s) \setminus \mathcal{L}((l-1)Q_s)$ . Otherwise,  $l$  is called a pole number of  $Q_s$ . Let  $S$  be the set of all gap numbers of  $Q_s$ . From the above basis of  $\mathcal{L}(mQ_s)$  given in (8), it is easy to check that

$$S = S_1 \cup S_2 \tag{12}$$

where

$$S_1 = \{aq + cs + b \mid 0 \leq a \leq s-2, 0 \leq c \leq t-2, a+1 \leq b \leq s-1\}$$

and

$$S_2 = \{aq + (t-1)s + b \mid 0 \leq a \leq s-3, a+1 \leq b \leq s-2\}.$$

This gap sequence plays a central role in determining the dimension and minimum distance of the ocd  $C_m$  as in the Hermitian case [11], [15].

**Remark 2** (a) If  $t=1$  (i.e.,  $s=q+1$ ) then  $F/K$  is the Hermitian function field and  $S = \{aq + b \mid 0 \leq a \leq q-2, a+1 \leq b \leq q-1\}$ . In particular, the length  $n$  of the code  $C_m$  is  $n=q^3$  in this case.

(b) If  $t=q+1$  (i.e.,  $s=1$ ) then  $F/K$  is a rational function field since  $F=K(x, y)=K(y)$ . In this case,  $Q_x$  does not have a gap number and  $n=q^2$ .

The dimension of the code  $C_m$  is easily determined from the above gap sequence given in (12). Let

$$I(m) := \{l \leq m \mid l = iq + js, i \geq 0, 0 \leq j \leq q-1\}. \quad (13)$$

Any integer  $m$  can be uniquely expressed as follows :

$$m = aq + cs + b \text{ with } a \geq 0, 0 \leq c \leq t-2, \text{ and } 0 \leq b \leq s-1 \quad (14)$$

or

$$m = aq + (t-1)s + b \text{ with } a \geq 0, \text{ and } 0 \leq b \leq s-2. \quad (15)$$

Using this expression and the basis of  $\mathcal{L}(mQ_x)$  in (8), it is easy to calculate  $|I(m)|$ , that is,

$$|I(m)| = \begin{cases} \frac{a(a+1)}{2}t + c(a+1) + \min\{a, b\} + 1 & \text{for } 0 \leq m \leq 2g-2, \\ m+1-g & \text{for } m > 2g-2. \end{cases} \quad (16)$$

**Proposition 3** Assume that  $0 \leq m < n = q((q-1)s + 1)$ . Then the dimension of  $C_m$  is given by

$$\dim C_m = |I(m)|.$$

*Proof.* It is obvious because

$$\dim C_m = \dim \mathcal{L}(mQ_x) = |I(m)|$$

for  $0 \leq m < n$ . □

#### IV. Minimum Distance of the Code $C_m$

In this section, we will completely determine the true minimum distance of the code  $C_m$  defined in (10) for any  $m < n$ . In the case of Hermitian code, Stichtenoth [11] determined the exact minimum distance of  $C_m$  for  $0 \leq m \leq q^3 - q^2$ . Yang and Kumar [15] determined it for any  $m$  with  $q^3 - q^2 \leq m \leq q^3 + q^2 - q - 2$ . If  $s=1$ , then  $F/K$  is a rational function field over  $K$ , and  $C_m$  is therefore MDS (maximum distance separable).

From here on, we focus on the case that  $1 < s < q + 1$  (equivalently,  $1 < t < q + 1$  since  $st = q + 1$ ). Since  $n = q(1 + (q-1)s)$ , we have  $q^2 < n < q^3$  in our case. By Goppa's lower bound, we have

$$d(C_m) \geq n - m.$$

For any integer  $m \geq 0$ , let  $\tilde{m}$  be the largest pole number of  $Q_x$  with  $\tilde{m} \leq m$ . Clearly,  $\mathcal{L}(mQ_x) = \mathcal{L}(\tilde{m}Q_x)$ . Therefore, we can assume in what follows that  $m$  is a pole number of  $Q_x$  without loss of generality.

**Theorem 4** Assume that  $0 \leq m < n = q(1 + (q-1)s)$  and that  $m$  is a pole number of  $Q_x$ . If  $n - m$  is a pole number of  $Q_x$  then

$$d(C_m) = n - m.$$

*Proof.* We divide our region into three cases.

Case (a).  $m = n - (s-1)q = q((q-2)s + 2)$ : Choose  $i := (q-2)s + 2$  distinct elements  $\alpha_1, \dots, \alpha_i \in U$ . Then the function

$$f := \prod_{v=1}^i (x - \alpha_v) \in \mathcal{L}(mQ_x)$$

has exactly  $iq = m$  distinct zeros in  $\text{supp}(D)$ . This implies that  $d(C_m) \leq n - m$ , so we are done.

Case (b).  $m < n - (s-1)q = q((q-2)s+2)$ : Since  $m$  is a pole number of  $Q_x$ , we write  $m = iq + js$  with  $i \geq 0$  and  $0 \leq j \leq q-1$ . Thus  $i \leq (q-2)s+1 = (q-1)s+1-s$ . Let  $A := \{x \in U \mid x^s \neq 1\}$ . Then  $|A| = (q-1)s+1-s \geq i$ , and we can choose  $x_1, \dots, x_i \in A$ . The element

$$z_1 := \prod_{v=1}^i (x - x_v)$$

has  $iq$  distinct zeros in  $\text{supp}(D)$ . Next we choose  $j$  distinct elements  $\beta_1, \dots, \beta_j \in K$  such that  $\beta^q + \beta = 1$  and let

$$z_2 := \prod_{\mu=1}^j (y - \beta_\mu).$$

Note that  $z_2$  has  $js$  distinct zeros in  $\text{supp}(D)$ . Then  $z := z_1 z_2 \in \mathcal{L}(mQ_x)$  has exactly  $m$  distinct zeros in  $\text{supp}(D)$ .

Case (c).  $n - (s-1)q < m < n$ : We have  $0 < n - m < (s-1)q < q((q-2)s+2)$ . Since  $n - m$  is a pole number of  $Q_x$  by assumption, there exists an element  $z \in \mathcal{L}((n-m)Q_x)$  with the divisor  $(z) = E - (n-m)Q_x$  where  $0 \leq E \leq D$  and  $\deg E = n - m$ . The element  $u := x^{(q-1)s+1} - x \in F$  has the divisor  $(u) = D - nQ_x$ , hence

$$(u/z) = (D - E) - mQ_x.$$

Hence, the codeword corresponding to  $u/z \in \mathcal{L}(mQ_x)$  has weight  $n - m$ .  $\square$

Using the gap sequence in (12), it is easy to check that  $n - m$  is not a pole number of  $Q_x$  if and only if

$$m = n - aq - cs + b \text{ with } 0 \leq a \leq s-2, 1 \leq c \leq t-1, \text{ and } 1 \leq b \leq s-1-a$$

or

$$m = n - aq - ts + b \text{ with } 0 \leq a \leq s-3, \text{ and } 2 \leq b \leq s-1-a$$

where  $ts = q + 1$ .

In the case that  $n - m$  is not a pole number of  $Q_x$ , the minimum distance of  $C_m$  is given in the following two theorems.

**Theorem 5** *If  $m = n - aq - cs + b$  with  $0 \leq a \leq s-2$ ,  $1 \leq c \leq t-1$  and  $1 \leq b \leq s-1-a$ , then*

$$d(C_m) = n - m + b = aq + cs.$$

*Proof.* Note that  $m > n - aq - cs$  and that both  $n - aq - cs$  and  $aq + cs$  are pole numbers of  $Q_x$ . By Theorem 4 we have

$$d(C_m) \leq d(C_{n-aq-cs}) = aq + cs.$$

In order to prove the equality, it suffices to show that any element  $f \in \mathcal{L}(mQ_x)$  has at most  $m - b = n - aq - cs$  distinct zeros in  $\text{supp}(D)$ . Suppose there is an element  $f \in \mathcal{L}(mQ_x)$  such that  $f$  has  $m'$  distinct zeros in  $\text{supp}(D)$  where  $n - aq - cs + 1 \leq m' \leq n - aq - cs + b = m$ . Then there exists an integer  $m''$  with  $m' \leq m''$  such that

$$(f) = E + J - m''Q_x,$$

with  $0 \leq E \leq D$ ,  $\deg(E) = m'$ ,  $J \geq 0$ ,  $\text{supp}(J) \cap \text{supp}(D) \subseteq \text{supp}(E)$ ,  $Q_x \notin \text{supp}(J)$ , and  $j := \deg(J) = \deg(f) = m'' - m'$ . Let  $u := x^{(q-1)s+1} - x$ . Then

$$\begin{aligned} (u/f) &= D - nQ_x - E - J + m''Q_x \\ &= (D - E) - J - (n - m'')Q_x. \end{aligned}$$

Now let  $\bar{K}$  be the algebraic closure of  $K = GF(q^2)$  and consider the constant field extension  $F' = F\bar{K}$  of  $F/K$ . Then  $F'/\bar{K}$  has the same genus and the same gap sequence at  $Q_x$ . The degree of any divisor in  $F'/K$  is preserved in  $F'/\bar{K}$  (see [12]). Our aim is to show that even in the extension field  $F'$ , such a function  $u/f$  with the divisor as given above does not exist. In  $F'/\bar{K}$ , all places (except  $Q_x$ ) are of degree one and correspond to points  $(x, \beta)$  with coordinates in  $\bar{K}$ . Given a place of degree one (a point)  $P_{x,\beta}$  contained in the support of  $J$ , we consider the the function  $y - \beta$

having  $P_{x,\beta}$  as a zero. Since  $(y-\beta) = P_{x,\beta} + I - sQ_x$ , where  $\beta^q + \beta = x^s$ ,  $I \geq 0$  and  $Q \notin \text{supp}(I)$ , we get  $P_{x,\beta} \sim -I + sQ_x$ . Replacing each place of degree one in the support of  $f$  with an equivalent divisor in this way, we get

$$f \sim -R + jsQ_x,$$

where  $R \geq 0$  and  $Q \notin \text{supp}(R)$ . Thus

$$(u/f) \sim D - E + R - (n - m'' + js)Q_x,$$

where  $D - E + R \geq 0$  and  $Q \notin \text{supp}(D - E) \cup \text{supp}(R)$ . Let  $m'' := n - aq - cs + b''$  and  $m' := n - aq - cs + b'$  where  $1 \leq b' \leq b'' \leq b$ . Then

$$\begin{aligned} n - m'' + js &= aq + cs - b'' + js \\ &= aq + (c + j - 1)s + s - b''. \end{aligned}$$

write  $c + j - 1 := kt + l$  with  $0 \leq l \leq t - 1$  and  $k \geq 0$ . Note that  $j \geq k$  since  $t > 1$  and  $1 \leq c \leq t - 1$ . Thus

$$\begin{aligned} n - m'' + js &= aq + kts + ls + s - b'' \\ &= (k + a) + ls + (s - b'' + k). \end{aligned}$$

Here we have

$$s - b'' + k \geq s - (s - 1 - a) + k = k + a + 1 > k + a$$

and

$$s - b'' + k = s - b' - (j - k) \leq s - b' \leq s - 1.$$

In particular, if  $l = t - 1$ , then  $j > k$  and

$$k + a < s - b'' + k \leq s - b' - 1 \leq s - 2.$$

This implies that  $n - m'' + js$  is a gap number of  $Q_x$ , and we get a contradiction to our equivalence of  $(u/f)$ . Hence, there is no function  $f \in \mathcal{L}(mQ_x)$  such that  $f$  has exactly  $m'$  distinct zeros in  $\text{supp}(D)$  with  $n - aq - cs + 1 \leq m' \leq m$ .  $\square$

**Theorem 6** *If  $m = n - ts - aq + b$  with  $0 \leq a \leq s - 3$ , and  $2 \leq b \leq s - 1 - a$ , then*

$$d(C_m) = n - m + b - 1 = ts + aq - 1 = (a + 1)q.$$

*Proof.* The same arguments in the Proof of Theorem 5 holds except that

$$n - aq - ts + 2 \leq m' \leq m'' \leq m = n - aq - ts + b$$

where  $2 \leq b \leq s - 1 - a$  and  $0 \leq a \leq s - 3$ . Then

$$\begin{aligned} n - m'' + js &= aq + ts - b'' + js \\ &= aq + (j + t - 1)s + (s - b''). \end{aligned}$$

If  $j = 0$  then  $s - 2 \geq s - b'' \geq s - (s - 1 - a) = a + 1 > a$ , and therefore  $n - m'' + js$  is a gap number of  $Q_x$ .

If  $j \geq 1$  then write  $j + t - 1 := kt + l$  with  $0 \leq l \leq t - 1$  and  $k \geq 0$ . Then we have

$$\begin{aligned} n - m'' + js &= aq + kts + ls + s - b'' = (a + k)q + ls \\ &+ (s - b'' + k). \end{aligned}$$

Here

$$s - b'' + k \geq s - (s - a - 1) + k = a + k + 1 > a + k$$

and

$$s - b'' + k = s - b' - (j - k) \leq s - b' \leq s - 2$$

since  $j \geq k$ . This implies that  $n - m'' + js$  is a gap number of  $Q_x$ . Hence, we have a contradiction.  $\square$

## V. Conclusion

An interesting family of geometric Goppa codes are studied here, which arise from some subfields of the Hermitian function field over  $GF(q^2)$  defined by  $y^q + y = x^s$  where  $s$  divides  $q + 1$ . These codes have large length  $n = q((q - 1)s + 1)$  compared with their alphabet size  $q^2$ , so they may be more attractive than the conventional Reed-Solomon codes. Their dimension and minimum distance are explicitly and completely given for any  $m < n$  where  $m$  is a parameter that governs

both dimension and minimum distance of the code

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