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통계적 모멘트에 의한 PSK 신호의 변조분류에 관한 연구

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A Study on Modulation Classification of PSK Signals Based on Statistical Moments

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要 約

통계적 모먼트(statistical moments)에 의한 변조형태 분류기(classifier)는 PSK 신호를 분류하는데 자주 이용되어 왔다. 이전에 사용된 분류기는 수신된 신호로부터 추출하기 어려운 신호위상 샘플의 통계적 모먼트를 이용하였으나, 본 논문에서는 확률변수변환을 통한 복조된 신호의 모먼트를 이용하여 PSK 신호를 분류하기 위한 새로운 분류기를 제안한다. 복조된 신호는 종래의 방법으로 쉽게 추출이 될 수 있다. PSK 신호에 대해 제안된 분류기의 성능평가는 복조된 신호의 정확한 위상분포를 사용하여 가산성 백색가우스잡음(AWGN)하에서 오분류확률(probability of misclassification)로 분석하였다. 분석결과 동기 시스템이 비동기 시스템보다 n이 4이고 오분류확률이 10^{-5}일때 BPSK에 있어서는 4dB, QPSK에 있어서는 3dB 더 우수함을 알 수 있었다.

ABSTRACT

Modulation type classifier based on statistical moments has been successfully employed to classify PSK signals. Previously, the classifier developed utilizes the statistical moment of samples of the received signal phase, which may be difficult to extract from received signal. In this paper we propose a new moments-based classifier to classify PSK signals by using the moments of the demodulated signal for PSK. The demodulated signal can be easily extracted from the conventional demodulation of PSK. The evaluation of the performance of the proposed classifier for PSK signals has been investigated in additive white Gaussian noise environment using the exact distribution of the demodulated signal. The performances of classifier in terms of probability of misclassification

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were evaluated. We found that the coherent system classifier gave 4dB improvement for BPSK and 3dB for QPSK over noncoherent system classifier, when the probability of misclassification is 10^{-5} and n equals to 4.

I. Introduction

Modulation type classifier plays an important role in some communications systems such as signal conformation, interference identification and monitoring. To obtain a rapid report of the modulation type of a received signal in modern dense communication environment, automatic modulation classification technique becomes necessary and therefore has recently received much attention[1]-[3].

According to an increasing demand of a digital communication, many kinds of modulation types have been proposed and made to practical use recently. However, these studies are on the assumption that the input signal type such as modulation type, carrier frequency and symbol rate are known at the receiver. Therefore, the demodulation can not be done when the input signal type is not known to the receiver as prior information.

Considering an increasing demand of the communication and the diversification of the communication methods, it can be easily expected that the communication system whose signal type is not known to the receiver as prior information will exist in the near future. We assume that the transmitted signals consist of continuous wave, BPSK signal and QPSK signal.

Preliminary we review the modulation classification method which uses the statistical moments of samples of the received signal phase described in [6].

Let the received signal r(t) consis of transmitted signal and additive white Gaussian noise in the form

$$r(t) = s(t) + n(t) \tag{1}$$

s(t) is M-ary phase signal and n(t) is Gaussian noise with mean zero and variance σ^2.

The phase bearing information of s(t) can be considerd as

$$\theta_M = \frac{2\pi}{M}(m-1), \qquad m = 1, 2, \dots M \tag{2}$$

The extracted phase is then sampled, the i-th phase sample $\psi(i)$ can be expressed as

$$\psi(i) = \phi_M(i) + \phi'(i), \qquad -\pi < \psi(i) < \pi \tag{3}$$

$\phi_M(i)$ is the i-th sampled phase component of s(t) and $\phi'(i)$ is the random phase component due to n(t). The phase component of r(t) may be extracted by means of I-Q channelization technique. Observe that M=1 corresponds to the continuous wave case, M=2 corresponds to BPSK and M=4 corresponds to QPSK.

Let $f_\Phi(\phi)$ be the probability density function of the phase fluctuation due to additive white Gaussian noise, then the probability density function of $f_\Psi(\psi; M)$ becomes as follows assuming the equiprobable M phase states [6].

$$f_\Psi(\psi; M) = \frac{1}{M}\sum_{m=1}^{M} f_\Psi\{\psi - \frac{2\pi}{M}(m-1)\},$$
$$m = 1, 2, \cdots, M \tag{4}$$

The n-th moment of the phase of a received signal is

$$m_n(M) = \int_{-\pi}^{\pi} \psi^n f_\Psi(\psi; M)\, d\psi \tag{5}$$

This values of moment are used in evaluation of the mean and variance of the probability density function of sampled moment.

The sampled moments are defined as

$$\overline{m_n}(M) = \frac{1}{L} \sum_{i=1}^{L} \psi^n(i, M) \tag{6}$$

where L is the number of samples taken from the phase of received signal. Since the samples are independent identically distributed, the probability density function of $f_{\overline{m_n}}\{\overline{m_n}(M)\}$ approaches to the Gaussian dendity by virtue of the central limit theorem.

It can be shown that the mean and variance of $f_{\overline{m_n}}\{\overline{m_n}(M)\}$ become respectively

$$\mu_n(M) = m_n(M)$$

$$\sigma_n^2(M) = \frac{m_{2n}(M) - m_n^2(M)}{L} \tag{7}$$

Then there remains formulation of modulation classification problem as stated in section 3.

The classifier developed in [6] for coherent system utilizes the statistical moments of samples of the received signal phase which is difficult to extract from the received signal. In this paper we consider modulation classifier for coherent and noncoherent systems by using the statistical moments of samples of the demodulated signal. We present a Bayes classifier to classify the PSK signals using the exact probability density function of demodulated signal. A Bayes test is based on two assumption. The first assumption is that a priori probability which represents the observer information about the source before the experiment conducted is completely given. The second assumption is that a cost is assigned to each possible course of the action. Then the Bayes criterion of minimum average cost results in a test of the likelihood ratio. The classifier proposed utilizes the sampled moments of demodulated signal which can be used as a sufficient statistics to recognizes modulation type of PSK signals. A hypothesis test is formulated based on this moment.

Ⅱ. Developement of classifier model

The received M-ary phase signal waveforms may be expressed as[4]

$$s(t) = \mathrm{Re}\,\{u(t)\exp[j(2\pi f_c t + \frac{2\pi}{M}(m-1) + \psi)]\},$$
$$m = 1, 2, \cdots, M \tag{8}$$

where Re{ · } denotes the real part of complex-valued quantity in the bracket, and ψ is an arbitrary initial phase which can be set to zero. f_c is the carrier frequency and $2\pi(m-1)/M$ represents the information bearing components of the signal phase. The pulse u(t) determines the spectral characteristics of the multiphase signal. If u(t) is a rectangular pulse of the form

$$u(t) = \sqrt{\frac{2E}{T}}, \qquad 0 \le t \le T \tag{9}$$

where T is symbol duration and E is symbol energy, the signal waveforms may be expressed as

$$s(t) = \sqrt{\frac{2E}{T}}\, A_{mc}\cos 2\pi f_c t - \sqrt{\frac{2E}{T}}\, A_{ms}\sin 2\pi f_c t \tag{10}$$

where

$$A_{mc} = \cos[\frac{2\pi}{M}(m-1) + \psi], \qquad m = 1, 2, \cdots, M$$
$$A_{ms} = \sin[\frac{2\pi}{M}(m-1) + \psi], \qquad m = 1, 2, \cdots, M \tag{11}$$

Thus, the signal given by eq. (10) is viewed as two quadrature carriers with amplitude A_{mc} and A_{ms}, which depend on the transmitted phase in each signaling interval. M=1 corresponds to the continuous wave(CW) case, while M=2 corresponds to binary PSK(BPSK) and M=4 corresponds to quadrature PSK(QPSK). The general form of the optimum demodulator for detecting one of M signals in an AWGN channel is one that computes the decision variables

$$U_m = \mathrm{Re}\{\int_0^T r(t)\, u^*(t) \exp[-j(\frac{2\pi}{M}(m-1)+\psi)]dt\} \quad (12)$$

where $u^*(t)$ is the conjugate of u(t), and selects the signal corresponding to the largest decision variable. Having described the form of the modulator and demodulator of PSK, we consider the probability density function of the phase fluctuation due to AWGN which is given by[4]

$$f_\Phi(\phi) = \frac{1}{2\pi}\exp(-R) + \frac{1}{2}\sqrt{\frac{R}{\pi}}\exp(-R\sin^2\phi)$$
$$\cos\phi[1+erf(\sqrt{R}\cos\phi)], \quad -\pi \le \phi \le \pi \quad (13)$$

where R is signal-to-noise ratio(SNR) and erf(·) is error function.

By applying the rule of transformation of the random variables, the probability density function of $x=\cos\phi$ can be expressed as

$$f_X(x) = \frac{\exp(-R)}{\pi\sqrt{1-x^2}} + \sqrt{\frac{R}{\pi(1-x^2)}}$$
$$\exp[-R(1-x^2)]x[1+erf(\sqrt{R}x)], \quad -1 \le x \le 1 \quad (14)$$

Figure 1 shows f(x) for several values of R. The pdf of x has the peak at x=1 because $\cos\phi$ occurs more often near 1 than any other values. It is observed that $f_X(x)$ becomes more peaked about x=1 as R increases.

The pdf of $y=\sin\phi$, $g_Y(y)$, is expressed as

$$g_Y(y) = \frac{\exp(-R)}{\pi\sqrt{1-y^2}} + \sqrt{\frac{R}{\pi}}\exp[-R\, y^2]$$
$$\mathrm{erf}[\sqrt{R(1-y^2)}], \quad -1 \le y \le 1 \quad (15)$$

Similarly, $g_Y(y)$ is sketched in figure 2, which will be used later in constructing the pdf of QPSK. It is observed that $g_Y(y)$ becomes narrower and more peaked about y=0 as R increases. The pdf of QPSK is obtained by adding that of $x=\cos\phi$

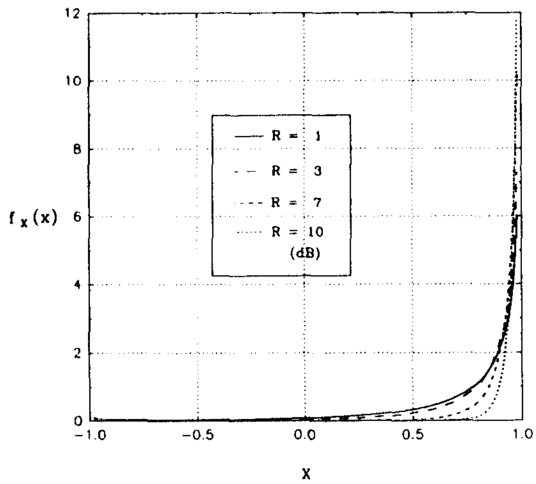

Fig. 1. Probability density function of $x=\cos\phi$ (Coherent System)

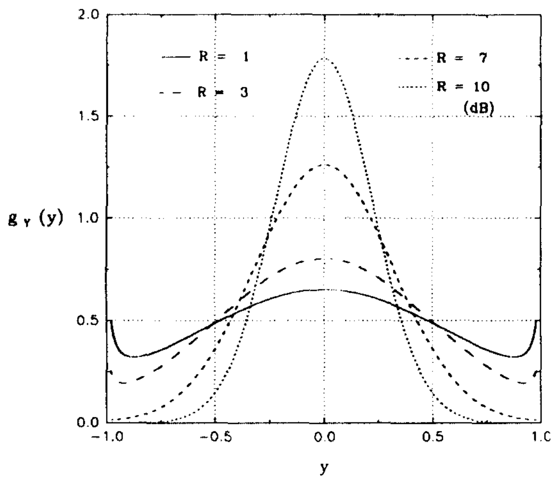

Fig. 2. Probability density function of $y=\sin\phi$ (Coherent System)

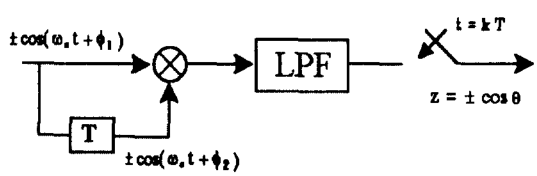

Fig. 3. Block diagram of differential BPSK receiver

and $y=\sin\phi$ with appropriate weights.

The block diagram of the differential demodulator for BPSK system is shown in figure 3. ϕ_1 and ϕ_2 are the phase errors due to AWGN and have the pdf of eq. (13). We assume that ϕ_1 and ϕ_2 are independent. θ is the phase difference and defined by

$$\theta=\phi_1-\phi_2 \tag{16}$$

The pdf of θ is given by[5]

$$f_\theta(\theta)=\frac{1}{2\pi}\exp(-R)[1+\frac{R}{2}\int_0^\pi(\sin\alpha+\cos\alpha)\exp(R\sin\alpha\cos\theta)\,d\alpha],\quad -\pi\le\phi\le\pi \tag{17}$$

For the same manners, we can obtain the new pdf of $z=\cos\theta$ from the transformation of random variables

$$f_Z(z)=\frac{\exp(-R)}{\pi\sqrt{1-z^2}}[1+\frac{R}{2}\int_0^\pi(\sin\alpha+z)\exp(Rz\sin\alpha)\,d\alpha],\quad -1\le z\le 1 \tag{18}$$

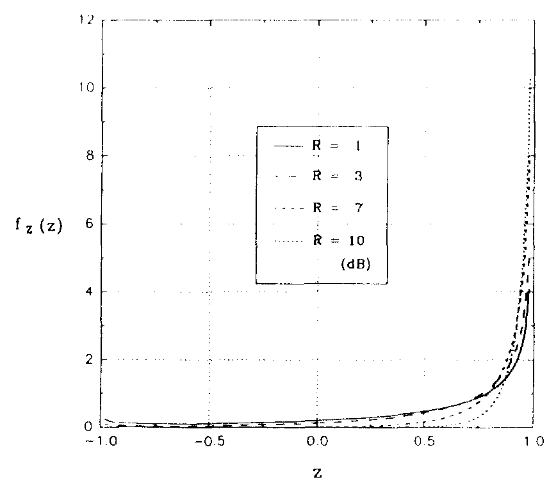

Fig. 4. Probability density function of $z=\cos\theta$ (Noncoherent System)

Figure 4 shows that the pdf $f_Z(z)$ becomes more peaked about $z=1$ as R increases. Similarly the pdf of $w=\sin\theta$, $g_W(w)$, is expressed as

$$g_W(w)=\frac{\exp(-R)}{\pi\sqrt{1-w^2}}+\frac{R\exp(-R)}{4\pi}\int_0^\pi\frac{\sin\alpha-\sqrt{1-w^2}}{\sqrt{1-w^2}}\exp(-R\sin\alpha\sqrt{1-w^2})\,d\alpha+\frac{R\exp(-R)}{4\pi}\int_0^\pi\frac{\sin\alpha+\sqrt{1-w^2}}{\sqrt{1-w^2}}\exp(R\sin\alpha\sqrt{1-w^2})\,d\alpha\qquad -1\le w\le 1 \tag{19}$$

The pdf of $w=\sin\theta$ is illustrated in figure 5 and $g_W(w)$ becomes narrower and more peaked about $w=0$ as R increases.

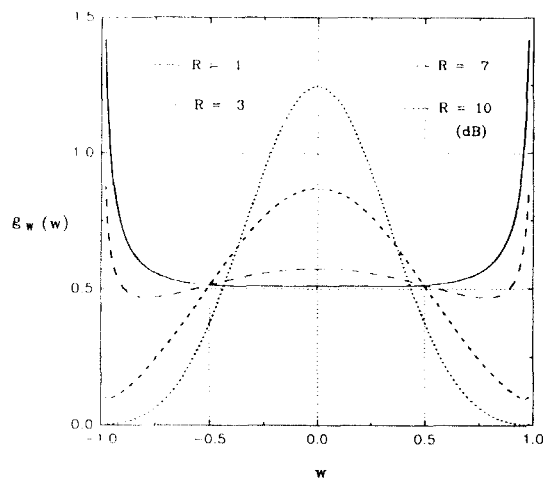

Fig. 5. Probability density function of $w=\sin\theta$ (Noncoherent System)

III. Calculation of moments and misclassification probability for coherent system

The n-the moments of the demodulated PSK signal is given by

$$m_n=\int_{-1}^{1}x^n f(x,M)\,dx \tag{20}$$

First, we consider a single random variable $x = \cos\phi$, characterized by its pdf $f(x)$ given by eq. (14). This case corresponds to CW signal. The derived general expression of moment for CW case is [see Appendix A]

$$m_n = \frac{\exp(-R)}{\sqrt{\pi}} \left\{ \frac{\Gamma[\frac{1}{2}+\frac{n}{2}]}{\Gamma[1+\frac{n}{2}]} + \sum_{k=0}^{\infty} \frac{2^{k+1}R^{k+1}\Gamma[\frac{3}{2}+k+\frac{n}{2}]}{(2k+1)!!\,\Gamma[2+k+\frac{n}{2}]} \right\}$$

; n = even number

$$m_n = \exp(-R)\sqrt{R}\ {}_1F_1[1+\frac{n}{2}, \frac{3}{2}+\frac{n}{2}, R] \frac{\Gamma[1+\frac{n}{2}]}{\Gamma[\frac{3}{2}+\frac{n}{2}]} \quad ; n = \text{odd number} \qquad (21)$$

where $(2k+1)!! = 1\cdot3\cdot5 \cdots (2k+1)$ and ${}_1F_1[a, b, z]$ is the hypergeometric function and defined as follows

$$ {}_1F_1[a, b, z] = \frac{\Gamma(b)}{\Gamma(b-a)\Gamma(a)} \int_0^1 \exp(zt)t^{a-1}(1-t)^{b-a-1}dt \qquad (22)$$

In the similar fashion, the general expressions of moments for BPSK and QPSK signals can be expressed in the closed form [see Appendix A]

(1) BPSK

$$m_n = \frac{\exp(-R)}{\sqrt{\pi}} \left\{ \frac{\Gamma[\frac{1}{2}+\frac{n}{2}]}{\Gamma[1+\frac{n}{2}]} + \sum_{k=0}^{\infty} \frac{2^{k+1}R^{k+1}\Gamma[\frac{3}{2}+k+\frac{n}{2}]}{(2k+1)!!\,\Gamma[2+k+\frac{n}{2}]} \right\}$$

; n = even number

$$m_n = 0 \quad ; n = \text{odd number} \qquad (23)$$

(2) QPSK

$$m_n = \frac{1}{2}\frac{\exp(-R)}{\sqrt{\pi}} \left\{ 2\left[\frac{\Gamma[\frac{1}{2}+\frac{n}{2}]}{\Gamma[1+\frac{n}{2}]}\right] + \sum_{k=0}^{\infty} \frac{2^{k+1}R^{k+1}}{(2k+1)!!\,\Gamma[2+k+\frac{n}{2}]} \cdot \left(\Gamma[\frac{3}{2}+k+\frac{n}{2}] + \Gamma[\frac{3}{2}+k]\Gamma[\frac{1}{2}+\frac{n}{2}]/\sqrt{\pi}\right) \right.$$

; n = even number

$$m_n = 0 \quad ; n = \text{odd number} \qquad (24)$$

We will follow the modulation classification scheme of reference[6] in applying the moments of eq. (23), (24). The n-th sampled moments, denoted by M_n, are definde as

$$M_n = \frac{1}{L}\sum_{j=1}^{L} x^n(j:M) \qquad (25)$$

where L is the number of samples observed. Since we can assume that samples are i.i.d. by virtue of the central limit theorem, the pdf of M_n approaches normal distribution as L becomes larger. Based on this assumption, the probability density function of M_n denoted by $f_{M_n}(M_n)$ is completely described by its mean and variance. That is

$$f_{M_n}(M_n) = N(\mu_n(M), \sigma_n^2(M)) \qquad (26)$$

where

$$\mu_n(M) = E\left[\frac{1}{L}\sum_{j=1}^{L} x^n(j:M)\right] = m_n(M) \qquad (27)$$

$$\sigma_n^2(M) = \left[\frac{1}{L^2}\sum_{j=1}^{L} E[\{x^n(j:M) - m_n(M)\}^2]\right. = \frac{m_{2n}(M) - m_n^2(M)}{L} \qquad (28)$$

From eq. (27), the mean of the sampled moment is the same as the ensemble moment. The sampled moment can be used as a sufficient statistic to recognize the modulation type of PSK signals.

The next case of interest is one in which we must classify one of M hypotheses using the sampled moments, we shall only consider the case for $M=1, 2, 4$. Under the 3 hypothese, we have

$$\begin{aligned} H_0 &: N(\mu_n(1), \sigma_n^2(1)), \text{ CW} \\ H_1 &: N(\mu_n(2), \sigma_n^2(2)), \text{ BPSK} \\ H_2 &: N(\mu_n(4), \sigma_n^2(4)), \text{ QPSK} \end{aligned} \tag{29}$$

The Bayes criterion leads to a likelihood-ratio test which is defined by

$$\frac{N(\mu_n(M), \sigma_n^2(M))}{N(\mu_n(2M), \sigma_n^2(2M))} \gtrless k \tag{30}$$

where k is a nonnegative constant. The design of a Bayes test requires knowledge of the costs and a priori probability. If we assign no costs for making right decision and equal costs for making either type of wrong decision, the likelihood-ratio test becomes minimum probability of error receiver. With this special cost assignment and equal a priori probability, k becomes unity. The threshold $\lambda_n(M, 2M)$ for n-th moment classifier can be easily derived from the eq. (30) with $k=1$. Based on Gaussian assumption, it can be derived that the threshold for the n-th moment classifier is

$$\lambda_n(M, 2M) = \frac{\sigma_n^2(2M)\mu_n(M) - \sigma_n^2(M)\mu_n(2M) + \sigma_n(M)\sigma_n(2M)s}{\sigma_n^2(2M) - \sigma_n^2(M)} \tag{31}$$

where

$$s = \sqrt{[\mu_n(2M) - \mu_n(M)]^2 + 2[\sigma_n^2(2M) - \sigma_n^2(M)]\ln[\sigma_n(2M)/\sigma_n(M)]}$$

if $M_n > \lambda_n(2,4)$ decide BPSK
if $M_n < \lambda_n(2,4)$ decide QPSK

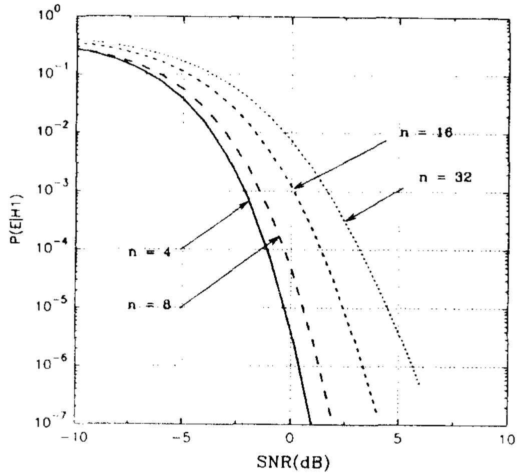

Fig. 6. Probability of misclassification for BPSK (Coherent System)

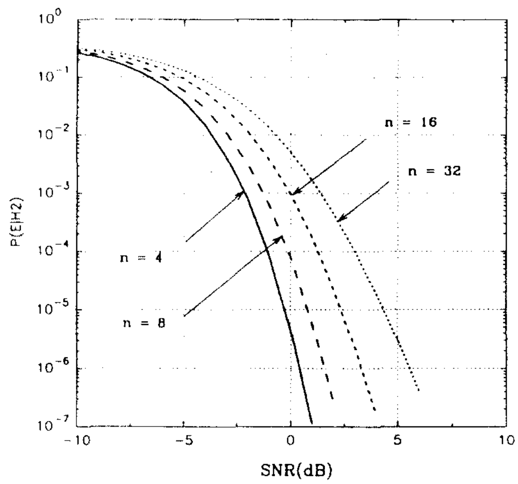

Fig. 7. Probability of misclassification for QPSK (Coherent System)

Having formulated the decision rule of classification for BPSK and QPSK, we evalutate the performance of misclassification an AWGN environment. The probability of misclassification given H_1 and H_2 are respectively

$$P(E \mid H_1) = P(M_n < \lambda_n(2,4)) = Q\left[\frac{\mu_n(2) - \lambda_n(2,4)}{\sigma_n(2)}\right] \tag{32}$$

$$P(E \mid H_2) = P(M_n > \lambda_n(2,4)) \qquad (33)$$

We shall demonstrate how the algorithm will perform in discriminating between BPSK and QPSK. Here L is assumed to be 128 samples. Figure 6 and 7 show the average probability of misclassification as a function of the SNR with the order of moment as a parameter. As in increases, the value of the moments becomes closer and hard to distinguish.

Ⅳ. Calculation of moments and misclassification probability for nonoherent system

From eq. (20) with f(x) given by eq. (18), the general expression of moment for CW case becomes [see Appendix B]

$$m_n = \frac{\exp(-R)\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\sqrt{\pi}\,\Gamma[1+\frac{n}{2}]} + \frac{1}{2}$$

$$\exp(-R)\sum_{k=\text{even}}^{\infty}\frac{R^{k+1}\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{1}{2}+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}$$

$$+\frac{1}{2}\exp(-R)\sum_{k=\text{odd}}^{\infty}\frac{R^{k+1}\Gamma[\frac{1}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{3}{2}+\frac{k}{2}+\frac{n}{2}]}$$

; n = even number

$$m_n + \frac{1}{2}\exp(-R)\sum_{k=\text{odd}}^{\infty}\frac{R^{k+1}\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{1}{2}+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}$$

$$+\frac{1}{2}\exp(-R)\sum_{k=\text{even}}^{\infty}\frac{R^{k+1}\Gamma[\frac{1}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{3}{2}+\frac{k}{2}+\frac{n}{2}]}$$

; n = odd number (34)

In the same manner, the derived general expressions of moments for BPSK and QPSK cases are [see Appendix B]

(1) BPSK

$$m_n = \frac{\exp(-R)\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\sqrt{\pi}\,\Gamma[1+\frac{n}{2}]} + \frac{1}{2}$$

$$\exp(-R)\sum_{k=\text{odd}}^{\infty}\frac{R^{k+1}\Gamma[\frac{1}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{3}{2}+\frac{k}{2}+\frac{n}{2}]}$$

$$+\frac{1}{2}\exp(-R)\sum_{k=\text{even}}^{\infty}\frac{R^{k+1}\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{1}{2}+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}$$

; n = even number

$m_n = 0$; n = odd number (35)

(2) QPSK

$$m_n = \frac{\exp(-R)\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\sqrt{\pi}\,\Gamma[1+\frac{n}{2}]} + \frac{1}{4}$$

$$\exp(-R)\sum_{k=\text{odd}}^{\infty}\frac{R^{k+1}\Gamma[\frac{1}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{3}{2}+\frac{k}{2}+\frac{n}{2}]}$$

$$+\frac{1}{4}\exp(-R)\sum_{k=\text{even}}^{\infty}\frac{R^{k+1}\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{1}{2}+\frac{k}{2}+\frac{n}{2}]}{k!\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[1+\frac{k}{2}+\frac{n}{2}]}$$

$$+\frac{1}{4}\frac{\exp(-R)}{\sqrt{\pi}}\sum_{k=0}^{\infty}R^{2k+1}\,\Gamma[\frac{n}{2}+\frac{1}{2}]$$

$$\{\frac{\Gamma[1+\frac{k}{2}]\,\Gamma[\frac{1}{2}]}{(2k!)\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[k+\frac{n}{2}+1]}$$

$$+\frac{R\Gamma[\frac{k}{1}+\frac{k}{2}]\,\Gamma[k+\frac{3}{2}]}{(2k+1)!\,\Gamma[1+\frac{k}{2}]\,\Gamma[k+\frac{n}{2}+2]}\}$$

$$; \; n = \text{even number}$$

$$m_n = 0 \qquad ; \; n = \text{odd number} \qquad (36)$$

The n-th sampled moments are given by eq. (25). In the similar fashion for coherent system, the samples are i.i.d. Therefore the pdf of M_n approaches normal distribution. Based on this assumption $f_{M_n}(M_n)$ is same as the eq. (26). The threshold for n-the moment classifier is expressed as the eq. (31). Therefore the probabilities of misclassification for BPSK and QPSK signals are given by eq. (32), (33) with different moment function. Figures 9, 10 show the probabilities of misclassification with the order of moment as a parameter.

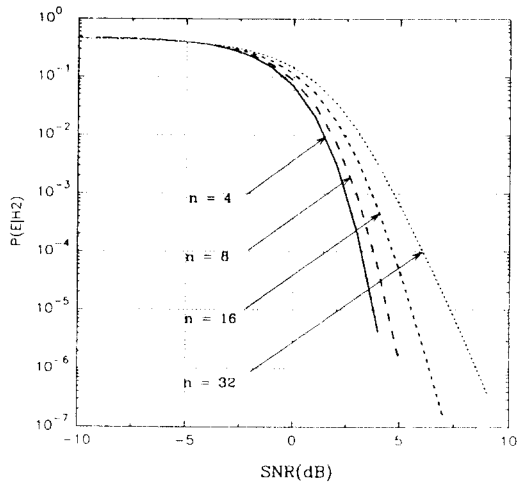

Fig. 8. Probability of misclassification for BPSK (Noncoherent System)

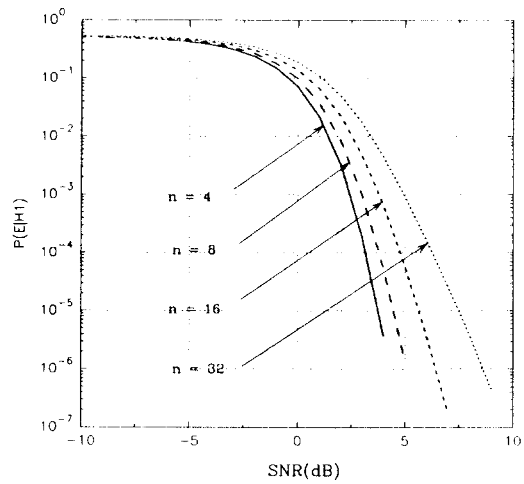

Fig. 9. Probability of misclassification for QPSK (Noncoherent System)

V. Conclusion

In this paper, a Bayes classifier to classify the BPSK and QPSK signals employing the exact pdfs of coherently and noncoherently demodulated signals are discussed in AWGN. The classifier utilizes the sampled moments of demodulated signal which can be used as a sufficient statistics to recognize modulation type of PSK signals. The performances of classifier in terms of probability of misclassification were evaluated. The coherent system classifier gave 4dB improvement for BPSK and 3dB for QPSK over noncoherent system classifier, when the probability of misclassification is 10^{-5} and n equals to 4. Compared with [6], we found that the moment classifier presented offered a 0.5dB improvement when the probability of misclassification is 10^{-3} and n is 4 for coherent QPSK.

Appendix A

The n-th moment of (20) is defined by

$$m_n = \int_{-1}^{1} f_X(x)\, x^n dx \qquad (A.1)$$

To integrate the first term of (A.1), we make use of integral formula[8] and even and odd function properties.

$$\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}}\, dx = \frac{\sqrt{\pi}\;\Gamma[\frac{1}{2}+n]}{2\Gamma[1+n]} \qquad (A.2)$$

We can rewrite the second term of (A.1) as follow

$$\sqrt{\frac{R}{\pi}}\ \exp(-R)\{\frac{\exp(R\,x^2)\,x^{n+1}}{\sqrt{1-x^2}} + \frac{\exp(R\,x^2)\,\mathrm{erf}(\sqrt{R}\ x)\,x^{n+1}}{\sqrt{1-x^2}}\} \qquad (A.3)$$

In (A.3), let A(x) and B(x) be

$$A(x)=\frac{\exp(R\,x^2)}{\sqrt{1-x^2}}\quad x^{n+1} \qquad (A.4)$$

$$B(x)=\frac{\exp(R\,x^2)\,\mathrm{erf}(\sqrt{R}\ x)}{\sqrt{1-x^2}}\quad x^{n+1} \qquad (A.5)$$

From the integral formula[8], we have

$$\int_{-1}^{1} A(x)\,dx = {}_1F_1[1+\frac{n}{2},\ \frac{3}{2}+\frac{n}{2},\ R]\ \frac{\Gamma[1+\frac{n}{2}]\sqrt{\pi}}{\Gamma[\frac{3}{2}+\frac{n}{2}]} \quad ;\ n = \text{odd number}$$

$$=0 \qquad ;\ n=\text{even number} \qquad (A.6)$$

To integrate B(x), we utilizes series expression which is gien by[7]

$$\mathrm{erf}(x)=\frac{1}{\sqrt{\pi}}\ \exp(-x^2)\sum_{n=0}^{\infty}\frac{2^{2n+2}x^{2n+1}(n+1)!}{(2n+2)!} \qquad (A.7)$$

Then, inserting series expression of (A.6) into B (x), we obtain

$$B(x)=\frac{1}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{2^{k+1}x^{2k+n+2}R^{\frac{2k+1}{2}}}{(2k+1)!!\,\sqrt{1-x^2}} \qquad (A.8)$$

The integral of B(x) is obtained from (A.2). Therefore the moment expression of (21) is obtained.

Similarly, the n-th moment of (15) is defined by (A.1). The integration of the first term is obtained from (A.2). Inserting the series expression of erf(x) into the second term, we have

$$\exp(-R\,x^2)\ \mathrm{erf}[\sqrt{R(1-x^2)}\,]\,x^n$$

$$=\exp(-R)\sum_{k=0}^{\infty}\frac{2^{k+1}R^{\frac{2k+1}{2}}}{\sqrt{\pi}\ (2k+1)!!}\ \sqrt{(1-x^2)^{2k+1}}\,x^n \qquad (A.9)$$

To integrate (A.9), we make use of integral formula[7].

$$\int_{-1}^{1}\sqrt{1-x^2}^{\,2k+1}\,x^n\,dx=\frac{\Gamma[k+\frac{1}{2}]\,\Gamma[\frac{n}{2}+\frac{1}{2}]}{\Gamma[k+\frac{n}{2}+1]} \quad ;\ n=\text{even number}$$

$$=0 \qquad ;\ n=\text{odd number} \qquad (A.10)$$

Therefore, the closed form for the moment of (15) can be expressed as follows

$$m_n=\frac{\exp(-R)}{\sqrt{\pi}}\ \{\frac{\Gamma[\frac{1}{2}+\frac{n}{2}]}{\Gamma[1+\frac{n}{2}]} + \sum_{k=0}^{\infty}\frac{2^{k+1}R^{k+1}\,\Gamma[\frac{3}{2}+k]\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\sqrt{\pi}\ (2k+1)!!\ \Gamma[\frac{n}{2}+k+2]}\} \quad ;\ n=\text{even number}$$

$$m_n=0 \qquad ;\ n=\text{odd number} \qquad (A.11)$$

The n-th moment of BPSK can be obtained from

$$m_n=\int_{-1}^{1}x^n\,\frac{1}{2}[f_X(x)+f_X(-x)]\,dx \qquad (A.12)$$

where $f_X(x)$ is (14). The (A.12) leads to (23).

Similary, the n-th moment of QPSK can be obtained from

$$m_n=\int_{-1}^{1}x^n\,\frac{1}{4}\,[f_X(x)+f_X(-x)+g_Y(y)+g_Y(-y)]dx \qquad (A.13)$$

where $f_X(x)$ is (14) and $g_Y(y)$ is (15). The (A.13) leads to (24).

Appendix B

Using the series expression of exp(x) and next integral formula[7]

$$\int_0^{\pi} \sin^n \alpha \, d\alpha = \frac{\sqrt{\pi}\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\Gamma[1+\frac{n}{2}]} \qquad (B.1)$$

we can rewrite the (18) as follows

$$f_Z(z) = \frac{\exp(-R)}{\pi\sqrt{1-z^2}} + \frac{1}{2}\frac{\exp(-R)}{\sqrt{\pi(1-z^2)}} \sum_{k=0}^{\infty} \frac{R^{k+1} z^k}{k!} \left\{ \frac{\Gamma[1+\frac{k}{2}]}{\Gamma[\frac{3}{2}+\frac{k}{2}]} + \frac{x\,\Gamma[\frac{1}{2}+\frac{k}{2}]}{\Gamma[1+\frac{k}{2}]} \right\} \qquad (B.2)$$

The moment calculation of (B.2) is obtained from (A.2) and then the general expression of (34) is obtained. To evaluate the n-th moment of (19), we make use of (B.1). Then, (19) lead to

$$g_W(w) = \frac{\exp(-R)}{\pi\sqrt{1-w^2}} + \frac{\exp(-R)}{2\sqrt{\pi}} \sum_{k=0}^{\infty} R^{2k+1}\sqrt{1-w^2} \left[\frac{\Gamma[1+\frac{k}{2}](1-w^2)^{-\frac{1}{2}}}{(2k)!\,\Gamma[\frac{3}{2}+\frac{k}{2}]} + \frac{R\sqrt{1-w^2}\,\Gamma[\frac{1}{2}+\frac{k}{2}]\sqrt{1-w^2}}{(2k+1)!\,\Gamma[1+\frac{k}{2}]} \right. \qquad (B.3)$$

The moment calculation of the first term in (B.3) is obtained from (A.2) and that of the second term is obtained from (A.10).

Therefore the moment expression of eq. (B.3) is obtained as follows.

$$m_n = \frac{\exp(-R)\,\Gamma[\frac{1}{2}+\frac{n}{2}]}{\sqrt{\pi}\,\Gamma[1+\frac{n}{2}]} + \frac{\exp(-R)}{2\sqrt{\pi}} \sum_{k=0}^{\infty} R^{2k+1}\,\Gamma[\frac{n}{2}+\frac{1}{2}] \left[\frac{\Gamma[1+\frac{k}{2}]\,\Gamma[k+\frac{1}{2}]}{(2k)!\,\Gamma[\frac{3}{2}+\frac{k}{2}]\,\Gamma[k+\frac{n}{2}+1]} + \frac{R\,\Gamma[\frac{1}{2}+\frac{k}{2}]\,\Gamma[k+\frac{3}{2}]}{(2k+1)!\,\Gamma[1+\frac{k}{2}]\,\Gamma[k+\frac{n}{2}+2]} \right.$$

; n = even number

$m_n = 0$; n = odd number (B.4)

The n-th moment of BPSK can be obtained from

$$m_n = \int_{-1}^{1} x^n \frac{1}{2}[f_Z(z)+f_Z(-z)]\,dx \qquad (B.5)$$

where $f_Z(z)$ is (B.2). The (B.5) leads to (35).

Similarly, the n-th moment of QPSK can be obtained from

$$m_n = \int_{-1}^{1} x^n \frac{1}{4}[f_Z(z)+f_Z(-z)+g_W(w)+g_W(-w)]\,dx \qquad (B.6)$$

were $f_Z(z)$ is eq. (B.2) and $g_W(w)$ is (B.3). The (B.6) leads to (36).

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