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A Discrete Time Priority Queueing Model with Bursty Arrivals

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Abstract

A queueing model with two input streams of different service priorities is studied. Specifically, IBP+BP/D/1 with head-of-line priority is analyzed. IBP and BP stand for Interrupted Bernoulli Process and Bernoulli Process respectively. The BP-stream customers have the higher service priority over the IBP-stream customers. An exact analysis of this priority queue is presented to derive the distributions of the state of the system at steady state, the waiting time distributions for each class of customers, and the interdeparture time distributions. The numerical results of the analysis are presented to show how the various parameters of the low and high priority arrival processes affect the performance of the system.

요 약

이 논문에서는 서비스 우선순위가 다른 2개의 독자적인 입력 스트림을 가지는 큐잉 모델을 연구하였다. 구체적으로 head-of-line 우선 순위가 적용되는 IBP+BP/D/1시스템을 분석하였는데, 여기에서 IBP는 Interrupted Bernoulli Process를 BP는 Bernoulli Process를 표시한다. BP-스트림은 IBP-스트림에 대하여 서비스 우선권을 가진다. 본 논문은 이 우선 순위 큐에 대하여 시스템 상태의 안정상태분포와 각 클래스 별 고객의 대기 시간분포 및 출발시간 간격분포를 구할 수 있는 정확한 분석방법을 제공하고 있다. 우선 순위가 다른 두 입력스트림의 다양한 파라미터들이 시스템의 성능에 어떠한 영향을 미치는지 보여줄 수 있는 수치적 예들도 제시하였다.

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I. Introduction

Mechanisms dealing with priorities among arrivals to a queueing system have been studied in literature. Generally, the priorities shown in the literature can be classified into two categories, service priorities and space priorities. In [1], Hebuterne and Gravey analyzed the case where an arriving high priority cell can take the place of a low priority cell already in the buffer if it finds the buffer full. Another mechanism for space priority scheme known as partial buffer sharing is studied in [2] by Garcia and Casals. In this scheme, both high and low priority packets share the buffer up to a threshold. After that only high priority packets are admitted. Comparison of the mechanisms reported in [1] and [2] is performed by Korner in [3]. A discrete time priority queueing system with a bursty arrival stream has been analyzed in [4]. Both a space and a service priorities are studied in that report. However, they only consider one arrival stream, and assumes that the arrival stream consists of two different priority classes of packets.

In this paper, it is assumed that customers with different service priorities come from separate input streams, which may simulate real situations more accurately in certain cases. Specifically, IBP+BP/D/1 system with head-of-line priority is analyzed. Bernoulli Process(BP)-stream customers have service priority over the Interrupted Bernoulli Process (IBP)-stream customers. An exact analysis of this priority queue is presented to derive the distributions of the state of the system at steady state and the waiting time distributions for each class of customers. The interdeparture time distributions of the departure processes are also obtained. The departure processes of queues are of special interest in the analysis of queueing networks because it can be the arrival processes to other queues. Interdeparture time distributions are useful information in characterizing the departure process [5].

This queueing system may readily find out its applications in communication systems. At the transport layer, for example, there are two traffic streams joining: the new packets from an application and the packets requested to be retransmitted. For a high speed network, a queueing network modeling a communication system to analyze the end-to-end delay achieved by the system must include a queueing system simulating the transport layer since the transport protocol processing overhead affects the performance of the system significantly [6-10].

IBP which captures the burstiness of a process is appropriate to model the packet arrivals from the applications of a high speed network [11]. Generally, there is certain randomness in the way that the transport layer retransmits lost or damaged packets, and thus BP may model the stream of packets to be retransmitted [12]. At the transport layer retransmission requests are usually taken care of immediately for the efficiency of the protocol processing. In our queueing model, this may be reflected by the head-of-line priority scheme. The analysis shown in this paper can be combined with the analysis of the intermediate queues in the queueing network to determine the end-to-end delay of a communication system.

In section II, the queueing model is defined, and the distributions of the state of the system at arbitrary time is obtained by deriving the z-transform of the distributions. Queue length distributions at IBP-stream customer arrival instances, from which mean waiting time of IBP-stream customers are readidly derived, is obtained in section III. In section IV, the departure processes of the system are analyzed. Some numerical examples are presented in section V. Finally, conclusions are given in section VI.

II. IBP+BP/D/1 at Arbitrary Time

The BP is defined over a slotted (discrete) time axis. The number of arrivals at each discrete time slot is 1 with the probability β and 0 with the probability $1-\beta$, that is, the interarrival times are independent with the same geometric distribution [12].

The IBP is also defined over a discrete time axis and it comprises two states, an active state and an idle state, which alternate. The transitions between states are governed by the two-state Markov chain as shown in Figure 1. Given that the process is in the active state(or the idle state) at slot i, it will remain in the same state in the next slot $i+1$ with probability p (or q), or will change to the idle state(or the active state) with probability $1-p$ (or $1-q$). During the active state, arrivals occur in a Bernoulli fashion with the probability that a slot contains an arrival to be α. No arrival occurs if the process is in the idle state. The "burstiness" of an arrival process is quantified by the squared coefficient of variations of inter-arrival times(C^2) which is equal to the variance over the mean square of the inter-arrival times. Equations for the z-transform of the probability distribution of the interarrival times, the mean interarrival time, and C^2 of an IBP are obtained in terms of the IBP parameters, p, q, and α, in [11].

Figure 1

The service priority is head-of-line with no preemption. The time required for a customer to pass through the server is equal to one slot. It is assumed that the slot length of each source of arrivals is the same as that of the server. In a slot, the IBP state change, an arrival from the IBP stream, an arrival from the BP-stream and/or a departure may occur. The order of these events in a slot is shown in Figure 2. A customer that arrives at an idle system in the middle of a slot is not served until the beginning of the next slot.

Figure 2

In order to obtain the steady state distribution of the state of the system, we observe the system at the slot boundaries. The state of the system is represented by (S, N), where S and N represent the state of the IBP and the number of customers in the system respectively. S is equal to 0 or 1, representing the idle or active state. $\pi_s(n)$ is defined to be the probability density function (p.d.f) of the state of the system, that is, $\pi_s(n) = \Pr ob[\ S=s, N=n]$, and $\pi(n) = \pi_0(n) + \pi_1(n)$. The state transition diagram of the system is shown in Figure 3.

Figure 3

The Global Balance Equations (GBEs) may be written down by applying flow conservation inspection method to the state transition diagram [13]. From those GBEs, we derive $\Pi_s(z)$ and $\Pi(z)$, the z transforms for $\Pi_s(n)$ and $\Pi(n)$ respectively. $\Pi_s(n)$ and $\Pi(n)$ are, then, obtained by inverting $\pi_s(z)$ and $\Pi(z)$. The GBEs and the derivation of $\Pi_s(z)$ and $\Pi(z)$ are given in the appendix.

III. Queue Length Distribution Observed at the Arrival Instances of Low Priority customers

In this section, we obtain the queue length distribution observed by the arrivals from the IBP stream [14]. This analysis will enable us to derive the mean waiting time of the IBP-stream customer.

We define a random variable $L_k^{(n)}$, the number of customers in the system immediately before the potential arrival point in the kth slot following the nth IBP-stream arrival given that the $(n+1)$st IBP stream arrival does not occur in the preceding

(k-1) slots. Let $L_0^{(n)}$ be the random variable denoting the number of customers in the system seen by the nth IBP-stream arrival. In the nth IBP-stream customer arriving slot, there are 1+B arrivals and a departure if $L_0^{(n)}$ is greater than 0. If $L_0^{(n)}$ is equal to 0, there is no departure. B is the random variable representing the number of arrivals from the BP-stream. Hence, $L_1^{(n)}$ is given by

$$L_1^{(n)} = \begin{cases} L_0^{(n)} + B & \text{if } L_0^{(n)} > 0 \\ B+1 & \text{if } L_0^{(n)} = 0 \end{cases} \tag{1}$$

Correspondingly, the number of customers in the kth slot following the IBP-stream arrival is given by

$$L_1^{(n)} = \begin{cases} L_0^{(n)} + B+1 & \text{if } L_{k-1}^{(n)>0}, \quad k=2,3,\cdots \\ B & \text{if } L_{k-1}^{(n)=0} \end{cases} \tag{2}$$

If the steady state exists for $L_0^{(n)}$ and $L_k^{(n)}$, then

$$\lim_{n\to\infty} L_k^{(n)} = L_k \text{ and } \lim_{n\to\infty} L_0^{(n)} = L_0 \tag{3}$$

Let $l_0(n)$ and $l_k(n)$ be the p.d.f.s of the random variables L_0 and L_k, respectively. An IBP-stream customer will arrive in the kth slot after the last IBP-stream arrival with probability $a(k)$ (=Prob [interarrival time of IBP-stream= k]) and find the queue length distribution $l_k(n)$. Hence, the p.d.f. $l_0(n)$ can be written as a function of the p.d.f. $l_k(n)$ as follows

$$l_0(n) = \sum_{k=1}^{\infty} a(k) l_k(n). \tag{4}$$

Let $L_0(z)$ and $L_k(z)$ be the z-transforms for L_0 and L_k, accordingly. Taking z-transform for the random variables described in relations (2) and (3) gives

$$L_k(z) = z\left(\beta + \frac{(1-\beta)}{z}\right)^k L_0(z) + (z-1)\sum_{i=1}^{k-1}\left(\beta+\frac{(1-\beta)}{z}\right)^i l_{k-1}(0) + z(z-1)\left(\beta+\frac{(1-\beta)}{z}\right)^k l_0(0), \quad k=2,3,\cdots \tag{5}$$

where β is the probability of an BP-stream arrival in a slot.

From (4), $L_0(z)$ can be written as a function of $L_k(z)$ as follows

$$L_0(z) = \sum_{k=1}^{\infty} a(k) L_k(z) \tag{6}$$

By substituting (5) into (6) one obtains

$$L_0(z) = zL_0(z)A\left(\beta+\frac{(1-\beta)}{z}\right) + (z-1) \cdot \sum_{i=1}^{\infty}\left(\beta+\frac{(1-\beta)}{z}\right)^i \sum_{k=i+1}^{\infty} a(k) l_{k-1}(0) + z(z-1)\sum_{i=1}^{\infty} a(k) l_0(0)\left(\beta+\frac{(1-\beta)}{z}\right)^k \tag{7}$$

where $A(z)$ is the z-transform of the p.d.f. of the interarrival times for the IBP-stream.

Obviously the term $\sum_{k=i+1}^{\infty} a(k) l_{k-1}(0)$ can be rewritten as $\sum_{k=1}^{\infty} a(k+i) l_k(0)$.

Let us define the sequence $x(i)$ as

$$x(i) = \sum_{k=1}^{\infty} a(k+1) l_k(0), \qquad i \geq 1 \tag{8}$$

and the $X(z)$ as the z-transform of the sequence $x(i)$.

$$X(z)=\sum_{i=1}^{\infty} x(i) z^{i}. \tag{9}$$

Then, equation (7) can be written as

$$\begin{aligned} L_0(z) = & zL_0(z) A\left(\beta+\frac{(1-\beta)}{z}\right) \\ & +(z-1) X\left(\beta+\frac{(1-\beta)}{z}\right) \\ & +z(z-1) l_0(0) A\left(\beta+\frac{(1-\beta)}{z}\right) \end{aligned} \tag{10}$$

From (10) one finally obtains

$$L_0(z)=\frac{(z-1)\left(X\left(\beta+\frac{(1-\beta)}{z}\right)+z l_0(0) A\left(\beta+\frac{(1-\beta)}{z}\right)\right)}{1-zA\left(\frac{\beta+(1-\beta)}{z}\right)} \tag{11}$$

It rematins to determeine the unknown function $X\left(\beta+\frac{(1-\beta)}{z}\right)$ and $l_0(0)$. We follow the procedure described in [15] to obtain those. The requirement for this method to be used is that $A(z)$ is a rational function in z.

The z-transform of interarrival time p.d.f. of IBP-stream, $A(z)$, can be written as

$$A(z)=\frac{a_2 z^2+a_1 z}{(1.0-\omega_1)(1.0-\omega_2)}, \tag{12}$$

where $1/\omega_1$ and $1/\omega_2$ are the zeroes of the denominator of the z transform of interarrival time p.d.f. for IBP.

Furthermore, $X(z)$ can also be written in a similar form.

$$X(z)=\frac{x_2^* z^2+x_1^* z}{(1.0-\omega_1)(1.0-\omega_2)}. \tag{13}$$

Now, let us define several functions as follow

$$E(z)=\beta+\frac{(1-\beta)}{z}, \tag{14}$$

$$P(z)=a_2(E(z))^2+a_1(E(z)), \tag{15}$$

$$\Gamma(z)=(1-\omega_1 E(z))(1-\omega_2 E(z)), \tag{16}$$

$$X^*(z)=x_2^*(E(z))^2+x_1^* E(z). \tag{17}$$

From (13)-(17), it is given that

$$X\left(\beta+\frac{(1-\beta)}{z}\right)=X(E(z))=\frac{X^*(z)}{\Gamma(z)} \text{ and} \tag{18}$$

$$A\left(\beta+\frac{(1-\beta)}{z}\right)=A(E(z))=\frac{P(z)}{\Gamma(z)}. \tag{19}$$

By substituting (18) and (19) into (11), one can obtain

$$L_0(z)=\frac{(z-1)(X^*(z)+z l_0(0) P(z))}{\Gamma(z)-zP(z)}. \tag{20}$$

The denominator of right hand side of the above equation has whenever the condition for the existence of a stochastic equilibrium is fulfilled exactly three zeros inside the unit disc of the complex plane, one of which is equal to unity. This can be shown by Rouche's theorem [15]. Since $L_0(z)$ is the z-transform of a p.d.f., $L_0(z)$ must be bounded in the range $|z| \le 1$. Therefore, the two zeros of the denominator inside the unit disc must also be the zeroes of the numerator in $L_0(z)$. This provides two linear equations with three unknowns, $l_0(0)$, x_1^* and x_2^*. The normalizing condition, $L_0(1)=1$, together with these two equations are used to determine the unknowns, and hence $X\left(\beta+\frac{(1-\beta)}{z}\right)$.

The waiting time is defined as the time since a customer arrives to the queue until the time the customer departs the system. Having determined the queue length distribution upon an IBP-stream arrival, it is now possible to calculate the mean waiting time of an IBP-stream customer. The waiting time of a high priority customer (BP-stream customer) is simply 1 since the server is deterministic with service time of one slot.

Let w_1 be the random variable denoting the IBP-stream customer waiting time given that the queue length seen at the given IBP-stream arrival instance is one. Whether that one customer is an BP-stream customer or IBP-stream customer does not affect the waiting time of the given IBP-stream customer. Similarly, when there are more than one customer in the queue at thd IBP-stream customer arrival instance, only the total number of customers seen by the IBP-stream arrival needs to be considered. The number of IBP-stream or BP-stream customers among those customers in the queue does not affect the waiting time of the arriving IBP-stream customer.

For simplicity, we assume that all the customers in the queue are low priority customers, the IBP-stream customers, at the IBP-stream arrival instance. First, let's define d_k as the time interval between the service start times of the consecutive customers(kth and (k+1)st customers) in the system at the IBP-stream arrival instance. If the number of customers in the system at an IBP-stream arrival instance is equal to m, d_m is the time interval between the service start time of the arriving IBP-stream customer and the last customer in the system at its arrival instance. w_m is defined as the random variable denoting the waiting time given that there are m customers in the system at the arrival instance of the IBP-stream customer. Then

$$w_1 = d_1 = 1 + i \quad \text{withprob.} \quad \beta^i(1-\beta),\ k>0 \text{ and} \tag{21}$$

$$w_m = \sum_{k=1}^{m} d_k \tag{22}$$

Let $w_m(n)$ be the p.d.f. of the IBP stream customer waiting time given that the queue length seen at the arrival instance is m, and $W_m(z)$ be the z-transform of $w_m(z)$. Then, from (21) and (22) one can obtain

$$W_1(z) = z\frac{1-\beta}{1-\beta z}, \tag{23}$$

$$w_m(n) = d_1 \otimes d_2 \otimes \ldots \otimes d_m = w_{m-1}(n) \otimes w_1(n), \tag{24}$$

$$W_m(z) = (W_1(z))^m = \left(\frac{z(1-\beta)}{1-\beta z}\right)^m, \quad m \geq 2. \tag{25}$$

Hence, the mean waiting time of an IBP-stream customer given that the queue length seen by the IBP-stream arrival is $m(\geq 2)$, $\overline{W_N}(m)$, is given by

$$\overline{W_N}(m) = \frac{dW_m(z)}{dz}\bigg| = \frac{m\beta}{1-\beta} + m, \quad m \geq 2. \tag{26}$$

Since we assumed that a customer that arrives at an idle system in the middle of a slot is not served until the beginning of the next slot, $\overline{W_N}(0)$ is equal to $\overline{W_N}(1)$. Now, we obtain $\overline{W_N}$, the mean waiting time of as IBP-stream customer, by removing the condition using $l_0(m)$, the p.d.f. of the queue length at IBP-stream arrival instances.

$$\overline{W_N} = \sum_{m=0}^{\infty} l_0(m)\, \overline{W_N}(m). \tag{27}$$

VI. The Departure Process of IBP+BP/D/1

The objective of this section is to obtain the p.d.f. of the interdeparture time distribution. Let us define $S(s,n)$ as the p.d.f. of the state of the system immediately after a departure. $S(s,n)$ can be obtained using $\pi_s(n)$, the p.d.f. of the state of the system at arbitrary time, as follows

$$S(0,0)=\frac{\pi_0(1)q(1-\beta)+\pi_1(1)(1-p)(1\beta)}{1-\pi_0(0)-\pi_1(1)}, \quad (28)$$

$$S(1,0)=\frac{\pi_0(1)(1-q)(1-\alpha)(1-\beta)+\pi_1(1)p(1-\alpha)(1-\beta)}{1-\pi_0(0)-\pi_1(1)}. \quad (29)$$

$$S(0,1)=(\pi_0(1)q\beta+\pi_1(1)(1-p)\beta+\pi_0(2)q(1-\beta)+\pi_1(2)(1-p)(1-\beta))/(1-\pi_0(0)-\pi_1(1))$$

$$\begin{aligned}S(1,1)=\ &(\pi_0(1)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta)\\&+\pi_1(1)p(\alpha(1-\beta)+(1-\alpha)\beta)\\&+\pi_0(2)(1-q)(1-\alpha)(1-\beta)\\&+\pi_1(2)p(1-\alpha)(1-\beta))\\&/(1-\pi_0(0)-\pi_1(1)),\end{aligned} \quad (31)$$

$$\begin{aligned}S(0,n)\ =\ &(\pi_0(n)q\beta+\pi_1(n)(1-p)\beta\\&+\pi_0(n+1)q(1-\beta)\\&+\pi_1(n+1)(1-p)(1-\beta))\\&/(1-\pi_0(0)-\pi_1(1))\end{aligned} \quad (32)$$

$$\begin{aligned}S(1,n)=\ &(\pi_0(n-1)(1-q)\alpha\beta+\pi_1(n-1)p\alpha\beta\\&+\pi_0(n)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta)\\&+\pi_1(n)p(\alpha(1-\beta)+(1-\alpha)\beta)\\&+\pi_0(n+1)(1-q)(1-\alpha)(1-\beta)\\&+\pi_1(n+1)p(1-\alpha)(1-\beta))\\&/(1-\pi_0(0)-\pi_1(1))\end{aligned} \quad (33)$$

where $n\geq 2$.

Let D be the interdeparture time between two successive customers, and D_i, $i=0$ or 1, be the time interval from the moment when the IBP is in state i until the instant when a departure occurs. Then, we have

$$D=\begin{cases}1 & \text{with prob. } 1-S(0,0)-S(1,0)\\ D_0 & \text{with prob. } S(0,0)\\ D_1 & \text{with prob. } S(1,0)\end{cases} \quad (34)$$

The time intervals D_0 and D_1 are given as follows

$$D_0=\begin{cases}1 & \text{with prob. } q\beta+(1-q)\cdot(1-(1-\alpha)(1-\beta))\\ 1+D_0 & \text{with prob. } q(1-\beta)\\ 1+D_1 & \text{with prob. } (1-q)(1-\alpha)(1-\beta)\end{cases} \quad (35)$$

$$D_1=\begin{cases}1 & \text{with prob. } (1-p)\beta+p(1-(1-\alpha)(1-\beta))\\ 1+D_0 & \text{with prob. } (1-p)(1-\beta)\\ 1+D_1 & \text{with prob. } p(1-\alpha)(1-\beta)\end{cases} \quad (36)$$

Let $D_0(z)$ and $D_1(z)$ be the z transform of D_0 and D_1 respectively. From (35) and (36), we have

$$\begin{aligned}D_0(z)\ =\ &(z(1-q)(1-(1-\alpha)(1-\beta))+q\beta)\\&+z^2\beta(1-\alpha)(1-\beta)(1-p-q))\\&/(1-z(1-\beta)(q+p(1-\alpha)\\&+z^2(p+q-1)(1-\alpha)(1-\beta)^2)\end{aligned} \quad (37)$$

$$\begin{aligned}D_1(z)=\ &(z(p(1-(1-\alpha)(1-\beta))+(1-p)\beta)\\&+z^2(1-(1-\alpha)(1-\beta))(1-\beta)(1-p-\\&/\ (1-z(1-\beta)(q+p(1-\alpha))\\&+z^2(p+q-1)(1-\alpha)(1-\beta)^2)\end{aligned}$$

Define $D(z)$ as the z-transform of the interdeparture time. From (28) it is given that

$$\begin{aligned} D(z) = \; & z(1-S(0,0)-s(1,0)) \\ & +zS(0,0)D_0(z) \\ & +zS(1,0)D_1(z). \end{aligned} \quad (39)$$

By inverting $D(z)$, we obtain the interdeparture time distribution.

V. Numerical Results

In this section, numerical examples are presented by employing the analytical approach presented above. Figures 4 and 5 show numerical results of the queue length distributions for the IBP+BP/D/1 system at arbitrary points in time and at the low priority customer (IBP-stream customer) arrival instances respectively. The value of α is set to 0.9, and the C^2 and the ρ (average arrival rate) of the IBP are equal to 10 and 0.4 respectively. The arrival rate of the BP(β) varies from 0.01 to 0.4. It is observed that the queue length distributions at the low priority customer arrival instances are affected by the amount of the BP stream traffic more sensitively.

Figure 4, 5

Numerical results of the mean waiting time of the low priority (IBP-stream) customers are shown in Figures 6(a) and 6(b). In both figures, β varies from 0.01 to 0.4. In Figure 6(a), two different values, 0.7 and 0.9, are used for α. The C^2 and ρ of the IBP are equal to 10 and 0.4 respectively. We observe that the waiting times are longer when α is larger (α=0.9) while β is small. After β reaches a certain value, however, the waiting times of the low priority customers for smaller α grow faster as β increases. α is set to 0.9 and two different values (10 and 20) of C^2 are used in Figure 6(b). The value of the ρ used in this figure is also 0.4. As expected, the waiting times of the low priority customers grow faster when C^2 is larger(C^2 = 20).

Figure 6(a), 6(b)

Figures 7(a) and 7(b) show numerical results of the interdeparture time distributions for the IBP+BP/D/1 system. In Figure 7(a), the vlaues of α and β are set to 0.9 and 0.1, and the C^2 and the ρ (average arrival rate) of the IBP are equal to 10 and 0.4 respectively. β varies from 0.1 to 0.4 in Figure 7(b). It shows how the interdeparture time distributions change as the amount of the high priority customer stream (BP stream) increases.

Figure 7(a), 7(b)

VI. Conclusions

In this paper, we have presented an exact analysis on IBP+BP/D/1 system with head-of-line priority. The queue length distributions at arbitrary points in time and at IBP stream customer arrival instances are analyzed. From the analysis on the queue length distributions at IBP-stream customer arrival instances, mean waiting time of the IBP-stream customers is obtained. The departure process of the system is also studied to obtain the interdeparture time distributions.

The numerical result of the analysis are presented to show how various parameters of the low and high priority arrival processes affect the performance of the system. It is shown that the queue length distributions at the low priority customer arrival instances are affected more sensitively than the queue length distributions at arbitrary point in time by the amount of the input from high priority stream. C^2 and the amount of the input from the high and low priority stream have different effect on the mean waiting time of the low priority customers.

The amount of the input from the high priority stream affects the characteristics of the interdeparture time distributions.

APPENDIX

By inspecting the Markov chain shown in Figure 3, we obtain the GBEs for the IBP+BP/D/1 as follows:

$$\pi_0(0)(1-q(1-\beta))=\pi_0(1)q(1-\beta) + \pi_1(0)(1-p)(1-\beta) + \pi_1(1)(1-p)(1-\beta) \quad (A.1)$$

$$\pi_0(1)(1-q\beta)=\pi_0(0)q\beta+\pi_0(2)q(1-\beta) + \pi_1(0)(1-p)\beta+\pi_1(1)(1-p)\beta + \pi_1(2)(1-p)(1-\beta), \quad (A.2)$$

$$\pi_0(2)(1-q\beta)=\pi_0(3)q(1-\beta)+\pi_1(2)(1-p)\beta + \pi_1(3)(1-p)(1-\beta) \quad (A.3)$$

$$\pi_1(0)(1-p(1-\alpha)(1-\beta))=\pi_0(0)(1-q)(1-\alpha)(1-\beta)+\pi_0(1)(1-q)(1-\alpha)(1-\beta) + \pi_1(1)p(1-\alpha)(1-\beta), \quad (A.4)$$

$$\begin{aligned}\pi_1(1)(1-p(\alpha(1-\beta)+(1-\alpha)\beta))= {} & \pi_0(0)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta) \\ & +\pi_0(1)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta) \\ & +\pi_0(2)(1-q)(1-\alpha)(1-\beta) \\ & +\pi_1(0)p(\alpha(1-\beta)+(1-\alpha)\beta) \\ & +\pi_1(2)p(1-\alpha)(1-\beta),\end{aligned} \quad (A.5)$$

$$\begin{aligned}\pi_1(2)(1-p(\alpha(1-\beta)+(1-\alpha)\beta))= {} & \pi_0(0)(1-q)\alpha\beta+\pi_0(1)(1-q)\alpha\beta \\ & +\pi_0(2)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta) \\ & +\pi_0(3)(1-q)(1-\alpha)(1-\beta)+\pi_1(0)p\alpha\beta \\ & +\pi_1(1)p\alpha\beta+\pi_1(3)p(1-\alpha)(1-\beta),\end{aligned} \quad (A.6)$$

$$\pi_0(n)(1-q\beta)=\pi_0(n+1)q(1-\beta)+\pi_1(n)(1-p)\beta + \pi_1(n+1)(1-p)(1-\beta), \quad (A.7)$$

$$\begin{aligned}\pi_1(n)(1-p(\alpha(1-\beta)+(1-\alpha)\beta))= {} & \pi_0(n-1)(1-q)\alpha\beta \\ & +\pi_0(n)(1-q)(\alpha(1-\beta)+(1-\alpha)\beta) \\ & +\pi_0(n+1)(1-q)(1-\alpha)(1-\beta) \\ & +\pi_1(n-1)p\alpha\beta+\pi_1(n+1)p(1-\alpha)(1-\beta),\end{aligned} \quad (A.8)$$

where $n \geq 3$.

From (A.1)-(A.6), we derive expressions for $\pi_0(2)$, $\pi_0(3)$, $\pi_1(0)$, $\pi_1(1)$, $\pi_1(2)$ and $\pi_1(3)$ in terms of $p, q, \alpha, \beta, \pi_0(0)$ and $\pi_0(1)$, as follows

$$\pi_1(0)=A_1\pi_0(0)+A_2\pi_0(1), \quad (A.9)$$

$$\pi_1(1)=A_3\pi_0(0)+A_4\pi_0(1), \quad (A.10)$$

$$\pi_0(2)=A_5\pi_0(0)+A_6\pi_0(1), \quad (A.11)$$

$$\pi_1(2)=A_7\pi_0(0)+A_8\pi_0(1), \quad (A.12)$$

$$\pi_0(3)=A_9\pi_0(0)+A_{10}\pi_0(1), \quad (A.13)$$

$$\pi_1(3)=A_{11}\pi_0(0)+A_{12}\pi_0(1), \quad (A.14)$$

There, A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}, A_{11} and A_{12} are defined as follows:

$$A_1=\frac{(1-\beta(1-p)-q(1-\beta))(1-\alpha)}{(1-p)},$$

$$A_2=\frac{(1-\beta)(1-p-q)(1-\alpha)}{(1-p)},$$

$$A_3=\frac{(1-p(1-\alpha)(1-\beta))(1-q(1-\beta))}{(1-p)(1-\beta)} - (1-\alpha)(1-\beta)(1-q),$$

$$A_4=-\frac{(1-p(1-\alpha)(1-\beta))q(1-\beta)}{(1-p)(1-\beta)} - (1-\alpha)(1-\beta)(1-q),$$

$$A_5=\frac{-(1-p)(2\alpha\beta-\alpha-\beta)(1-q)-(1-\alpha)\beta pq}{(1-\alpha)(1-\beta)(p+q-1)} + \frac{(1-p)p\alpha(1-\beta)}{(1-\alpha)(1-\beta)(p+q-1)}A_1 - \frac{(1-p)(p\alpha(\beta-1)+1)}{(1-\alpha)(1-\beta)(p+q-1)}A_3,$$

$$A_6 = (-(1-p)(2\alpha\beta-\alpha-\beta))(1-q) -(1-\alpha)\beta pq+p(1-\alpha)) /((1-\alpha)(1-\beta)(p+q-1))$$

$$+\frac{(1-p)p\alpha(1-\beta)}{(1-\alpha)(1-\beta)(p+q-1)}A_2$$

$$-\frac{(1-p)(p\alpha(\beta-1)+1)}{(1-\alpha)(1-\beta)(p+q-1)}A_4,$$

$$A_7=\frac{\alpha q(1-q)}{(1-\alpha)(1-p-q)}$$

$$+\frac{(1-\alpha)\beta(p+q-1)+(1-\beta)\alpha pq}{(1-\alpha)(1-\beta)(1-p-q)}A_1,$$

$$+\frac{(1-\alpha)\beta(p+q-1)+(1-\beta)\alpha pq-q}{(1-\alpha)(1-\beta)(1-p-q)}A_3,$$

$$A_8=\frac{(1-\alpha)(1-q)+(1-\beta)\alpha q(1-q)}{(1-\alpha)(1-p-q)}$$

$$+\frac{(1-\alpha)\beta(p+q-1)+(1-\beta)\alpha pq}{(1-\alpha)(1-\beta)(1-p-q)}A_2,$$

$$+\frac{(1-\alpha)\beta(p+q-1)+(1-\beta)\alpha pq-q}{(1-\alpha)(1-\beta)(1-p-q)}A_4,$$

$$A_9=\frac{(1-p)\alpha\beta(1-q)}{(1-\alpha)(1-\beta)(p+q-1)}$$

$$+\frac{P(1-p)\alpha\beta}{(1-\alpha)(1-\beta)(p+q-1)}(A_1+A_3)$$

$$+\frac{p(1-\alpha)(\beta q-1)+(1-p)(1-q)(2\alpha\beta-\alpha-\beta)}{(1-\alpha)(1-\beta)(1-p-q)}A_5,$$

$$+\frac{(1-\alpha)\beta(1-p)p+(1-p)(1-p(\alpha+\beta-2\alpha\beta))}{(1-\alpha)(1-\beta)(1-p-q)}A_7,$$

$$A_{10}=\frac{(1-p)\alpha\beta(1-q)}{(1-\alpha)(1-\beta)(p+q-1)}$$

$$+\frac{p(1-p)\alpha\beta}{(1-\alpha)(1-\beta)(p+q-1)}(A_2+A_4)$$

$$+\frac{p(1-\alpha)(\beta q-1)+(1-p)(1-q)(2\alpha\beta-\alpha-\beta)}{(1-\alpha)(1-\beta)(1-p-q)}A_6,$$

$$+\frac{(1-\alpha)\beta(1-p)p+(1-p)(1-p(\alpha+\beta-2\alpha\beta))}{(1-\alpha)(1-\beta)(1-p-q)}A_8,$$

$$A_{11}=\frac{\alpha\beta q(1-q)}{(1-\alpha)(1-\beta)(1-p-q)}$$

$$+\frac{\alpha\beta pq}{(1-\alpha)(1-p)(1-p-q)}(A_1+A_3)$$

$$+\frac{\alpha\beta q(q-1)+q(2\alpha-1-\alpha q)+1-\alpha}{(1-\alpha)(1-\beta)(1-p-q)}A_5,$$

$$+\frac{\beta(1-\alpha)(p-1)-\alpha\beta q(1+p)-q(1-\beta-\alpha q)}{(1-\alpha)(1-\beta)(1-p-q)}A_7,$$

$$A_{12}=\frac{\alpha\beta q(1-q)}{(1-\alpha)(1-\beta)(1-p-q)}$$

$$+\frac{\alpha\beta pq}{(1-\alpha)(1-p)(1-p-q)}(A_2+A_4)$$

$$+\frac{\alpha\beta q(q-1)+q(2\alpha-1-\alpha q)+1-\alpha}{(1-\alpha)(1-\beta)(1-p-q)}A_6,$$

$$+\frac{\beta(1-\alpha)(p-1)-\alpha\beta q(1+p)-q(1-\beta-\alpha p)}{(1-\alpha)(1-\beta)(1-p-q)}A_8,$$

We multiply the equations given in (A.7) and (A.8) by z^n and then sum over all applicable n. This yields

$$\Pi_0(z)\left(B_{01}+\frac{B_{02}}{z}\right)=\Pi_1(z)\left(B_{03}+\frac{B_{04}}{z}\right)$$
$$+\pi_0(0)\left(B_{01}+\frac{B_{02}}{z}\right)+\pi_0(1)(B_{01}z+B_{02})$$
$$+\pi_0(2)B_{02}z+\pi_1(0)\left(B_{03}+\frac{B_{04}}{z}\right)$$
$$+\pi_1(1)(B_{03}z+B_{04})+\pi_1(2)B_{04}z, \qquad (A.15)$$

$$\Pi_1(z)\left(B_{11}z+B_{12}.\frac{B_{13}}{z}\right)$$
$$=\Pi_0(z)\left(B_{14}z+B_{15}+\frac{B_{16}}{z}\right)$$
$$+\pi_0(0)\left(B_{14}z+B_{15}+\frac{B_{16}}{z}\right)$$
$$+\pi_0(1)(B_{14}z^2+B_{15}z+B_{16})$$
$$+\Pi_0(2)(B_{15}z^{2+}B_{16}z)$$
$$+\pi_0(3)B_{16}z^2+\pi_1(0)\left(B_{11}z+B_{12}+\frac{B_{13}}{z}\right)$$

$$+\pi_1(1)(B_{11}z^2+B_{12}z+B_{13}),$$
$$+\pi_1(2)(B_{12}z^2+B_{13}z)+\pi_1(3)B_{13}z^2. \quad \text{(A.16)}$$

There, B_{0i} and B_{1j}, i=1 to 4 and j=1 to 6, are defined as follows:

$B_{01}=1-q\beta$
$B_{02}=-q(1-\beta)$,
$B_{03}=-(1-p)\beta$,
$B_{04}=-(1-p)(1-\beta)$,
$B_{11}=-p\alpha\beta$,
$B_{12}=1-p(\alpha(1-\beta)+(1-\alpha)\beta)$,
$B_{13}=-p(1-\alpha)(1-\beta)$,
$B_{14}=-(1-q)\alpha\beta$,
$B_{15}=-(1-q)(\alpha(1-\beta)+(1-\alpha)\beta)$,
$B_{16}=-(1-q)(1-\alpha)(1-\beta)$.

By substituting (A.9) to (A.14) into (A.15) and (A.16), one can obtain

$$x_1(z)\Pi_0(z)+y_1(z)\Pi_1(z)$$
$$=f_1(z)\pi_0(0)+g_1(z)\pi_0(1), \quad \text{(A.17)}$$

$$x_2(z)\Pi_0(z)+y_2(z)\Pi_1(z)$$
$$=f_2(z)\pi_0(0)+g_2(z)\pi_0(1), \quad \text{(A.18)}$$

Where $x_i(z)$, $y_i(z)$, $f_i(z)$ and $g_i(z)$, i=1 and 2, are defined as follows

$$x_1(z)=B_{01}+\frac{B_{02}}{z},$$

$$y_1(z)=B_{03}+\frac{B_{04}}{z},$$

$$f_1(z)=x_1(z)+y_1(z)A_1$$
$$+z(y_1(z)A_3+B_{02}A_5+B_{04}A_7),$$

$$g_2(z)=zx_1(z)+y_1(z)A_1$$
$$+z(y_1(z)A_4+B_{02}A_6+B_{04}A_8),$$

$$x_2(z)=zB_{14}+B_{15}+\frac{B_{16}}{z},$$

$$y_2(z)=zB_{11}+B_{12}+\frac{B_{13}}{z},$$

$$f_2(z)=x_2(z)+y_2(z)A_1$$
$$+z(y_2(z)A_3+(B_{15}+B_{16})A_5+(B_{12}+B_{13})A_7),$$
$$+z^2(B_{16}A_9+B_{13}A_{11})$$

$$g_2(z)=zx_2(z)+y_2(z)A_2$$
$$+z(y_2(z)A_1+(B_{15}+B_{16})A_6+(B_{12}+B_{13})A_8),$$
$$+z^2(B_{16}A_{10}+B_{13}A_{12})$$

From (A.17) and (A.18), $\Pi_i(z)$, $i=0$, i=0 and 1, are derived as follows

$$\Pi_0(z)=((f_1(z)y_2(z)-f_2(z)y_1(z))\pi_0(0)$$
$$+(g_1(z)y_2(z)-g_2(z)y_1(z))\pi_0(1)) \quad \text{(A.19)}$$
$$/\ (x_1(z)y_2(z)-x_2(z)y_1(z)),$$

$$\Pi_1(z)=((f_1(z)x_2(z)-f_2(z)x_1(z))\pi_0(0)$$
$$+(g_1(z)x_2(z)-g_2(z)x_1(z))\pi_0(1)) \quad \text{(A.19)}$$
$$/\ (x_2(z)y_1(z)-x_1(z)y_2(z)),$$

Since $\Pi(z)=\Pi_0(z)+\Pi_1(z)$, by substituting (A.19) and (A.20) into $\Pi_0(z)$ and $\Pi_1(z)$, $\Pi(z)$ is obtained as

$$\Pi(z)=((f_1(z)(y_2(z)-x_2(z))-$$
$$-f_2(z)(y_1(z)-x_1(z)))\pi_0(0)$$
$$\cdot(g_1(z)(y_2(z)-x_2(z))$$
$$-g_2(z)(y_1(z)-x_1(z)))\pi_0(1))$$
$$/\ (x_1(z)y_2(z)-x_2(z)y_1(z)) \quad \text{(A.21)}$$

To obtain $\Pi(z)$, $\Pi_0(z)$and $\Pi_1(z)$, it remains to determine two unknowns, $\pi_0(0)$ and $\pi_0(1)$. For that, we need two linear equations with those two unknowns. Since Π is the z-transform a p.d.f., $\Pi(z)$must be bounded in the range $|z| < 1$. Therefore, a zero of the denominator inside the unit disc must also be a zero of the numerator in $\Pi(z)$. This provides on linear equation with those two unknowns. The normalizing condition, $\Pi(1)=1$, together with that equation are used to determine the unknowns, and thus $\Pi(z)$, $\Pi_0(z)$ and $\Pi_1(z)$.

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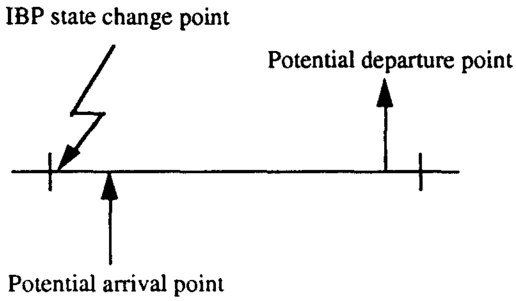

Figure 1 : Order of events in a slot

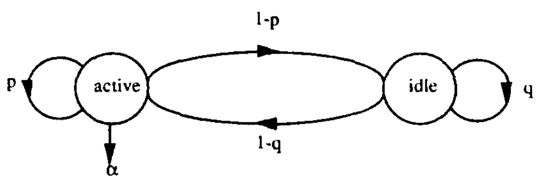

Figure 2 : The Markov Chain for an IBP

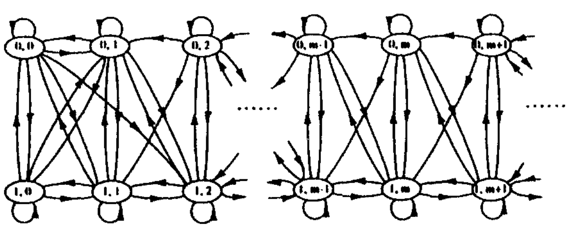

Figure 3 : The two dimensional Markov chain of IBP+BP/D/1

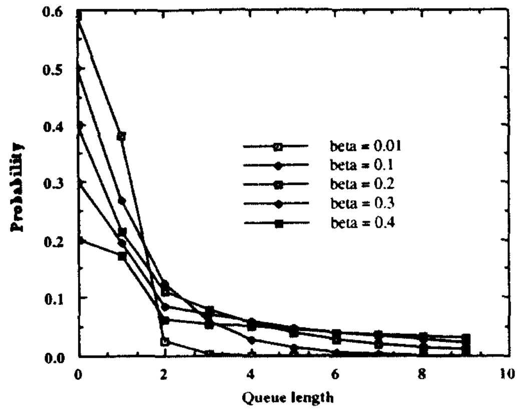

Figure 4 : Queue length distributions for IBP+BP/D/1 at arbitrary points in time w.r.t. the amount of BP stream (β).

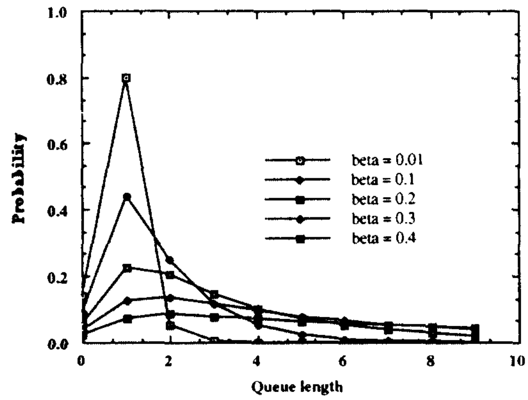

Figure 5 : Queue length distributions for IBP+BP/D/1 at low priority customer arrival instances w.r.t. the amount of BP stream (β).

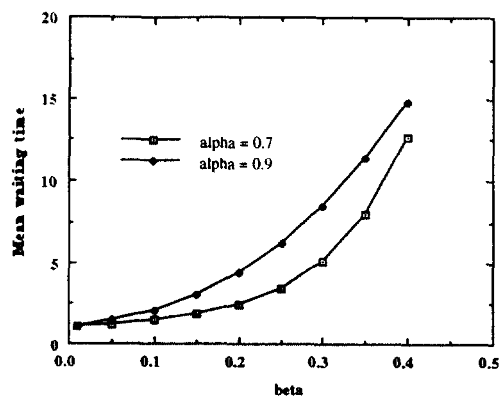

Figure 6(a) : Mean waiting time of low priority customers (IBP-stream customers) w.r.t. the arrival rate (α) during the busy period of the IBP.

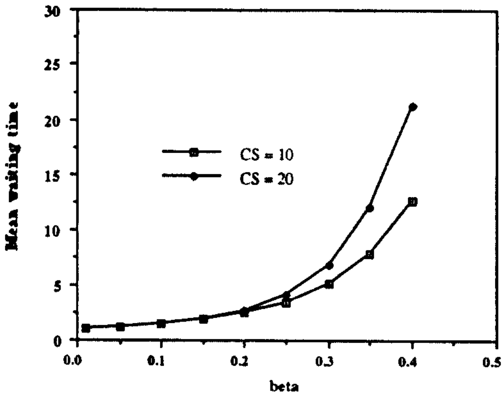

Figure 6(b) : Mean waiting time of low priority customers (IBP-stream customers) w.r.t. the degree of the burstiness (C^2) of the IBP.

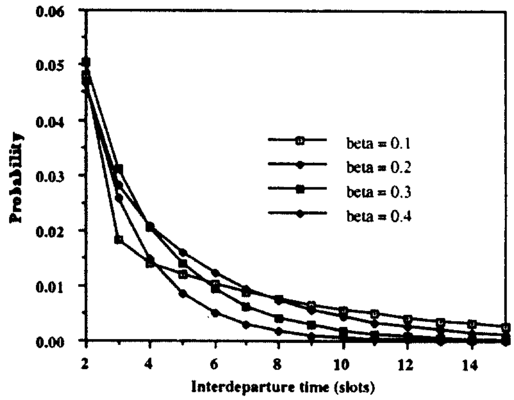

Figure 7(b) : Interdeparture time distributions for IBP+BP/D/1 w.r.t. the amount of the BP-stream (β).

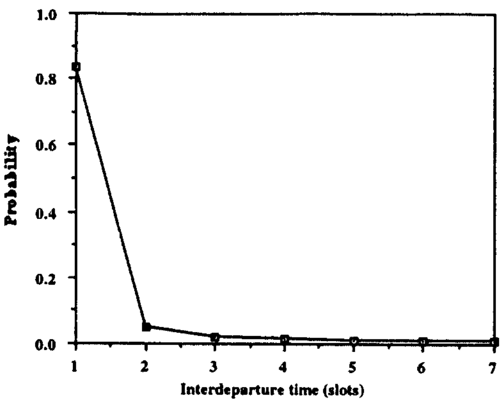

Figure 7(a) : Interdeparture time distribution for IBP+ BP/D/1.

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