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길쌈부호화된 Noncoherent DS/CDMA 시스템의 성능분석

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Performance Analysis of Convolutionally coded Noncoherent DS/CDMA System

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要約

본 논문에서는 AWGN 채널환경에서 다수의 사용자에 대해 Noncoherent M-ary 직교변조와 길쌈부호가 결합된 DS/CDMA 시스템의 성능을 분석하였다. 이러한 분석을 위해 비트오류확률의 상한식에 사용되는 Pairwise오류확률을 유도하였다. 그리고, 컴퓨터 모의실험을 통해 분석적 결과를 검증하였다. 또한 길쌈부호가 결합되었을 때와 결합되지 않았을 때의 DS/CDMA 시스템의 성능을 비교하였고, 그 결과로 길쌈부호와 직교변조를 함께 사용한 경우, 다중화 환경에서의 길쌈부호이득은 에러울 10^4 이하에서 1.5dB이상임을 알 수 있었다.

ABSTRACT

In this paper, we present a performance analysis of the DS/CDMA(Direct Sequence/Code Division Multiple Access) system equipped with a noncoherent M-ray orthogonal modulation scheme combined with a convolutional code, and operating in multiuser environments over an AWGN(Additive White Gaussian Noise) channel. Analytical expressions that can be used to compute the upper bound of the bit error probability are derived. The validity of the analytical results is then verified by extensive computer simulations. Performances of the DS/CDMA system with and without the convolutional codes are also compared, and we have found that the convolutional coding gain in multi-user situations is greater than about 1.5dB at or below the bit error probability 10⁴.

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I. Introduction

The CDMA system using the direct sequence spread spectrum technique has received a tremendous amount of attention during the last few wears. One of the interesting features of the DS/CDMA system is that in the forward link a pilot signal is used so as to provide a reference phase to a mobile station, while in the reverse link the pilot signal is not often available due to the power efficiency problem(1). The transmitter of the reverse link employs an M-ary orthogonal modulation instead, so that a noncoherent receiver structure is feasible [2]-[4]. The noncoherent M-ary orthogonal modulation technique reduces the interference coming from other sources, but at the expense of the performance degradation while maintaining the noncoherent scheme.

The analysis of the DS/CDMA system with the noncoherent M-ary orthogonal modulation in an AWGN channel were presented independently by Kim (2) and Bi (4). They derived the closed form equations for the bit error probability and verified the results by simulations. Bi (5) extended his previous work in (4) by analyzing the performance of the DS/CDMA system in a fading channel having three different kinds of multipathes. All these results were, however, for the DS/CDMA system without he convolutional codes.

Ling and Falconer [6] considered a family of the orthogonal/ convolution codes in a single-user environment. The design and implementation aspects were discussed. It was also shown that it is desirable to use a code with higher convolution coding rate and a larger orthogonal code size for the noncoherent DS/CDMA system. But, this time, the closed form or the upper bound equations for the bit error probability were not found.

This paper extends the results in [2] and [6] by investigating the effect of the convolutional codes, the M-ary orthogonal modulation, and multi-user interference of the DS/CDMA system, all in the unified fashion, over the AWGN channel. In particular, we derive expressions that can be used to compute the upper bound of the bit error probability. The validity of he analytical results is demonstrated by computer simulations. Performances of the DS/CDMA system with and without the convolutional codes are also compared.

This paper is organized as follows. Section I contains a brief description of the DS/CDMA system and a review of its performance when the convolutional codes are not employed. In Section II, we present a performance analysis of the Convolutional coded DS/CDMA system with the noncoherent M-ary orthogonal modulation in multi-user environments over the AWGN channel. Experimental results are given in Section IV to demonstrate the validity of our derivations. Finally, concluding remarks are made in Section V.

I. System Configuration

In the transmitter part of the DS/CDMA system, the vocoder output is passed through the convolutional encoder of the rate R and the constraint length K to generate the code symbols. Every log₂M code symbols are then modulated by the M-ary Walsh orthogonal modulator to obtain the Walsh symbols. Each Walsh symbol is spread by the LPN(Pseudo-Noise) chips that are obtained from the long code generator. The resultant signal goes into the in-phase(I) and quadrature(Q) channels simultaneously, and the signal on each channel is spread once again by the short PN sequence. The final spread sequences are modulated by the QPSK

(Quadrature Phase Shift Keying) procedure.

The block diagram of the receiver this system is shown in Figure 1. The received signal from all different users is first down-converted to the baseband. They are then despread and detected by a bank of the noncoherent M matched filters. The detector outputs are used as a metric for the soft decision of the Viterbi algorithm.

The transmitted QPSK signal of the i-th user during one Walsh symbol interval $T_{\mathbf{w}}$ in of the form .

$$s_{i}(t) = \sqrt{P} W^{j}(t) c_{i}(t) p_{Q}(t) \sin \omega_{c} t +$$

$$\sqrt{P} W^{j}(t) c_{i}(t) p_{I}(t) \cos \omega_{c} t,$$

$$0 \le t \le T_{w}$$
(1)

where P is the transmitted power per an Walsh symbol, ω_c is the carrier frequency, and $W^i(t)$ is the transmitted j-th M-ary Walsh symbol, $j=1,2,\cdots,M$. In (1), $c_i(t)$ is the spreading waveform of the long PN sequence for the i-th user, and $p_i(t)$ and $p_o(t)$ are he spreading wave-

forms of the I- and Q-channels short PN sequences, respectively. The spreading signals $c_i(t)$, $p_i(t)$ and $p_Q(t)$ are the sequences of unit amplitude(positive and negative) rectangular pulses (chips) of the duration T_c . The chip amplitudes are all independent, and identically distributed random variables with probability 1/2 of being ± 1 .

Assuming no path loss and the same power for all users, the received signal is given by

$$r(t) = \sum_{i=1}^{N} s_{i}(t - \tau_{i}) + n(t),$$
 (2)

where N is the number of users, τ_i is a random delay of the i-th user, and n(t) is a narrowband noise obtained from the zero-mean AWGN with the two-sided power spectral N_o/2. After the bandpass filter of the bandwidth B, the AWGN becomes a narrowband noise n(t) that can be represented as

$$n(t) = n_c(t)\cos\omega_c t + n_c(t)\sin\omega_c t, \qquad (3)$$

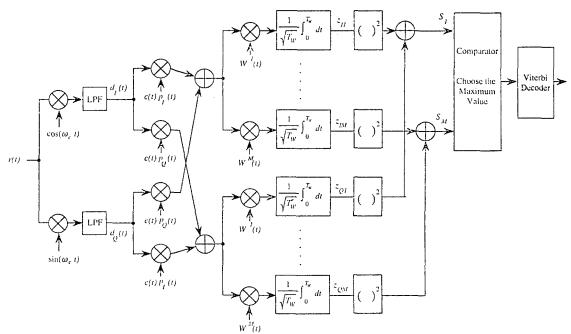


Figure 1. The block diagram of the receiver of the DS/CDMA system with noncoherent detection

where $n_c(t)$ and $n_s(t)$ are zero-mean and lowpass Gaussian random processes with the variance N_oB .

The output of the lowpass filter on the I- and the Q-channel is given by

$$\begin{aligned} d_{I}(t) &= LPF \left\{ r(t) \cos \omega_{c} t \right\} \\ &= \sum_{i=1}^{N} \sqrt{P} W^{J}(t-\tau_{i}) c_{i}(t-\tau_{i}) & (4) \\ \left[p_{I}(t-\tau_{i}) \frac{\cos \theta_{i}}{2} + p_{Q}(t-\tau_{i}) \frac{\sin \theta_{i}}{2} \right] + \frac{n_{c}(t)}{2}, \\ d_{Q}(t) &= LPF \left\{ r(t) \sin \omega_{c} t \right\} \\ &= \sum_{i=1}^{N} \sqrt{P} W^{J}(t-\tau_{i}) c_{i}(t-\tau_{i}) \\ \left[p_{I}(t-\tau_{i}) \frac{\sin \theta_{i}}{2} + p_{Q}(t-\tau_{i}) \frac{\cos \theta_{i}}{2} \right] + \frac{n_{s}(t)}{2}, \end{aligned}$$

$$(5)$$

respectively, where $\theta_i = v_c \tau_i$. Let the k-th user be the user of interest. For simplicity, we assume the perfect synchronism for the PN chips. Let

$$z_{1}^{k} = d_{1}(t) c_{k}(t - \tau_{k}) p_{1}(t - \tau_{k}) + d_{Q}(t) c_{k}(t - \tau_{k}) p_{Q}(t - \tau_{k}),$$
(6)

then the output of the m-th correlator on the I-channel is given by

$$z_{lm}^{k} = \frac{1}{\sqrt{T_{w}}} \int_{0}^{T_{w}} z_{l}^{k}(t) W^{m}(t - \tau_{k}) dt.$$
 (7)

Substituting (5) and (6) in (7), we have

$$z_{\text{lm}}^{k} = \begin{cases} \sqrt{E_{w}} \cos \theta_{k} + \sum_{i=1,i\neq k}^{N} I_{i}^{k,i} + N_{i}^{k}, & m = j \\ \sum_{i=1,i\neq k}^{N} I_{i}^{k,i} + N_{i}^{k}, & m \neq j \end{cases}$$
(8)

where E_w =PT $_w$ is the Walsh symbol energy, $I_i^{\kappa, J}$ is the interference from other users, and N_i^{κ} is the term due to the narrowband Gaussian noise. Similarly, the output of the m-th correlator on the Q-channel, Z_{Qm}^{κ} , becomes

$$z_{Qm}^{k} = \begin{cases} \sqrt{E_{w}} \sin \theta_{k} + \sum_{j=1, j \neq k}^{N} I_{Q}^{k,j} + N_{Q}^{k}, & m = j \\ \sum_{k=1}^{N} I_{Q}^{k,j} + N_{Q}^{k}, & \end{cases}$$
(9)

 $m \neq j$

We form the decision variables $S_m^k(m=1,2,\cdots,M)$ for the k-th user by noncoherently combining the I- and Q-channel outputs in such a way that

$$S_m^k = (z_{lm}^k)^2 + (z_{Qm}^k)^2 \tag{10}$$

The receiver uses the maximum likelihood decision rule to select the maximum value among the decision variables S_m^k . The index of the largest decision variables will represent the transmitted Walsh symbol.

The interference $I_I^{k,j}$ and $I_Q^{k,j}$ are zero-mean and Gaussian random variables with the same variance $E_w/2L$ (2). It is shown in (2) that the noise term on the I-channel can be expressed as

$$N_{I}^{k} = \frac{1}{2\sqrt{T_{w}}} \int_{0}^{T_{w}} c_{k}(t - \tau_{k}) W^{m}(t - \tau_{k}) \times \left[n_{c}(t) p_{I}(t - \tau_{k}) + n_{s}(t) p_{Q}(t - \tau_{k}) \right] dt,$$
(11)

which is a zero-mean Gaussian random variable with the variance $N_{\rm e}/2$. Consequently, the noise on the Q-channel, $N_{\rm Q}^{\rm t}$, is also zero-mean and Gaussian with the same variance, and the expression for $N_{\rm Q}^{\rm t}$ is of the form as in (11) after switching the positions of the two terms, $n_{\rm e}(t)$ and $n_{\rm e}(t)$. Thus, $Z_{\rm kim}$ and $Z_{\rm kQm}$ become Gaussian random variables with the variance σ^2 such that

$$\sigma^2 = \frac{E_w(N-1)}{2L} + \frac{N_o}{2}$$
 (12)

Note that σ^2 is the sum of the interference noise power from other users and the additive noise power.

Without employing the convolutional codes in the DS/CDMA system, the bit error probability P_b is expressed as [2]

$$P_{b} = \frac{M}{2(M-1)} \sum_{n=1}^{M-1} (-1)^{n-1} {M-1 \choose n} \frac{1}{n+1}$$

$$\exp\left(-\frac{n\gamma_{w}}{2(n+1)}\right)$$
(13)

Here, γ_w is the ratio between the Walsh symbol energy and the overall noise power given by

$$\gamma_{w} = \frac{E_{w}}{\sigma^{2}} = \left(\frac{N-1}{2L} + \frac{N_{o}}{2E_{w}}\right)^{-1},$$
 (14)

and $E_w = \log_2(M)E_b$, with E_b being denoted as the information bit energy.

II. Performance Analysis

In this section, based on the results in the previous section, we derive analytical expressions that can be used to compute the upper bound of the bit error probability for the Convolutional coded DS/CDMA system with the noncoherent M-ary orthogonal modulation in multi-user environments.

Let $y_n^{(r)}$, $n=1,2,\cdots$ represent the decision variables, i.e. branch metric, for n-th branch of the r-th path through the trellis. Suppose that the first Walsh symbol, i.e., the all-zero code word, is transmitted. Let the case r=1 denote the first all-zero path. Then based on the decision variables given in (10), the two input variables to the decoder are now

$$y_n^{(1)} = S_{m=1}^k, \qquad n = 1, 2, \cdots$$
 (15)

$$y_n^{(2)} = \begin{cases} S_{m=1}^k, & \text{if the Walsh symbol of the n-} \\ & \text{th branch of the second path} \\ & \approx \text{the Walsh symbol of the first} \\ & \text{path} & \text{n=1,2,\cdots} \end{cases}$$
(16)

Let,
$$y_n^{(2)'} = S_{mx_1}^k$$

Define the decision variable for the r-th path consisting of B branches through the trellis as

$$U^{(r)} = \sum_{n=1}^{B} y_n^{(r)} \tag{17}$$

Let the case r=1 denote the first B-branch all-zero path, and let the case r=2 denote the second B-branch path that begins in the initial state(all-zero) and remerges with the all-zero B transitions. Therefore, the decision variables for the first and the the second paths consisting of B branches are given as

$$U^{(1)} = \sum_{n=1}^{B} y_n^{(1)} , \qquad (18)$$

and

$$U^{(2)} = \sum_{n=1}^{d} y_{n}^{(2)} + \sum_{n=d+1}^{B} y_{n}^{(1)},$$
 (19)

respectively, where d is the Hamming distance between two paths. The difference between $U^{\scriptscriptstyle (1)}$ and $U^{\scriptscriptstyle (2)}$ is then

$$U^{(1)} - U^{(2)} = \sum_{n=1}^{d} (y_n^{(1)} - y_n^{(2)'}).$$
 (20)

For convenience, we now redefine the decision variables in (18) and (19), respectively, as

$$U^{(1)} = \sum_{n=1}^{d} y_n^{(1)}, \qquad (21)$$

and

$$U^{(2)} = \sum_{n=1}^{d} y_n^{(2)}$$
 (22)

Note that U⁽¹⁾ is described statistically as a non-central chi-square random variable with 2d degrees of freedom and noncentrality parameter

$$z^2 = \sum_{m=1}^{d} E_m = d E_m, \qquad (23)$$

where E_w =Rlog₂(M) E_b , with R being denoted as a convolutional code rate. Note also that unlike the uncoded DS/CDMA system as in the

last part of the previous section, the convolutional code rate R is included in the expression of E_{w} . The probability density function of $U^{(i)}$ can be written as

$$p(u^{(1)}) = \frac{1}{2\sigma^2} \left(\frac{u^{(1)}}{z^2}\right)^{(d-1)/2}$$

$$\exp\left(-\frac{z^2 + u^{(1)}}{2\sigma^2}\right) I_{d-1} \left(\frac{z\sqrt{u^{(1)}}}{\sigma^2}\right), \quad u^{(1)} \ge 0$$
(24)

where $I_d(\cdot)$ is the modified Bessel function of the d-th order and σ^2 is given in (12).

On the other hand, U⁽²⁾ is statistically a central chi-square random variable, having 2d degrees of freedom with the probability density function as

$$p(u^{(2)}) = \frac{1}{(2\sigma^2)^d (d-1)!} u^{(2)^{n-1}}$$

$$\exp\left(-\frac{u^{(2)}}{2\sigma^2}\right), \qquad u^{(2)} \ge 0$$
(25)

The probability of error in pairwise comparison of the $U^{(i)}$ and $U^{(2)}$ is

$$P_{2}(d) = P(U^{(2)} - U^{(1)} \ge 0) = \int_{0}^{\infty} P(U^{(2)} \ge u^{(1)} | U^{(1)}$$

$$= u^{(1)}) \ p(u^{(1)}) \ du^{(1)},$$
(26)

where from (8)

$$P(U^{(2)} \ge u^{(1)} | U^{(1)} = u^{(1)}) = \int_{u^{(1)}}^{\infty} p(u^{(2)}) du^{(2)}$$

$$= \int_{u^{(1)}}^{\infty} \frac{1}{(2\sigma^2)^d (d-1)!} u^{(2)^{n-1}} \exp\left(-\frac{u^{(2)}}{2\sigma^2}\right) du^{(2)}$$

$$= \exp\left(-\frac{u^{(1)}}{2\sigma^2}\right) \sum_{k=0}^{d-1} \frac{1}{k!} \left(\frac{u^{(1)}}{2\sigma^2}\right)^k$$
(27)

Hence, using (27) in (26) yields

$$P_2(d) = \int_0^{\infty} \left[\exp\left(-\frac{u^{(1)}}{2\sigma^2}\right) \sum_{l=0}^{d-1} \frac{1}{l!} \left(\frac{u^{(1)}}{2\sigma^2}\right)^l \right]$$

$$\times \frac{1}{2\sigma^{2}} \left(\frac{u^{(1)}}{z^{2}}\right)^{(d-1)/2} \exp\left(-\frac{z^{2} + u^{(1)}}{2\sigma^{2}}\right)$$

$$I_{d-1} \left(\frac{z\sqrt{u^{(1)}}}{\sigma^{2}}\right) du^{(1)}$$
(28)

It is shown in Appendix that (28) can be simplified as

$$P_{2}(d) = 2^{-d} \exp(-\gamma d) \sum_{l=0}^{d-1} {d+l-1 \choose d-1}_{1}$$

$$F_{1}(d+l, d; \gamma d/2) 2^{-l},$$
(29)

where

$$\gamma = \left(\frac{1}{R\log(M)\gamma_h} + \frac{N-1}{L}\right)^{-1} \tag{30}$$

and $\chi=E_b/N_0$. Derivation of equations (29) and (30) are the major contribution of theis paper.

We are now ready to compute analytically the upper bound for the probability of bit error with (29) and (30) using the expression for the upper bound derived in [7]. We reproduce the expression here for convenience:

$$P_b < \sum_{d=d}^{\infty} \beta_d P_2(d) \tag{31}$$

Note that the distance in (31) must be measured by the unit of Walsh symbols. Note also that d_{free} is the minimum weight(in term of the number of nonzero symbols) of the nonzero path. For the convolutional code of the rate R=1/3 and the Walsh modulation with M=64, for example, d_{free} =5. The information weight spectrum β_d is the total number of nonzero information bits for all the inputs that have the path weight equal to d. Here, a computer search technique is used to find the first few terms of β_d .

W. Experimental Results

This section presents experimental results to demonstrate the validity of our analytical expressions. For this, we consider two kinds of DS/CDMA system with a PN chip rate of 128 chips per information bit.

First, we have selected the coded DS/CDMA system parameters as follows: the convolutional code rate R=1/3: the constraint length K=9, M=64(i.e., 64-ary orthogonal modulation): and L=256. Note that every 6 code symbols are modulated by the 64-ary Walsh orthogonal modulator, and each Walsh chip is spread by the 4 long code PN chips. the resultant signal is then spread by two short PN sequences in both I-and Q-channels, separately.

Second, we have selected the uncoded DS/CDMA system parameters as follows: M=64: and L=256. Note that every 6 code symbols are modulated by the 64-ary Walsh orthogonal mod-

Fig. 2. Bit error probability versus E_k/N₀: R=1/3, K=9, M=64, L=256, and N=1: Curve A=theoretical upper bound of the coded system, Curve B=simulation curve of the coded system, Curve D=simulation curve of the uncoded system, and Curve D=simulation curve of the uncoded system.

ulator, and each Walsh chip is spread by the 12 long code PN chips. The resultant signal by two short PN sequences in both I- and Q-channels, separately.

The generator polynomials for the user long PN sequence and the two short quadrature PN sequences are used as suggested in [9]. We assume the perfect synchronism of the PN chips throughout the experiments.

In Figure $2\sim5$, the bit error probabilities are plotted as functions of Eb/No(or γ b) for the cases when the number of users N=1,3,5, and 7, respectively. In each figure, curve A represents the numerical values of the analytical upper bound given in (31) by employing our results in (29) and (30), and curve B represents the simulation counterparts. Also plotted in each figure is curve C to represent the numerical values of (13) that are for the uncoded DS/CDMA system with the noncoherent 64-ary orthogonal modulation and curve D to represent

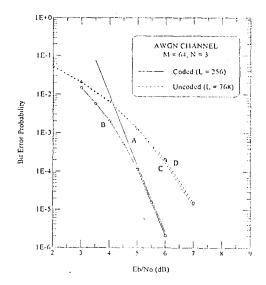


Fig. 3. Bit error probability versus E_b/N₀: R=1/3, K=9, M=64, L=256, and N=3: Curve A=theoretical upper bound of the coded system. Curve B=simulation curve of the coded system, Curve C=theoretical curve of the uncoded system, and Curve D=simulation curve of the uncoded system.

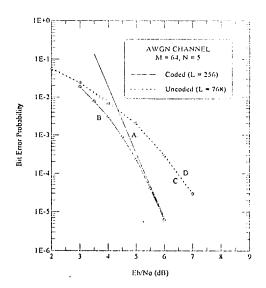


Fig. 4. Bit error probability versus E_b/N_0 : R=1/3, K=9, M=64, L=256, and N=5: Curve A=theoretical upper bound of the coded system, Curve B=simulation curve of the coded system, Curve C=theoretical curve of the uncoded system, and Curve D=simulation curve of the uncoded system.

the simulation counterparts. The solid curves in each figure gives the bit probability at L=256 and the dashes curve in each figure is obtained for L=768.

The following two observations can be made: First, our analytical results for the convolutional coded DS/CDMA system DS/CDMA system with and without the convolution codes are also compared, and we have found that the convolutional coding gain in multi-user situations is greater than about 1.5dB at or below the bit error probability 10⁻⁴.

Figure 6 show the bit error probability versus the number of users when E_b/N_0 is fixed at 5 dB. In the figure, curve A represents the numerical values of the analytical upper bound given in (31) by employing our results in (29) and (30) at L=256, and curve B represents the numerical values of (13) at L=768. For the case of the bit error probability 10-4, it is observed that number of possible users in the convolu-

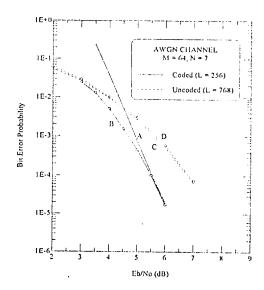


Fig. 5. Bit error probability versus E_b/N₀:
R=1/3, K=9, M=64, L=256, and N=7:
Curve A=theoretical upper bound of the coded system,
Curve B=simulation curve of the coded system,
Curve C=theoretical curve of the uncoded system, and
Curve D=simulation curve of the uncoded system.

tional coded DS/CDMA system is about 10 while only 2 in the uncoded DS/CDMA system.

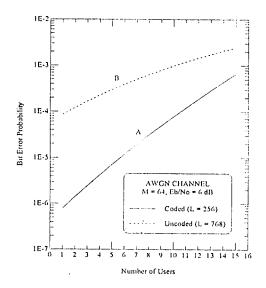


Fig. 6. Bit error probability versus number of users: R=1/3, K=9, M=64, Eb/N0=6(dB), and L=256: Curve A=theoretical upper bound of the coded system, and Curve B=simulation curve of the uncoded system,

Therefore, we can see that the convolutional coded system increases the user capacity dramatically in multi-user environments.

V. Conclusions

In the paper, we have presented a performance analysis of the DS/CDMA system equipped with the noncoherent M-ary orthogonal modulation scheme combined with the convolutional code, and operating in multi-user environments over the AWGN channel. The expressions that can be used to evaluate the upper bound of the bit error probability have been derived. Extensive computer simulations were performed to demonstrate the validity of our derivations. We have seen that our analytical agree well with the simulation counterparts, particularly when the bit error probability is less than 10⁴. We have also found that the convolutional code always enhances the overall performance of the DS/CDMA system and at the convolutional code always enhances the overall performance the overall performance of the DS/CDMA system and at the same time that it enables the large user capacity in the multi-user environment.

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Appendix

A Derivation of Equation (29)

This appendix provides a derivation of the expression given in (29) from (28). Let

$$v = u^{(1)} / 2\sigma^2 \tag{A1}$$

Form this, the equation (28) becomes

$$P_{2}(d) = \int_{0}^{\infty} \left[\exp(-v) \sum_{l=0}^{d-1} \frac{1}{l!} (v)^{l} \right] \times \left(\frac{v \sigma^{2}}{z^{2}} \right)^{(d-1)/2}$$

$$\exp\left(-\frac{z^{2}}{2 \sigma^{2}} - v \right) I_{d-1} \left(2 \sqrt{\frac{z^{2} v}{2 \sigma^{2}}} \right) dv.$$
(A2)

Let

$$\gamma = \left(\frac{N_o}{E_w} + \frac{N-1}{L}\right)^{-1},\tag{A3}$$

then $\frac{z^{2}}{2\sigma^{2}} = \frac{E_{\omega} d}{2} \left(\frac{N_{o}}{2} + \frac{E_{\omega} (N-1)}{2L} \right)^{-1}$ $= d \left(\frac{N_{o}}{E_{\omega}} + \frac{(N-1)}{L} \right)^{-1} = d\gamma$ (A4)

Using (A4) in (A2), we obtain

$$P_{2}(d) = \int_{0}^{\infty} \left[\sum_{l=0}^{d-1} \frac{1}{l!} v^{l} \right] \left(\frac{v}{\gamma d} \right)^{(d-1)/2}$$
(A5)

$$\exp(-\gamma d - 2\nu) I_{d-1}(2\sqrt{\gamma \nu d}) d\nu$$

where L(x) can be represented by the infinite series

$$I_{\alpha}(x) = \sum_{n=0}^{\infty} \frac{\left(\frac{x}{2}\right)^{\alpha+2n}}{n! \Gamma(\alpha+n+1)} \qquad x \ge 0.$$
 (A6)

Now, using (A6) in the last term of (A5), we get

$$I_{d-1}(2\sqrt{\gamma \nu d}) = \sum_{n=0}^{\infty} \frac{\left(\sqrt{\gamma \nu d}\right)^{d-1+2n}}{n!\Gamma(d+n)} = \left(\gamma d\right)^{d-\frac{1}{2}}$$

$$\sum_{n=0}^{\infty} \frac{\left(\gamma d\right)^{n}}{n!\Gamma(d+n)} \nu^{(n+d-\frac{1}{2})},$$
(A7)



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where Γ (p) is the gamma function. Substitutiong (A7) in (A5) gives

$$P_2(d) = \exp(-\gamma d) \sum_{l=0}^{d-1} \frac{1}{l!} \sum_{m=0}^{\infty} \frac{(\gamma d)^m}{n! \Gamma(d+n)}$$

$$\int_{0}^{\infty} v^{(l+n+d-1)} \exp(-2v) dv$$

$$= \exp(-\gamma d) \sum_{l=0}^{d-1} \frac{1}{l!} \sum_{n=0}^{\infty} \frac{(\gamma d)^{n}}{n! \Gamma(d+n)} \frac{\Gamma(l+n+d)}{2^{(l+n+d)}}$$

$$= 2^{-d} \exp(-\gamma d) \sum_{l=0}^{d-1} \frac{2^{-l}}{l!} \sum_{n=0}^{\infty} \frac{\Gamma(l+n+d)(\gamma d)^{n}}{n! \Gamma(d+n) 2^{n}}.$$
(A8)

Here, the last term of (A8) can be expressed

$$\sum_{n=0}^{\infty} \frac{\Gamma(l+n+d)(\gamma d)^n}{n! \Gamma(d+n) 2^n} = \frac{\Gamma(d+l)}{\Gamma(d)} 1$$

$$F_1(d+l,d; \gamma d/2). \tag{A9}$$

where ${}_{1}F_{1}(\alpha, \beta; \mathbf{x})$ is the confluent hypergeometric function defined as

$$_{1}F_{1}(\alpha,\beta;x) = \sum_{k=0}^{\infty} \frac{\Gamma(\alpha+k)\Gamma(\beta)x^{k}}{\Gamma(\alpha)\Gamma(\beta+k)k!}$$
 (A10)

$$\beta \neq 0, -1, -2, \cdots$$

Therefore, equation (29) is obtained by using (A9) in (A8).



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