論文 95-2-2-25

# 도파관 및 파의 산란해석을 위한 요소기법

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# Techniques of Element Scheme of Solving Waveguides and Wave Scattering

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## 要 約

도파관 및 산란체 해석시 많이 이용되는 모멘트법의 적용시 향상된 기법인 요소기법을 제안한다. 이 기법은 종래의 모멘트법에서는 전개함수를 절점단위로 취급한 대신, 전개함수를 요소단위로 취급하므로서 적분결과식의 단순화 및 임의 절점배치의 구현으로 접합이 있는 구조물의 해석을 용이하게 하였다.

요소기법의 유용성을 보이기 위하여, 이 기법을 엄밀해를 가진 셉텀 도파관의 모드 계산과 엄밀해가 없는 쾨드리지 원통의 레이다 단면적 계산에 적용하였다.

## ABSTRACT

This paper presents improving techniques of waveguides and wave scattering by utilizing element scheme in method of moments. The introduced element scheme expands the expansion function over an element segment instead of a node as done in ordinary moment methods. This scheme lends to accept the treatment of junction structures by offering simplified formulation and random node numbering feature as well.

To show the effectiveness of element scheme, waveguide modes of a septum guide with known solutions is determined and also this scheme is applied to a quadridge cylinder to calculate its radar cross section.

## Ⅰ. INTRODUCTION

Method of moments has been used to obtain the numerical solutions in waveguide and scattering problems of various structures in shape[1,2]. Specializing moment methods to an integro-differential equation subject to boundary conditions reduces a matrix equation, from which the

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論文番號 : 94309

接受日字 : 1994年 11月 7日

unknown physical quantities may be determined. However, in applying this method to arbitrarily shaped multiply connected structures this node scheme suffers in random node numbering feature especially for structures with junctions because a testing point is straddled with two elements. To remedy this problem and also to have a more systematic coding benefit, introduced element scheme expands the current over an element segment as often done in finite element methods[3]. The resulting each matrix elements is based on element by element rather than node by node as in ordinary node scheme, which simplifies the formulation due to its cut downed integration range together with benefit of random node assignment.

To show the usefulness of introduced element scheme in treating multiply connected cross sections, waveguide modes of a conducting septum waveguide for known solutions are calculated and compared to each other. And this scheme is applied to two dimensional conducting quadridge cylinder of unknown solutions to obtain its radar cross section.

## Ⅱ. FORMULATION OF THE PROBLEM

For a given electric wall current $\bar{J}$ on the boundary, the electric field $\bar{E}$ can be represented by the scalar and vector potentials, $\phi$ and $\bar{A}$ as[2]

$$L(\underline{J}) = \underline{E}^{inc} = j\omega\underline{A} + \nabla\phi \qquad (1)$$

where

$$\underline{A} = \frac{\mu}{4j}\oint_c \underline{J}H_o^{(2)}(kR)\,dl \qquad (2)$$

$$\phi = \frac{1}{4\omega\varepsilon}\oint_c (\nabla\cdot\underline{J})H_o^{(2)}(kR)\,dl \qquad (3)$$

and $H_o^{(2)}$ is Hankel function, and $\bar{E}^{inc}$ represents the incident electric field for scattering problem and sets to zero for waveguide mode computation in element scheme. k represents the wavenumber and R the distance between field point and source point, respectively. TE wave is assumed both for scattering and waveguide problem and axial direction in cylinder along the z-axis, hence the wall currents flow along the circumferential direction in scattering and also parallels to this direction for cutoff currents in waveguide problem. Especially, for modes in waveguide those k appeared in eq. (2) and (3) become the cutoff wavenumbers[4].

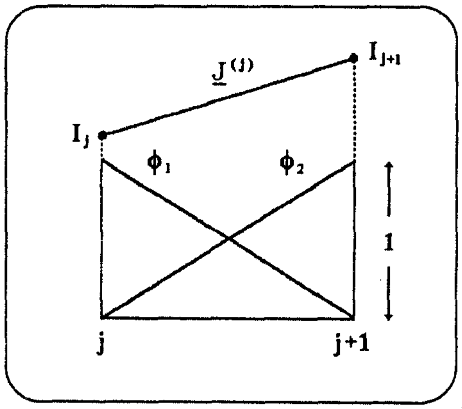


Figure 1. Current expansion over an element

To specialize the moment methods to eq. (1) based on element scheme, wall current $\bar{J}$ is superposed by the jth element current $\bar{J}^{(j)}$ over an element segment in terms of chosen linear basis function set $\{\phi_1, \phi_2\}$, as shown in Fig. 1.

$$\underline{J} = \sum_{j=1}^{N} \underline{J}^{(j)} \qquad (4)$$

where

$$\underline{J}^{(j)} = \hat{u}_j[I_j^{(j)}\phi_1 + I_{j+1}^{(j)}\phi_2] \qquad (5)$$

$\hat{u}_j$ denotes the unit circumferential vector tangent to a boundary C in the jth element, $I_j^{(j)}$ and

$I_{j+1}^{(j)}$ are coefficients at node point j and j+1 belong to the same jth element. Substituting eq. (4) into (1) and taking test with each $\phi_1$ and $\phi_2$ on the ith element of the contour gives

$$(\hat{u}_i \cdot \hat{u}_j)\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_j^{(j)} \\ I_{j+1}^{(j)} \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (6)$$

where

$$Z_{mn} = \oint_c \phi_m L(\phi_n)\, dl \quad (7)$$

$$V_1 = \oint_c (\hat{u}_i \cdot \underline{E}^{inc})\phi_1 dl \quad (8)$$

$$V_2 = \oint_c (\hat{u}_i \cdot \underline{E}^{inc})\phi_2 dl \quad (9)$$

In eq. (7), m and n are equal to 1 or 2 depending on the index of Z-matrix element. It is noted that two respective testing on each element have been required due to two unknown coeffcients in eq. (5). Also note that, in eq. (7), (8), and (9), integration interval has been restricted to an element segment reduced in half compared to those of node scheme for shown linear basis function in Fig. 1. Evaluation of these integrals may be performed following standard procedure encountered in ordinary moment methods, but detailed calculations are not shown in here for brief presentation. This element based integrals enables us to treat the multiply connected cross sections quite systematically when combined with matrix assemby, since one pays attention only to element integrals not to other integrals straddled with different element numbers as appeared in the node scheme.

To construct a complete system matrix, a current continuity condition at each node sharing an interelement, known as interelement boundary condition, is applied. Applying this interelement boundary condition is called matrix assembly in finite element methods, which is written as[3]

$$I_{j+1}^{(j)} = I_{j+1}^{(j+1)}. \quad (10)$$

Imposing the constraint condition eq. (10) to (6) simply adds rows i and j and columns i and j in the Z-matrix, and rows i and j in the V-matrix. The assembled matrix is shown in eq. (11)

$$[Z_{ij}][I_j] = [V_i] \quad (11)$$

where index i and j run from 1 to total number of node, and matrix element $Z_{ij}$ and $V_i$ are given in eq. (12) and (13), respectively.

$$\begin{aligned} Z_{ij} &= Z_{ij} + Z_{11} \\ Z_{i,j+1} &= Z_{i,j+1} + Z_{12} \\ Z_{i+1,j} &= Z_{i+1,j} + Z_{21} \\ Z_{i+1,j+1} &= Z_{i+1,j+1} + Z_{22} \end{aligned} \quad (12)$$

and

$$\begin{aligned} V_i &= V_i + V_1 \\ V_{i+1} &= V_{i+1} + V_2. \end{aligned} \quad (13)$$

In eq. (12), the four Z-matrix elements are defined by eq. (7). The system matrix in eq. (11) obtained by element scheme is just equal to that matrix obtained by ordinary moment methods.

## Ⅲ. ILLUSTRATION

To treat junctions, for instance waveguide of a septum with zero thickness shown in Fig. 2, Kirchhoff's current law(KCL) should be satisfied at each junction point. It indicates that three expansion functions at a junction, in Fig. 2, can not be treated separately under the KCL condition. Hence expansion function from DEG and DEB is overlapped to satisfy the KCL at junction point[5]. This overlapping makes the current at point E equal to the sum of two currents at point A and F. The headed arrow indicates the positive direction of current. The waveguide shown in Fig. 2 supports dominant mode $TE_{\frac{1}{2},1}$ of exact cutoff wavenumber 1.166, whereas the numerical solution was 1.1843 for matrix size of 40. The numerically computed wall curent of this dominant mode also well agreed with the known exact solution.

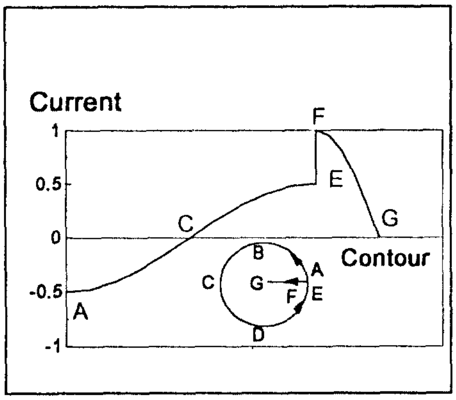


Figure 2. Wall current of septum waveguide

Fig. 3 shows the radar cross section for a conducting quadridge cylinder, for which a plane wave incidence has been assumed and the scattered field has been found in terms of boundary current calculated by element scheme, by using (14)[6]

(14)

In Fig. 3, ka and kL are equal to unity and 0.5 respectively, and the quadridge has the equal length L of arms. The incident plane wave is impinging on $\phi$ = 180°. Calculated radar cross section shows symmetry pattern along circumferential direction due to geometrical symmetry in $\phi$ axis, as expected.

## Ⅳ. CONCLUSION

To facilitate the analysis of multiply connected cross sections, element scheme in moment mothods has been introduced. This scheme showed its usefulness especially for the structures with junctions by allowing random node numbering feature and simplified integral formulations. Its validity was tested by calculating waveguide modes of a septum waveguide and radar cross section of a quadridge cylinder, based on employed element scheme.

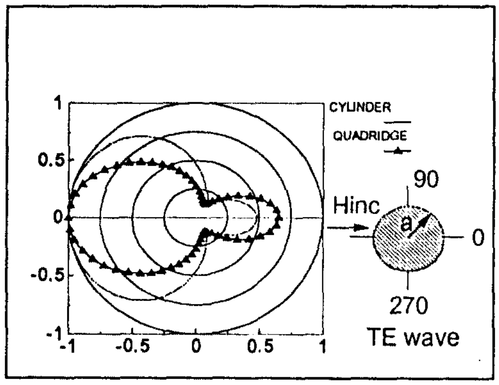


Figure 3. RCS of conducting quadridge cylinder

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