論文 95-10-10-6

Integral Nulling for Linearly Constrained Antenna Arrays in the Presence of Random Errors

Byong Kun Chang* Regular Members

불규칙 오류가 존재하는 선형조건이 주어진 안테나 어레이를 위한 적분영점 방법

正會員 張 炳 健*

ABSTRACT

The concept of optimum pattern integral in an linear array with equispaced antennas subject to random variations of array weight and antenna position has been introduced in a constrained broadband null synthesis problem. To this end, the expected integration of perturbed array factor and its power response with respect to spatial variable are analyzed and used in finding an optimum weight vector to eliminate broadband interferences. Geometery of the optimum weight vector is discussed in the weight vector space. Computer simulation results are presented to show the average array performance for different values of random errors.

要 約

최적적분 패턴의 개념이 어레이 계수나 안테나 위치의 불규칙 오류가 있는 균일분포 선형 안테나 어레이에서 제약조건이 있는 경우에 광대역 영점을 형성하는 문제에 사용하는 것을 소개하였다. 이를 위하여 무작위 오류가 개입된 어레이 팩터(array factor)와 전력 응답을 공간 변수에 의하여 분석하고, 광대역 방해신호를 제거 하기위한 최적계수 벡터를 발견하는데 사용하였다. 최적 계수 벡터의 기하학적 모형을 계수벡터 공간에서 토의했고, 다른 불규칙 오류들에 대한 어레이의 평균적 성능을 컴퓨터 실험결과를 통하여 제시하였다.

* 인천대학교 전기공학과
Department of Electrical Engineering.
論文番號 : 95038-0203
接受日字 : 1995年 2月 3日

1. Introduction

When random variations of array weight, element (i.e., antenna) position, or incoming signal wavefront exist, array performance is expected to be degraded. It was shown [1,2] that for an equispaced endfire array subject to random variations of array weight and element position, the expected directivity or signal-to-noise ratio (SNR) improves as the random errors decrease. It was also shown [3] that the expected power pattern of an arbitray antenna array with directional element, which is subject to random variations of weights and element positions above their mean values, resulted in a nominal (i.e., with no random errors) power pattern superimposed by a power level which is proportional to the power pattern of directional element. This proportionality depends on the product of the sum of variance of the relvant random errors and the sum of the squared value of the expected weights. Using this result, the problem of maximizing directivity has been discussed with a constraint to the susceptibility of the power pattern to the random errors. Here, it was assumed that the distribution of the relevant random variations is statistically independent and normal with mean zero and spherically symmetric. If the correlations existing among weights and mutual coupling between elements are assumed to be negligible, the assumption of statistically independence is reasonable [4].

In this paper, a pattern integral is used in an optimal sense to achieve a broadband gain response in a linear array with equispaced isotropic elements which is subjected to random variations of array weights and element positions. The expectation of the integral of array factor and power response are analyzed and employed to find an optimum weight vector which generates a broadband gain response in a specified spatial region. A narrowband gain within a specified region, pattern derivative or integral as a linear constraint may be used [5-7] to achieve a braodband gain response. Without loss of generality, it is assumed that: the expected element positions are confined to a one-dimensional space; the directions of incoming signals are confined to a two-dimensional space; and the element positions vary randomly in a three-dimensional space.

2. Randomly Perturbed Linear Array

Consider a narrowband linear array with N equispaced isotropic elements on the x-axis in a three-dimensional space. Each element is followed by a complex weight as shown in Fig. 1. Assume that the array is subjected to independent random variations of the array weights and element positions. Then we have the array weights and element positions as

$$w_n = c_n + \kappa_n \qquad (1)$$

and

$$d_n = ndu_x + \rho_n , \quad \text{for } 1 \le n \le N, \qquad (2)$$

where c_n are the expected values of randomly perturbed complex weights w_n; ndu_x are the expected element positons; d is inter-element spacing with no random error; d_n is randomly perturbed element spacings; u_x is a unit vector on the x-axis; ρ_n are independent complex random variables with mean zero; and ρ_n are independent random vectors with all having the same statistical distributions. If the array is free of random variations of array weight and element position, the array factor is given by

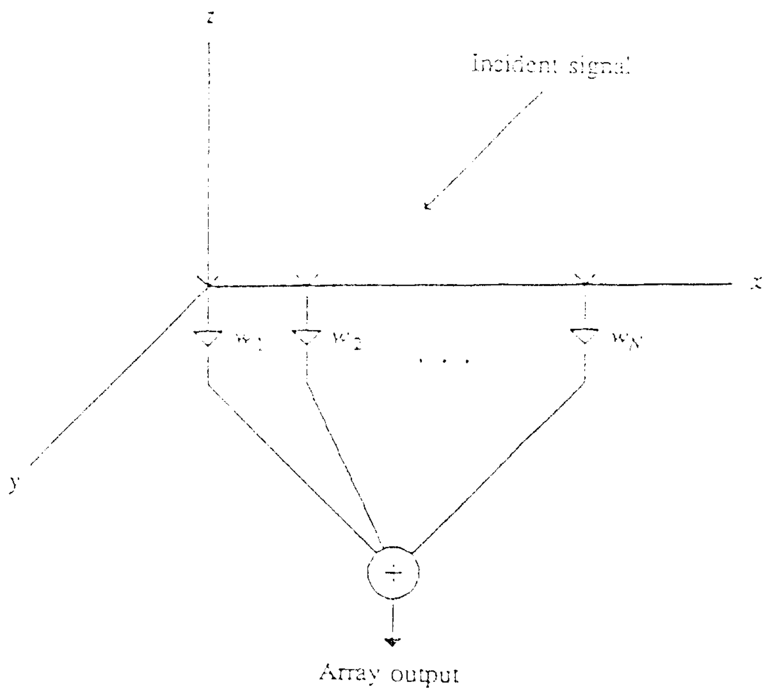


Fig. 1. Narrowband linear array.

$$H_n(u,f) = p^H c, \tag{3}$$

where c is given by

$$c = [c_1\ c_2\ \cdots\ c_N]^T$$

and (4)

$$p = [e^{jafdu}\ \cdots\ e^{jafNdu}]^T, \tag{5}$$

where $u = \cos\theta$, θ is the angle from the line of elements; f is frequency variable; $a=2\pi/\nu$, ν is wave propagation velocity; $j=\sqrt{-1}$, and $*$, T, and H denote complex conjugate, transpose, and complex conjugate transpose, respectively.

Also, the nominal power response of the array is given by

$$|H_n(u,f)|^2 = c^H P\ c, \tag{6}$$

where the nth row and mth column component of the $N\times N$ matrix P is given by

$$[P]_{nm} = e^{jaf(n-m)du},\ \text{for } 1\le n,\ m\le N. \tag{7}$$

In practice, some errors by the random variations of array weights and element positions are expected due to array imperfections resulting from manufacturing process or external circumstances [8]. As a result, the nominal array factor and power response are affected such that the p vector and P matrix are perturbed in some ways. We assume that: distribution of $\rho_n=(\rho_{xn}, \rho_{yn}, \rho_{zn})$ is independent of n ; its distribution is spherically symmetric; and each cartesian component of ρ is of independent normal distribution with mean zero and variance $\sigma^2/3$. If random variations are present with respect to array weight and element position, the array factor can be expressed as

$$H(u,f)=\sum_{n=1}^{N} w_n e^{ja f(nd u+\rho_{xn} u+\rho_{zn}\sqrt{1-u^2})}. \quad (8)$$

It can be shown that the expected array factor is expressed in terms of the nominal array factor as

$$E\,[H(u,f)] = \frac{1}{\sqrt{1+\delta^2}} H_n(u,f). \quad (9)$$

where

$$\delta^2 = e^{\frac{(af)^2\sigma^2}{2}} - 1. \quad (10)$$

In taking the exepectation of perturbed power response, we assume that the variance of x_n is proportional to the power of c_n wih same raito of γ^2 for all n. This implies that if the element has an ohmic resistance, the variance of its weight variation is proportional to the amount of power loss consumed in the element. Thus the variance of x_n can be express as

$$\sigma_{xn} = \gamma^2 |c_n|^2, \text{ for } 1 \le n \le N. \quad (11)$$

Then the expected power response is given by

$$E\,[|H(u,f)|^2] = \frac{1}{1+\delta^2}|H_n(u,f)|^2 + \left[\gamma^2 + \frac{\delta^2}{1+\delta^2}\right]\sum_{n=1}^{N}|c_n|^2. \quad (12)$$

3. Expectations of Integral Array Factor and Power Response

The integral of the nominal array factor is

$$\int_{u_l}^{u_u} H_n(u,f)\,du = \sum_{n=1}^{N} c_n \frac{e^{-jafndu_u} - e^{-jafndu_l}}{-jafnd}, \quad f \neq 0, \quad (13)$$

where $u_l = \cos\theta_l$ and $u_u = \cos\theta_u$ are the lower and upper limits of a specified angular region. Note that the subscript u denotes the upper limit. Now, the integration of the array factor of a randomly perturbed linear array is given by

$$\int_{u_l}^{u_u} H(u,f)\,du = \sum_{n=1}^{N} w_n \left\{ \frac{e^{-jaf(ndu_u+\rho_{xn}u_u+\rho_{zn}\sqrt{1-u_u^2})}}{-jafnd\left[1+\rho_{xn}u_u+\rho_{zn}\frac{u_u}{\sqrt{1-u_u^2}}\right]} - \frac{e^{-jaf(ndu_l+\rho_{xn}u_l+\rho_{zn}\sqrt{1-u_l^2})}}{-jafnd\left[1+\rho_{xn}u_l+\rho_{zn}\frac{u_l}{\sqrt{1-u_l^2}}\right]} \right\}. \quad (14)$$

If it is assumed that ρ_{xn} and ρ_{zn} are small compared to element spacing d, the denominator in (14) can be approximated using the following simple approximation for a binomial series.

$$(1+x)^{-1} \approx 1-x \quad \text{for } |x|<1. \quad (15)$$

Taking the expectatoin of the approximated version of (14), we get the following relationship.

$$E\left[\int_{u_l}^{u_u} H(u,f)du\right] \approx \frac{1}{\sqrt{1+\delta^2}}\int_{u_l}^{u_u} H_n(u,f)du. \quad (16)$$

Also, the expectatoin of perturbed power response can be represented as

$$E\left[\int_{u_l}^{u_u}|H(u,f)|^2 du\right] \approx \frac{1}{1+\delta^2}\int_{u_l}^{u_u}|H_n(u,f)|^2 du + \left[\frac{\delta^2}{1+\delta^2}+\gamma^2\right](u_u-u_l)\sum_{n=1}^{N}|c_n|^2 = c^H \hat{P} c, \quad (17)$$

where the nth row and mth column component of the $\hat{P}$ matrix is given by

$$[\tilde{P}]_{nm} = \begin{cases} (1+\gamma^2)(u_u - u_l) & \text{for } n=m \\ \dfrac{e^{jaf(n-m)du_u} - e^{jaf(n-m)du_l}}{jaf(n-m)d(1+\delta^2)} & \\ \text{for } n \neq m \quad 1 \leq n, m \leq N. \end{cases} \tag{18}$$

Based on the above results, we derive an optimum weight vector for a set of broadband nulls for eliminating jamming signals with a constraint gain at the direction of a desired signal.

4. Optimum Weight Vector

We consider the case of K optimum integral gains of α_k at (u_{ok}, f_{ok}), $1 \leq k \leq K$ with a constraint gain β at (u_c, f_c). It is assumed that α_k and β are positive real numbers with the range of $0 \leq \alpha_k$, $\beta \leq 1$. The error between the desired gain and the related array factor at the kth point is given by

$$\epsilon(u_{ok}, f_{ok}) = \alpha_k - p_{ok}^H w, \tag{19}$$

where

$$w = [w_1 \ w_2 \ \cdots \ w_N]^T, \tag{20}$$

$$p_{ok} = \left[e^{jaf_k(du_{ok} + \rho_{1xk}u_{ok} + \rho_{1zk}\sqrt{1-u_{ok}^2})} \ \cdots \right.$$
$$\left. e^{jaf_k(Ndu_{ok} + \rho_{Nxk}u_{ok} + \rho_{Nzk}\sqrt{1-u_{ok}^2})} \right]^T, \tag{21}$$
$$\text{for } 1 \leq k \leq K,$$

$u_{ok} = \cos\theta_{ok}$, θ_{ok} is the angluar variable of the kth optimum integral gain, and ρ_{nxk} and ρ_{nzk} denote the x and z components of ρ_n at the kth point, respectively.

The constrained optimization problem of minimizing the expected integration of the squared errors within the angluar region of u_{okl} and u_{oku} with a constraint gain β at (u_c, f_c) can be formulated as

$$\min_{c} \ E\left[\sum_{k=1}^{K} \int_{u_{okl}}^{u_{oku}} |\epsilon(u_{ok}, f_{ok})|^2 du_{ok} \right]$$

$$\text{subject to} \quad \tilde{p}_c^H c = \sqrt{1+\delta^2}\,\beta, \tag{22}$$

$$\tilde{p}_c = [e^{jaf_c du_c} \cdots e^{jaf_c N du_c}]^T. \tag{23}$$

Using the method of Lagrage multipliers [9] and assuming that c and its complex conjugate c* are independent each other [10], we have the optimum weight vector as

$$c_{u,opt} = \frac{1}{\sqrt{1+\delta^2}} \left[I - \frac{P_o^{-1}\tilde{p}_c \tilde{p}_c^H}{\tilde{p}_c^H P_o^{-1} \tilde{p}_c} \right] P_o^{-1} \tilde{p}_o \alpha$$
$$+ \frac{\beta\sqrt{1+\delta^2} P_o^{-1} \tilde{p}_c}{\tilde{p}_c^H P_o^{-1} \tilde{p}_c}. \tag{24}$$

where

$$\alpha = [\alpha_1, \alpha_2 \ldots \alpha_K]^T, \tag{25}$$

$$\tilde{p}_o = [\tilde{p}_{o1} \ \tilde{p}_{o2} \ \cdots \ \tilde{p}_{oK}], \tag{26}$$

$$\tilde{p}_{ok} = \left[\frac{e^{jafdu_{oku}} - e^{jafdu_{okl}}}{jafd} \ \cdots \right.$$
$$\left. \frac{e^{jafNdu_{oku}} - e^{jafNdu_{okl}}}{jafNd} \right]^T. \tag{27}$$

$$P_o = \frac{1}{\sqrt{1+\delta^2}} \tilde{P}_o + \left[\frac{\delta^2}{1+\delta^2} + \gamma^2 \right]$$
$$\sum_{k=1}^{K} (u_{oku} - u_{okl}) I. \tag{28}$$

$$\tilde{P}_o = \begin{cases} \sum_{k=1}^{K} (u_{oku} - u_{okl}) & \text{for } n=m \\ \sum_{k=1}^{K} \dfrac{e^{jaf_k(n-m)du_{oku}} - e^{jaf_k(n-m)du_{okl}}}{jaf(n-m)d} & \\ \text{for } n \neq m \quad 1 \leq n, m \leq N, \end{cases} \tag{29}$$

and I is an $N \times N$ identity matrix.

It is assumed that P_o is nonsingular. It can be shown that the unconstrained optimum weight vector is given by

$$c_{u,opt} = \frac{1}{\sqrt{1+\delta^2}} P_o^{-1} \tilde{p}_o \alpha. \tag{30}$$

Equation (30) will be used to analyze the geometry of the constrained optimum weight vector.

5. Geometry of Optimum Weight Vector

The vector space which does not include the zero vector is called 'affine subspace', which is translated by a fixed vector from the origin [11]. To find the translation from the origin to the constrained surface, which is an affine subspace, we formulate the following constrained optimization problem.

$$\min_{c} \; c^H c$$
$$\text{subject to} \quad \tilde{p}_c^{\,H} c = \sqrt{1+\delta^2}\,\beta. \tag{31}$$

Using the method of Language multipliers, we get the optimum solution as

$$c_t = \frac{\sqrt{1+\delta^2}\,\beta}{N}\,\tilde{p}_c, \tag{32}$$

which is perpendicular to the constrained surface. To find the geometrica property of the optimum weight vector in (24), we denote the $N \times N$ projection matrix as J, i.e.,

$$J = I - \frac{P_o^{-1}\tilde{p}_c\,\tilde{p}_c^{\,H}}{\tilde{p}_c^{\,H} P_o^{-1}\tilde{p}_c}. \tag{33}$$

It can be shown that J is idempotent, i.e., $J = J^2$, but it is not Hermitian. Thus, J is a nonorthogonal projection matrix which projects a vector onto the range of J. If we define a subspace S_o as

$$S_o = \{c\,;\quad \tilde{p}_c^{\,H} c = 0\}, \tag{34}$$

S_o is parallel to the constrained surface S_a. Thus we have the following relationship.

$$S_a = \{c + c_t;\ c \in S_o\}. \tag{35}$$

Let y be the range of J. Since y satisfies

$$\tilde{p}_c^{\,H} y = 0, \tag{36}$$

any vector projected by J is on the S_o. Let d be given by

$$d = \frac{\beta\sqrt{1+\delta^2}\,P_o^{-1}\tilde{p}_c}{\tilde{p}_c^{\,H} P_o^{-1}\tilde{p}_c}. \tag{37}$$

Then the constrained optimum weight vector in (24) can be written as

$$c_{c,opt} = J c_{u,opt} + d. \tag{38}$$

From (38), it is shown that the unconstrained optimum weight vector is nonorthogonally projected onto the subspace S_o by J. Then the projected vector is added by d to yield the constrained optimum weight vector. Since d satisfies the constraint, so does $c_{c,opt}$.

6. Simulation Results

A linear array with equispaced isotropic elements has been simulated to find the performance of the integral null. It is assumed that the element spacing is half the wavelength of array center frequency and the frequencies for the constraint gain and the region of optimum integral gains are the same as the array center frequency. Also, It is assumed that the random variation of the element positions is small and thus δ^2 is negligible such that sqrt$\sqrt{1+\delta^2}$ in (22) is approximated as 1. The constraint gain is assumed to be unit in the direction of a desired signal. Thus the optimum weight vector is given by

$$c_{c,opt} = \frac{P_o^{-1}\tilde{p}_c}{\tilde{p}_c^{\,H} P_o^{-1}\tilde{p}_c}. \tag{39}$$

Multiple integral nulls are experimented for the case of three integral nulls at 50°-52°, 110°-112°, and 150°-152° as displayed in Fig. 2 for $\delta^2+\gamma^2 = 0.001$. It is observed that the null depths are approximately-60 ㏈. It is shown that the null width and depth become narrower and not deeper as the null gets closer to the location of the constrained direction (i.e., θ = 90°). Two very broadband nulls of width 85° each have been formed symmetrically with respect to the broadside direction using two integral nulls over 0°-85° and 95°-180° in a 41-element linear array for $\delta^2+\gamma^2$ =0.001 as shown in Fig. 3, where the beam pattern of uniform broadside linear array with no random errors is overlapped for comparison. It is observed that the sidelobe level has been remarkably reduced by about 23 dB compared with that of the broadside linear array while the main beam width becomes broader. The null depth is shown to be approximately -50 dB over the angular region related to the two integral nulls.

Also, the performance of the integral null has been evaluated by processing a narrowband desired signal and two narrowband jamming signals which are assumed to be all complex. The real parts of the complex envelope representations of the desired and two jamming signals have been generated by processing three independent white Gaussian random sequences through an 8th-order Butterworth bandpass filter. The lower and upper cutoff frequencies are 19.5 Hz and 20.5 Hz respectively, when the sampling frequency is 160 Hz. The imaginary parts have been generated by performing Hilbert transform on the real part signals. The complex envelopes of the desired and two jamming signals are given by

$$s(k)=a_s(k)e^{j(2\pi f_c kT_c+\phi_s(k))} \quad (40)$$

and

$$r_i(k)=a_{ri}(k)e^{j(2\pi f_c kT_c+\phi_{ri}(k))}, \text{ for } i=1, 2, \quad (41)$$

where f_c = 20 Hz, T_s is the sampling inteval of 1/160 second, $E[a_s^2(k)]=0.01$, $E[a_{ri}^2(k)]=0.1$, $\phi_s(k)$ and $\phi_{ri}(k)$, i = 1, 2 are the phases of the desired and jamming signals respectively, and k is a discrete time index. The phases

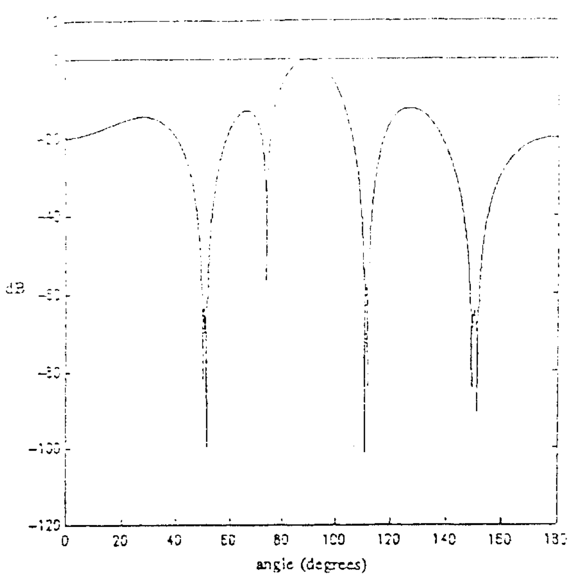


Fig. 2. Beam pattern with three integral nulls over 50°-52°, 110°-112°, and 150°-152° for r=0.001 in an 8-element linear array.

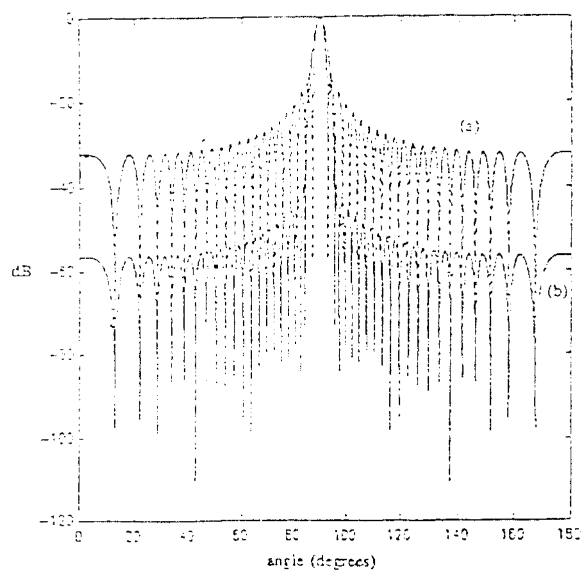


Fig. 3. Overlapped beam patterns for a 41-element linear array: (a) uniform broadside with no error (b) linear array with two optimum integral nulls over 0°-85° and 95°-180° for r=0.001

are assumed to be zero in the simulation. The constraint unit gain is set at the broadside direction and a broadband null has been formed between 48° and 54° using an integral null. The desired signal is incident at the broadside direction and two jamming signals are incident at 50° and 52° to simulate a broadband jamming environment. To generate the signals with noninteger samples of time delay, the interpolation method in [12] was used. After processing the generated input signals through an optimized 4-element linear array, the real part of the complex output signal was plotted in Fig. 4 with that of the desired signal and output signal from a broadside uniform linear array for $\delta^2+\gamma^2$ = 0.01 and 0.001. The corresponding estimation error signals, which are given by subtracting the real part of the output signal from that of s(k), are shown in Fig. 5. It is observed

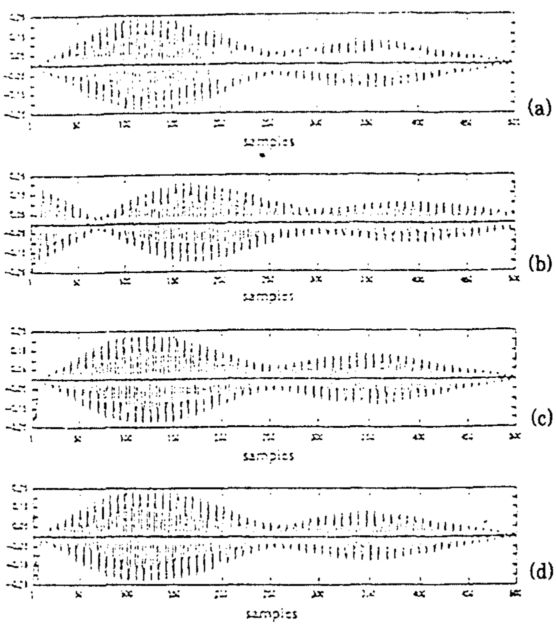


Fig. 4. (a) Desired signal; output signals by (b) broadside linear array (c) an integral null with r=0.1 (c) with r=0.0001.

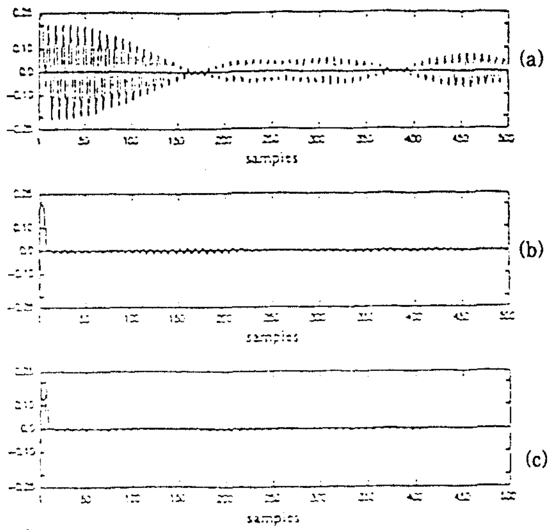


Fig. 5. Error signals; (a) broadside linear array (b) an integral null with r=0.1 (c) with r=0.001

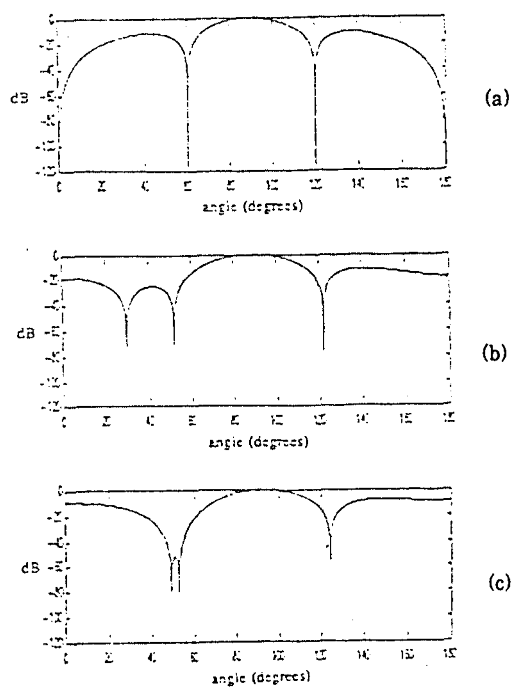


Fig. 6. Beam patterns; (a) broadside linear array (b) an integral null with r=0.1 (c) with r=0.001

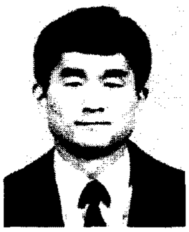
that the two jamming signals are successfully eliminated with smaller random errors. The estimation error decreases for smaller random error while the array with integral nulls performs much better than the uniform broadside linear array. The beam patterns of the broadside linear array and the array with integral null are shown in Fig. 6.

7. Conclusions

The pattern integral was introduced in a randomly perturbed linear array with equispaced isotropic element to synthesize a broadband null. The effects of random variations of array weights and element positions are analyzed with respect to the expected integrations of array factor and power response. It was assumed that the random variation of element positions is small compared to the element spacing and also the random variations of array weights and element positions are independently distributed. The nulling performance improved as the random variations decreased such that a broadband jamming signal was successfully eliminated. It was demonstrated that the optimum integral null performed well in forming a broadband null to counteract incoming broadband jamming signals.

REFERENCES

1. F. I. Tseng and D. K. Cheng. "Gain optimization for arbitrary antenna arrays subject to random fluctuations," IEEE Trans. Antennas Propagat., vol. AP-15, no. 3, pp.356-366, May. 1967.
2. D. K. Cheng and F. I. Tseng, "Optimum spatial processing in a noisy environment for arbitrary antenna arrays subject to random errors," IEEE Trans. Antennas Propagat., vol. AP-16, pp.164-171, March. 1968.
3. E. N. Gilbert and S. P. Morgan, "Optimum design of directive antenna arrays subject to random variations," Bell Syst. Tech. J., 34, pp.637-663, May. 1955.
4. L. L. Bailin and M. J. Ehrlich, "Factors affecting the performance of linear arrays," I.R.E Trans., vol. PGAP-1, pp.85-106, February. 1952.
5. H. Steyskal, "Synthesis of antenna patterns with prescribed nulls," IEEE Trans. Antennas Propagat., vol. AP-30, no. 2, pp.273-279, March. 1982.
6. M. H. Er and A. Cantoni, "Derivative constraints for broad-band element space antenna array processors," IEEE Trans. Acoust., Speech, Signal Processing, vol. ASSP-31, no. 6, pp.1378-1393, December. 1983.
7. B. K. Chang and N. Ahmed, "Adaptive broadband beamforming: an integral nulling approach," Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc., Glasgow, Scotland, pp.2776-2778, April 1989.
8. J. Ruze, "The effect of aperture errors on the antenna radiation pattern," Nuovo Cimento, vol. 9, no. 3, pp.364-370, 1952.
9. O. L. Frost, Ⅲ, "An algorithm for linearly constrained adaptive array processing," Proc. IEEE, vol. 60, no. 8, pp.926-935, August 1972.
10. Claerbout, Fundamental of Geophysical Data Processing: With Applications to Petroleum processing, McGraw-Hill, New York, 1976.
11. J. M. Ortega, Matrix Theory, Plenum Press, New York, 1987.
12. D. H. Youn, N. Ahmed, and G. C. Carter, "A method for generating a class of time-delayed signals," Proc. IEEE Int. Conf. Acoust., Speech, and Signal Proc., Atlanta, GA, March. 1981.



張 炳 健(Byong Kun Chang) 정회원

1975년 2월 : 연세대학교 전자공학과(공학사)
1985년 5월 : 미국 Iowa 주립대 전기공학과(공학석사)
1991년 5월 : 미국 New Mexico 주립대 전기공학과(공학박사)
1990년 8월~1994년 2월 : 미국 Nevada 주립대 전기공학과 조교수
1994년 3월~현재 : 인천대학교 전기공학과 부교수