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Analysis of Decorrelating Detector in the Presence of the Residual MAI

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殘餘 다원접속간섭을 고려한 逆相關 검출기의 성능분석

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※본 논문은 정보통신연구관리단의 대학기초연구지원사업의 지원에 의하여 수행한 연구임.

ABSTRACT

This paper analyzes decorrelating detector for synchronous packet CDMA communications where a set of quasiorthogonal code waveforms are generated from a common code by assigning distinct initial code phases to all users. In this analysis, we characterize the residual multiple-access interference (MAI) caused by possible timing offsets when synchronous and simultaneous packet transmissions are on the reverse link of centralized networks. Also, to show feasibility of decorrelating detector employing the common code, we further investigate its robustness against the multipath channel. It is demonstrated that the decorrelating detector greatly reduces the residual MAI to the order of N^{-2}, N number of chips/bit, and yields significant performance gain compared to the single user detector.

요 약

본 논문은 모든 시스템 사용자에게 상이한 초기 부호 위상을 할당하여 단일 확산부호에서 의사직교 부호파형의 집합을 생성하는 패킷 동기식 CDMA 통신을 위한 역상관 검출기를 제안하고 성능 분석을 수행한다.

집중형 통신망의 역방향 링크에서 동시에 동기식으로 전송되는 여러 패킷이 존재하는 경우 가능한 시간 오프셋을 고려하여 이에 의한 잔여 다원접속간섭(MAI)의 통계적 특성을 평가한다.

아울러, 단일 부호를 채택는 역상관 검출기의 구현 가능성을 보이기 위해 다경로 채널에서 제안된 검출기의 robustness 를 조사한다.

성능분석을 통하여 제안된 역상관 검출기는 종래의 단일 사용자 검출기에 비해 N^{-2}에 반비례하여 잔여 다원접속간섭을 감소시키므로써 상당한 성능이득을 얻을 수 있음이 입증된다.

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Ⅰ. Introduction

Motivated by the need to increase capacity in code-division multiple-access (CDMA) channels, much interests focused on the development of multiuser detection which provides a significant performance gain by exploiting the structure of the multiuser interference. Initially, the optimum multiuser detector was obtained which consists of a bank of matched filters and a (vector) Viterbi decoder [1]. It was shown to yield the best possible performance achievable in Gaussian noise channels and be near-far resistant under dissimilar received energies, but requires computational complexity which is exponential in the number of users. In order to reduce the exponential complexity while maintaining superior performance over the conventional single-user detector, several multiuser detectors were proposed and analyzed, one of which is the decorrelating detector of Lupas and Verdu [2], [3] whose complexity is only linear in the number of users, and whose error probability is independent of the interfering signal powers.

In this paper, we analyze the decorrelating detector based on common code scheme requiring distinct initial code phases for all users in synchronous packet CDMA channels where their symbol epochs roughly coincide at a single central receiver. In such synchronous channels with some timing offsets among users, low sidelobe values for all possible distinct code phases may be useful in case of decorrelating the sum of the simultaneously transmitted signals because of the residual MAI. We consider a slotted packet network in which the assignment of distinct code phases is made through the reservation using a request packet before transmitting a packet of information. The request packet will be transmitted in random access (slotted ALOHA) mode in a randomly selected reservation slot while the packet of information transmitted in synchronous CDMA channels without contention. With this network model, we may exploit the side information on the channel delays acquired from those request packets in properly assigning initial code phases to be used for transmission of the following packets of information.

Ⅱ. Multiuser Communication Model

We consider the reverse link of centralized networks in which K user terminals communicate with a single central receiver through synchronous packet CDMA channels. We here assume near-synchronous channels at the symbol level to account for the possible timing offsets among all user transmissions. A set of quasi-orthogonal signature waveforms is generated from a common code by differentiating its initial code phases. The common code waveform can be expressed by

$$c(t)=\sum_{j=0}^{N-1} c_j\phi(t-jT_c),\ t\in[0,T) \tag{1}$$

where $\{c_j\}$ denotes a signature sequence of elements taking values of ±1, $\phi(t)$ is a chip waveform of duration T_c with unit power, i.e., $\int_0^{T_c}\phi^2(t)dt=T_c$, and T is the bit duration assumed to be equal to NT_c for a code period N. By circularly shifting the initial code phase, we can generate the kth user's signature waveform to encode the packet of information in a given slot, that is,

$$c_k(t)=\sum_{j=0}^{N-1} c_{j\oplus n_k}\phi(t-jT_c),\ t\in[0,T),\ k=1,2,\ldots,K. \tag{2}$$

Here $j\oplus n$ is defined by $j+n$ modulo N, assuming $n_k\neq n_{k'}$ for $k\neq k'$.

The kth user's signature waveform $c_k(t)$ can be related to the common code waveform $c(t)$ through the formula

$$c_k(t)=c(t\oplus n_kT_c),\ t\in[0,T),\ k=1,2,\ldots,K \tag{3}$$

where $t\oplus\tau$ is equivalent to $t+\tau$ modulo T.

If all users transmit their packets of information using the quasiorthogonal signature waveforms $\{c_k(t)\}$ through roughly symbol-synchronous CDMA channels,

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the input signal at the central receiver can be written as

$$r(t)=\sum_{l=-L}^{L}\sum_{k=1}^{K} b_k(l)s_k(t-lT-\delta_k)+n(t) \quad (4)$$

where $\{b_k(l); l=-L,...,L\}$ represents the sequence of information bits of the kth user, assumed i.i.d. random variables taking values of ± 1 with equal probability, during the packet of length $2L+1$, δ_k indicates the timing offset associated with the kth user, $n(t)$ is white Gaussian noise with power spectral density σ_N^2, and the time-limited bandpass signal $s_k(t-lT-\delta_k)$ is given by

$$s_k(t-lT-\delta_k)=\sqrt{2E_kT^{-1}}\,c[(t-lT-\delta_k)\oplus(n_k+w_l)T_c] \cos(\omega_c t+\theta_k),\ t\in[lT+\delta_k,(l+1)T+\delta_k). \quad (5)$$

In the above, E_k is the received energy of the kth user's signal, w_l is the random chip delay inserted to purely randomize the statistics of the residual MAI at the symbol level, ω_c is the carrier frequency, and θ_k denotes the unknown carrier phase of the kth user.

Consider the sampled output of the normalized matched filter for the lth bit of the ith user in the presence of the timing offsets $\{\delta_k\}$, which is given by

$$\begin{aligned}\tilde{y}_i(l)&=\int_{lT}^{(l+1)T} r(t)\tilde{s}_i(t-lT)dt\\ &=b_i(l)\sqrt{E_i}\,[(1-\tilde{\delta}_i)-\frac{\tilde{\delta}_i}{N}(1+\alpha_{i,i}(l))]\\ &-\frac{1}{N}\sum_{\substack{k=1\\k\neq i}}^{K} b_k(l)\sqrt{E_i}\,(1+\tilde{\delta}_k\alpha_{i,k}(l))\cos(\theta_k-\theta_i)\\ &+\tilde{n}_i(l).\end{aligned} \quad (6)$$

Here the normalized (unit-energy) signal $\tilde{s}_i(t-lT)$ is defined by $s_i(t-lT)/\sqrt{E_i}$ for which $s_i(t-lT)$ is found in (5) with $\delta_i=0$ for $k=i$, provided perfect estimation of the carrier phase is available at the receiver to focus on the effect of timing offsets. As for the common code, $\{c_j\}$ is assumed to be a maximal-length sequence because of good correlation properties, that is, $1/T\int_0^T c_k(t)c_i(t)dt=-1/N$ if $k\neq i$ and 1 if $k=i$. In the near symbol-synchronous channels, $\{\delta_k\}$ are assumed a small fraction of T_c, so $\tilde{\delta}_k=\delta_k/T_c$ being much less than one, and the partial correlation term $\alpha_{i,k}$ due to the timing offset δ_k has the expression

$$\alpha_{i,k}(l)=c_{n_i\oplus w_l}\,[c_{n_k\oplus(w_l-1)}-b_k(l-1)c_{n_k\oplus(w_{l-1}-1)}] \quad (7)$$

where $b_k(l-1)b_k(l)$ is statistically treated like $b_k(l-1)$. The normalized output noise $\tilde{n}_i(l)$ is zero-mean Gaussian with variance σ^2.

Next, in order to estimate the effect of multipath on the decorrelating detector employing the common code, if the multipath channel is modeled as one having two significant rays, the received signal $r(t)$ in (4) can be rewritten as

$$\begin{aligned}r(t)=&\sum_{l=-L}^{L}\sum_{k=1}^{K} b_k(l)s_k(t-lT)\\ &+\sum_{l=-L}^{L}\sum_{k=1}^{K}\lambda_k b_k(l)\hat{s}_k(t-lT-\tau_k)+n(t)\end{aligned} \quad (8)$$

where we have assumed that the symbol epochs of all primary signals $\{s_k(t)\}$ coincide at the receiver, i.e., symbol-synchronous channels, while the scattering signals $\{\hat{s}_k(t)\}$ are shifted by the channel delays $\{\tau_k\}$. It is also assumed that the initial code phases $\{n_k\}$ were selected to meet the collision-free condition of $\min_l|(n_k-n_{k'})T_c+lT|>\tau_k$ for all $k\neq k'$. Here λ_k represents the relative intensity of the kth scattering signal $\hat{s}_k(t-lT-\tau_k)$ which is given by (5) with (δ_k,θ_k) replaced by $(\tau_k,\hat{\theta}_k)$, respectively.

Based on this channel model, the normalized matched filter output $\tilde{y}_i(l)$ in (6) takes the form

$$\begin{aligned}\tilde{y}_i(l)=&\,b_i(l)\sqrt{E_i}-\frac{1}{N}\sum_{\substack{k=1\\k\neq i}}^{K} b_k(l)\sqrt{E_k}\cos(\theta_k-\theta_i)\\ &-\frac{1}{N}\sum_{k=1}^{K}\lambda_k b_k(l)\sqrt{E_i}\,(1+\tilde{\tau}_k\hat{\alpha}_{i,k}(l))\cos(\hat{\theta}_k-\theta_i)+\tilde{n}_i(l)\end{aligned} \quad (9)$$

where the partial correlation term $\hat{\alpha}_{i,k}(l)$ due to the channel delay τ_k is given by

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$$\hat{\alpha}_{i,k}(l) = \frac{1}{\tilde{\tau}_k} \sum_{j=0}^{\tilde{\tau}_k - 1} c_{n_i \oplus (w_l + j)} [c_{n_k \oplus (w_l - \tilde{\tau}_k + j)} - b_k(l-1) c_{n_k \oplus (w_{l-1} - \tilde{\tau}_k + j)}]. \quad (10)$$

In the above we have assumed a discrete-time channel for simple analysis such that $\{\tau_k\}$ are an integer multiple of T_c, i.e., $\tilde{\tau}_k = \tau_k / T_c$ taking an integer.

Ⅲ. The Decorrelating Detector

In synchronous CDMA channels, the multiuser detector processes the received waveform $r(t)$, $t \in [lT, (l+1)T)$ with a bank of matched filters, which produces a normalized vector of observables $\tilde{y}_i(l)$, $i = 1, \dots, K$, in (6) [2]

$$\tilde{\mathbf{y}} = \mathbf{R}\mathbf{E}^{1/2}\mathbf{b} + \mathbf{n} \quad (11)$$

where for $\delta_k = 0$ $(k = 1, \dots, K)$, i.e., symbol-synchronous, $\mathbf{E}^{1/2} = diag\{\sqrt{E_1}, \dots, \sqrt{E_K}\}$, $\mathbf{b} = [b_{(1)}(l), \dots, b_K(l)]^T$, $\mathbf{n} = [n_1(l)$, $\mathbf{n} = [n_1(l), \dots, n_K(l)]^T$ is a zero-mean Gaussian vector with covariance matrix equal to $\sigma_N^2 \mathbf{R}$ and $\mathbf{R}$ is the normalized (unit-energy) crosscorrelation matrix whose coefficients are

$$\tilde{\rho}_{i,k} = \int_{lT}^{(l+1)T} \tilde{s}_k(t - lT)\tilde{s}_i(t - lT)\,dt \quad (12)$$

$$= \begin{cases} -\dfrac{1}{N}\cos(\theta_k - \theta_i) & \text{if } k \neq i, \\ 1 & \text{if } k = i. \end{cases} \quad (13)$$

Consider the decorrelating detector which recovers the input data vector $\mathbf{b}$ in the presence of the correlated noise vector $\mathbf{R}^{-1}\mathbf{n}$ by eliminating the multiuser interference through the matrix filter $\mathbf{R}^{-1}$. Hence a set of decision variables is given by the vector

$$\mathbf{R}^{-1}\tilde{\mathbf{y}} = \mathbf{E}^{1/2}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n} \quad (14)$$

where $\mathbf{R}^{-1}\mathbf{n}$ becomes a zero-mean Gaussian vector with covariance matrix $\sigma_N^2 \mathbf{R}^{-1}$. In case of symbol-synchronous systems, the error probability of the ith user's data is simply of the form

$$P_i(\epsilon) = Q\left(\sqrt{\frac{E_i}{\sigma_N^2 (\mathbf{R}^{-1})_{ii}}}\right) \quad (15)$$

where the Q-function is defined by $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-y^2/2}\,dy$.

Now, if we account for the effect of the timing offsets $\{\delta_k\}$ on the decorrelating detector, (14) can be rewritten as

$$\mathbf{R}^{-1}\tilde{\mathbf{y}} = \mathbf{E}^{1/2}\mathbf{b} + \mathbf{R}^{-1}\mathbf{R}(\delta)\mathbf{E}^{1/2}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n}. \quad (16)$$

Here $\mathbf{R}(\delta)$ is the partial correlation matrix caused by $\delta = (\delta_1, \dots, \delta_K)$ whose coefficients are

$$\tilde{\rho}_{i,k}^{(l)}(\delta_k) = \int_{lT}^{(l+1)T} [\tilde{s}_k(t - (l-1)T - \delta_k) + \tilde{s}_k(t - lT - \delta_k)]\tilde{s}_i(t - lT)\,dt - \tilde{\rho}_{i,k} \quad (17)$$

$$= \begin{cases} -\dfrac{\tilde{\delta}_k}{N}\alpha_{i,k}(l)\cos(\theta_k - \theta_i) & \text{if } k \neq i, \\ -\tilde{\delta}_i[1 + \dfrac{1}{N}(1 + \alpha_{i,i}(l))] & \text{if } k = i. \end{cases} \quad (18)$$

Note that $\tilde{\rho}_{i,k}^{(l)}(\delta_k)$ becomes zero when $\delta_k = 0$. Thus, if the timing offsets exist, i.e., $\tilde{\rho}_{i,k}^{(l)}(\delta_k) \neq 0$ $(k = 1, \dots, K)$, then the second term in (16) forms the residual multiple-access interference degrading the performance of the decorrelating detector.

Similarly, if we account for the effect of the multipath with only two significant rays, the decorrelating detector produces the decision vector which has the form

$$\mathbf{R}^{-1}\tilde{\mathbf{y}} = \mathbf{E}^{1/2}\mathbf{b} + \mathbf{R}^{-1}\hat{\mathbf{R}}(\tau)\Lambda\mathbf{E}^{1/2}\mathbf{b} + \mathbf{R}^{-1}\mathbf{n} \quad (19)$$

where $\Lambda = diag\{\lambda_1, \dots, \lambda_K\}$ and $\hat{\mathbf{R}}(\tau)$ is the partial correlation matrix due to the channel delays $\tau = (\tau_1, \dots, \tau_K)$ whose coefficients are

$$\hat{\tilde{\rho}}_{i,k}^{(l)}(\tau_k) = \int_{lT}^{(l+1)T} [\hat{\tilde{s}}_k(t - (l-1)T - \tau_k) + \hat{\tilde{s}}_k(t - lT - \tau_k)]\hat{\tilde{s}}_i(t - lT)\,dt \quad (20)$$

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$$= -\frac{1}{N}(1+\tilde{\tau}_k\hat{\alpha}_{i,k}(l))\cos(\hat{\theta}_k-\theta_i). \quad (21)$$

Here the normalized (unit-energy) signal $\tilde{s}_k(t-lT-\tau_k)$ is simply related to $\hat{s}_k(t-lT-\tau_k)/\sqrt{E_k}$. After decorrelating process, we see that the scattering signals reduce to the residual multiple-access interference which is the second term in (19).

Ⅳ. Analysis of Residual MAI

All user terminals initiate their packet transmissions at the beginning of a slot to maintain symbol-synchronous channels, but with the centralized network considered here, the central receiver may observe the sum of those packet signals whose symbol epochs exhibit nonnegligible differential delays in the presence of the additive noise. In this near symbol-synchronous system, the decorrelating receiver will experience somehow deterioration of the multiuser performance because of the residual MAI, that is,

$$\mathbf{MAI}(\delta,\theta)=\mathbf{R}^{-1}\mathbf{R}(\delta)\mathbf{E}^{1/2}\mathbf{b} \quad (22)$$

where $\theta=(\theta_1,\ldots,\theta_K)$ implies explicitly the dependence of $\mathbf{R}^{-1}$ and $\mathbf{R}(\delta)$ on these parameters.

The worst-case assumption on the residual MAI occurs in a mean-square sense when all unknown signal phases $\{\theta_k\}$ are aligned, i.e.,

$$\max_{\theta}\mathbf{E}\{[\mathbf{MAI}(\delta,\theta)]_i^2\}=\mathbf{E}\{[\mathbf{MAI}(\delta,\theta=\theta_i\cdot\mathbf{1})]_i^2\} \quad (23)$$

for $i=1,\ldots,K$ and a unit vector $\mathbf{1}=(1,\ldots,1)$.

Based on the worst-case assumption, the matrix filter $\mathbf{R}^{-1}$ can be evaluated as follows. First, the coefficients $\tilde{\rho}_{i,k}$ of the crosscorrelation matrix $\mathbf{R}$ are simply given by $-1/N$ for $k\neq i$ and 1 for $k=i$. Denote by Δ_n the determinant of the nxn matrix having the above coefficients. By observing the structure of the matrix $\mathbf{R}$, it is easy to show that for $n=2,\ldots,K$,

$$\Delta_n=\left(1+\frac{1}{N}\right)\Delta_{n-1}-\frac{1}{N}\left(1+\frac{1}{N}\right)^{n-1} \quad (24)$$

where the initial condition becomes $\Delta_1=1$. From the above recursion, we find that the determinant of $\mathbf{R}$ has the expression

$$\Delta_K=\left(1+\frac{1}{N}\right)^{K-1}\left(1-\frac{K-1}{N}\right). \quad (25)$$

Next, we proceed to determine the matrix $\mathbf{Adj(R)}$, the adjoint of $\mathbf{R}$, whose elements are defined by

$$(\mathbf{Adj(R)})_{ik}=(-1)^{k+i}\det(\mathbf{M}_{ki}(\mathbf{R})). \quad (26)$$

Here $\mathbf{M}_{ki}(\mathbf{R})$ is the $(K-1)\mathrm{x}(K-1)$ submatrix of $\mathbf{R}$ obtained by deleting row k and column i of $\mathbf{R}$. After a few manipulations, it can be shown that

$$(\mathbf{Adj(R)})_{ik}=\begin{cases}\frac{1}{N}\left(1+\frac{1}{N}\right)^{K-2} & \text{if } k\neq i,\\ \Delta_{K-1} & \text{if } k=i.\end{cases} \quad (27)$$

Hence, the inverse of $\mathbf{R}$ is calculated directly through the formula $\mathbf{R}^{-1}=\Delta_K^{-1}\mathbf{Adj(R)}$ by combining (25) and (27).

Now, if we substitute the inverse of $\mathbf{R}$ into (22) along with $\mathbf{R}(\delta)$ where $\cos(\theta_k-\theta_i)=1$ is assumed in (18), the residual MAI in view of the ith user is equal to

$$[\mathbf{MAI}(\delta)]_i=PU_i(\delta_i)+SU_{k,i}(\delta) \quad (28)$$

where $[\mathbf{MAI}(\delta)]_i$ denotes the residual MAI according to the worst-case assumption, and

$$PU_i(\delta_i)=-\frac{\sqrt{E_i}\,b_i(l)}{\Delta_K}\left[\Delta_{K-1}\tilde{\delta}_i[1+\frac{1}{N}(1+\alpha_{i,i}(l))]\right.$$
$$\left.+\frac{1}{N^2}\left(1+\frac{1}{N}\right)^{K-2}\sum_{\substack{k=1\\k\neq i}}^{K}\tilde{\delta}_i\alpha_{k,i}(l)\right] \quad (29)$$

$$SU_{k,i}(\delta)=\sum_{\substack{k=1\\k\neq i}}^{K}\frac{\sqrt{E_k}\,b_k(l)}{\Delta_K}\left[\frac{\Delta_{K-1}}{N}\tilde{\delta}_k\alpha_{i,k}(l)+\frac{1}{N}\right.$$
$$\left.\left(1+\frac{1}{N}\right)^{K-2}\tilde{\delta}_k[1+\frac{1}{N}(1+\alpha_{k,k}(l))]\right]$$

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$$+\sum_{\substack{k=1\\k\neq i}}^{K}\sum_{\substack{j=1\\j\neq i,\,j\neq k}}^{K}\frac{\sqrt{E_j}\;b_j(l)}{\Delta_K}\frac{1}{N^2}\left(1+\frac{1}{N}\right)^{K-2}\tilde{\delta}_j\alpha_{k,\,j}(l). \tag{30}$$

In the above, we see that the first term $PU_i(\delta_i)$ is essentially due to the unknown random variables of the ith user, namely, $\tilde{\delta}_i$, $b_i(l)$ and $b_i(l-1)$. Thus, it can be interpreted as the self-interference which is different from the second term $SU_{k,\,i}(\delta)$ caused by other users. We refer to it as the inter-user interference in view of the ith user.

Conditioned on $b_i(l)$, the self-interference $PU_i(\delta_i)$ has the mean value

$$\mathbf{E}\{PU_i(\delta_i)\,|\,b_i(l)\}=-\sqrt{E_i}b_i(l)\tilde{\delta}_k\frac{\Delta_{K-1}}{\Delta_K}\;(1+\frac{1}{N}) \tag{31}$$

where we have assumed $\mathbf{E}\{\alpha_{k,\,i}(l)\}=0$ $(k=1,\dots,K)$ for large N because of the pseudo randomness of $\{c_j\}$ and the random chip delays $\{w_l\}$ varying from bit to bit. It remains to evaluate the inter-user interference $SU_{k,\,i}(\delta)$ which can be effectively modeled as the Gaussian random variable for large K. We then evaluate the second-order moment $\sigma_I^2(\delta)$

$$\sigma_I^2(\delta)=\mathbf{E}\{SU_{k,\,i}^2(\delta)\}$$

$$=\left[\sum_{\substack{k=1\\k\neq i}}^{K}E_k\tilde{\delta}_k^2\right]\cdot\frac{1}{(N+1)^2}\left(2+\frac{(N+3)^2-2(K+2)}{(N-K+1)^2}\right). \tag{32}$$

This result implies that the residual MAI caused by the timing offsets is approximately in inverse proportion to N^2 for given system parameters. When compared to that of the conventional single-user detector, we have drastically reduced MAI for larger N even in the presence of the timing mismatches at the decorrelating detector, since the conventional MAI decreases linearly with N [4].

We now turn to the performance analysis of the decorrelating detector based on the worst-case residual MAI. The probability of error at decision upon $b_i(l)$ of the decision vector $\mathbf{R}^{-1}\tilde{\mathbf{y}}$ in (16) is expressed by

$$P_i(\epsilon\,|\,\delta)=\Pr[(\mathbf{R}^{-1}\tilde{\mathbf{y}})_i\neq\sqrt{E_i}b_i(l)\,|\,\delta] \tag{33}$$

$$\cong Q\left(\sqrt{\frac{E_i(1-\tilde{\delta}_i\Delta_{K-1}/\Delta_K(1+1/N))^2}{\sigma_I^2(\delta)+\sigma_N^2\,\Delta_{K-1}/\Delta_K}}\right). \tag{34}$$

When the common code assignment having distinct code phases is employed for a single-cell CDMA network, the multipath has a crucial impact on the multiuser performance of the decorrelating detector. In this case, it is necessary to properly estimate the effect of multipath by characterizing the residual MAI that results, which is

$$\mathbf{MAI}(\tau,\,\theta,\,\hat{\theta})=\mathbf{R}^{-1}\hat{\mathbf{R}}(\tau)\Lambda\mathbf{E}^{1/2}\mathbf{b}. \tag{35}$$

Here $\hat{\theta}=(\hat{\theta}_1,\dots,\hat{\theta}_K)$ implies explicitly the dependence of $\hat{\mathbf{R}}(\tau)$ on these parameters. For simple analysis, we have assumed that the multipath channel has two significant resolvable rays with the signal intensity vector Λ.

The worst-case assumption on the residual MAI occurs in a mean-square sense when all unknown signal phases $\{\theta_k\}$ and $\{\hat{\theta}_k\}$ are aligned, i.e.,

$$\max_{\theta,\,\hat{\theta}}\mathbf{E}\{[\mathbf{MAI}(\tau,\,\theta,\,\hat{\theta}_k)]_i^2\}=\mathbf{E}\{[\mathbf{MAI}(\tau,\,\theta=\theta_i\cdot\mathbf{1},\,\hat{\theta}=\theta)]_i^2\} \tag{36}$$

for $i=1,\dots,K$.

Since the inverse of $\mathbf{R}$ remains unchanged under the worst-case assumption, substitution of both $\mathbf{R}^{-1}$ and $\hat{\mathbf{R}}(\tau)$ into (35) yields the residual MAI in view of the ith user

$$[\mathbf{MAI}(\tau)]_i=-\sum_{k=1}^{K}\frac{\lambda_k\sqrt{E_k}\;b_k(l)}{\Delta_K}\left[\frac{\Delta_{K-1}}{N}\;(1+\tilde{\tau}_k\hat{\alpha}_{i,\,k}(l)\right.$$

$$\left.+\frac{1}{N^2}\left(1+\frac{1}{N}\right)^{K-2}\sum_{\substack{j=1\\j\neq i}}^{K}(1+\tilde{\tau}_k\hat{\alpha}_{j,\,k}(l))\right] \tag{37}$$

where $[\mathbf{MAI}(\tau)]_i$ represents the worst-case residual MAI and $(\hat{\theta}_k-\theta_i)=1$ is assumed in (21).

Then the second-order moment $\hat{\sigma}_I^2(\tau)$ can be obtained

$$\hat{\sigma}_I^2(\tau)=\mathbf{E}\{[\mathbf{MAI}(\tau)]_i^2\}$$

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$$\tilde{=} \sum_{k=1}^{K} \lambda_k^2 E_k \cdot \left[\frac{1}{(N-K+1)^2} + 2\tilde{\tau}_k \left(\frac{1}{(N+1)^2} + \frac{2}{(N+1)^2(N-K+1)} + \frac{K}{(N+1)^2(N-K+1)^2} \right) \right]. \quad (38)$$

We observe the similar behavior of the residual MAI, namely, approximately the square-law inverse of N, which allows to mitigate the worse effect of multipath in case of small channel delays for larger N.

Thus, the probability of error at decision upon $b_i(l)$ of the decision vector $\mathbf{R}^{-1}\tilde{\mathbf{y}}$ in (19) becomes

$$P_i(\epsilon|\tau) \tilde{=} Q\left(\sqrt{\frac{E_i}{\hat{\sigma}_I^2(\tau) + \sigma_N^2 \Delta_{K-1}/\Delta_K}}\right). \quad (39)$$

V. Numerical Results

To investigate the error rate behavior of the decorrelating detector combined with common code assignment, we first present approximate and exact values on the second-order moments of the residual MAI caused by timing offsets or multipath in Table 1, in which the first entry corresponds to the approximation obtained from (32) and (38). Here we have used the m-sequence of period $N=127$ to compute the true second-order moment which is generated by the polynomial $g(x)=x^7+x+1$ with initial loading 1101101 [5]. It is shown that the approximation based on the randomization $\{w_l\}$ can be used with good accuracy for analysis of the decorrelating detector.

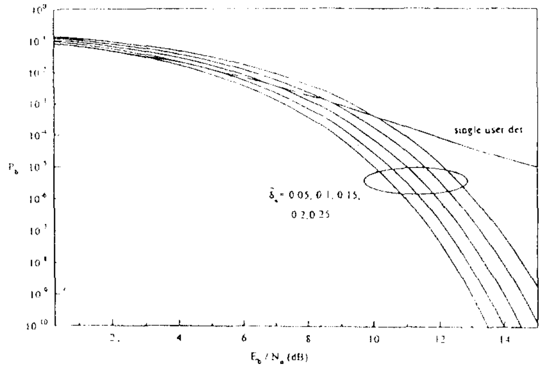

Fig 1. Error rate $P_i(\epsilon|\delta)$ against timing offsets when $K=16$, $N=127$.

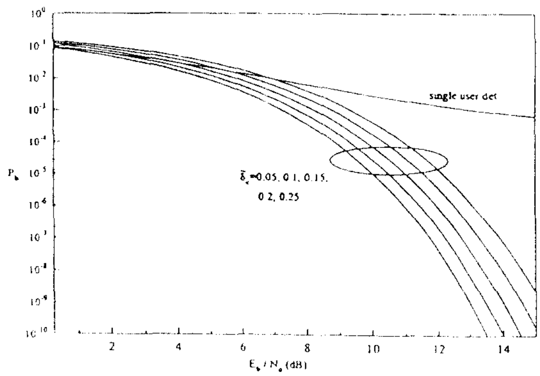

Fig. 2. Error rate $P_i(\epsilon|\delta)$ against timing offsets when $K=32$, $N=127$.

Table 1. The normalized second-order moments (x10^{-3}) of the worst-case residual MAI caused by timing offsets or multipath (equal energy, offsets, and delays assumed for all users).

channel delays	$K=16$, $N=127$				$K=32$, $N=127$			
	$\sigma_I^2(\delta)/E_k\tilde{\delta}_k^2$		$\hat{\sigma}_I^2(\tau)/\lambda_k^2 E_k$		$\sigma_I^2(\delta)/E_k\tilde{\delta}_k^2$		$\hat{\sigma}_I^2(\tau)/\lambda_k^2 E_k$	
$\tilde{\tau}_k=1$	3.062	3.067	3.266	3.264	7.239	7.184	7.473	7.409
$\tilde{\tau}_k=2$	–	–	5.256	5.169	–	–	11.475	11.408
$\tilde{\tau}_k=3$	–	–	7.247	7.125	–	–	–	–
$\tilde{\tau}_k=4$	–	–	9.237	9.248	–	–	–	–
$\tilde{\tau}_k=5$	–	–	11.228	11.080	–	–	–	–

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Figs. 1 and 2 show the error rate $P_i(\epsilon|\delta)$ for various timing offsets ($\tilde{\delta}_k = .05, .1, .15, .2, .25$) when $K = 16$, 32 and $N = 127$, assumed equal received energy and timing offset for all users. For comparison, the single user detector is also analyzed and plotted, in which the decorrelating detector yields considerable performance gain even with timing offsets. We see that the performance gain is slightly large for $K = 32$ because the increased MAI severely degrades the single user detector rather than the decorrelating detector.

Next, we look into the behavior of the error rate $P_i(\epsilon|\tau)$ in Figs. 3 and 4 which accounts for the multipath, assuming several channel delays $\tilde{\tau}_k = 1$, 2, 3, 4, 5, ($n_k = 8\text{x}(k-1)$) and $\tilde{\tau}_k = 1$, 2 ($n_k = 4\text{x}(k-1)$) when $K = 16$ and $K = 32$ for $N = 127$, respectively. Here we have assumed that the multipath intensity exhibits exponential decay, namely, $\lambda_k^2 = \exp(-\tau_k/\Delta)$ with typical delay spreads $\Delta = 10T_c$ or $5T_c$, for example, in the mobile satellite channels. It is observed that the performance degradation is very little even with large $\tilde{\tau}_k$, showing greater robust against the multipath than the timing offsets.

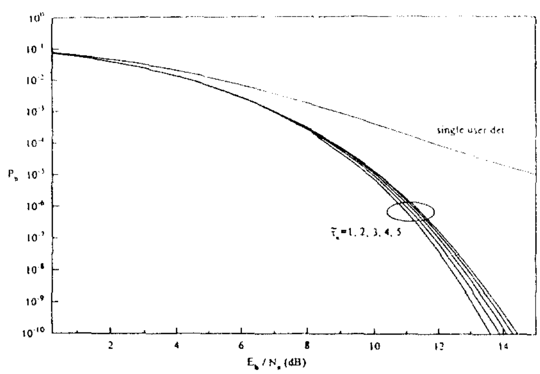

Fig. 3. Error rate $P_i(\epsilon|\tau)$ against multipath when $K = 16$, $N = 127$.

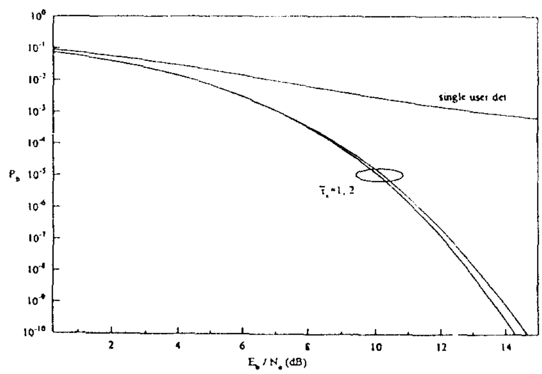

Fig. 4. Error rate $P_i(\epsilon|\tau)$ against multipath when $K = 32$, $N = 127$.

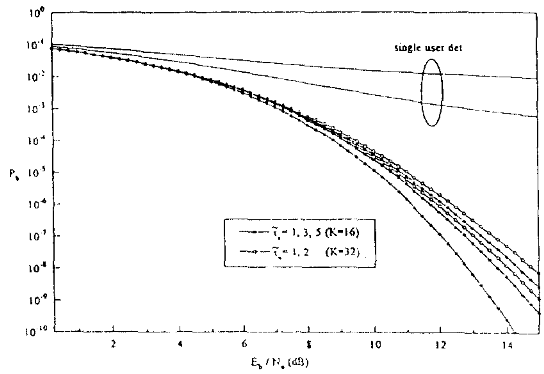

Fig. 5. Error rate $P_i(\epsilon|\tau)$ against unequal received energy when $K = 16$, 32, $N = 127$.

Finally, we examine the sensitivity of $P_i(\epsilon|\tau)$ against unequal received energy in Fig. 5, assuming the same set of parameters above only except $E_k = 2E_i$ for all $k \neq i$. The error performance is still within acceptable range, while the single user detector is severely degraded to show undesirable flat error rate. This implies that the decorrelating detector yields certain performance gain even facing with the near-far problem as well as the channel impairment of timing offsets or multipath.

Ⅵ. Conclusion

We have proposed the decorrelating detection scheme based on the common code assignment in which near-orthogonal code sequences being used are discernible by their distinct initial code phases. To show feasibility of the proposed scheme, we have

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characterized the worst-case residual MAI caused by the timing offsets existing in the reverse link of centralized networks or the multipath with small delay spread, which may be observed in the mobile satellite channels. In this analysis, we found that the residual MAI can be reduced to the order of N^{-2}, so the performance gain is sufficiently large enough to mitigate the above channel impairments when compared to the single user detector. It is also shown that the decorrelating detector employing common code gives better performance even facing with both multipath and near-far situation.

REFERENCES

1. S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Trans. Inform. Theory*, vol. IT-32, pp. 85-96, Jan. 1986.
2. R. Lupas and S. Verdu, "Linear multiuser detectors for synchronous code-division multiple-access channels," *IEEE Trans. Inform. Theory*, vol. IT-35, pp. 123-136, Jan. 1989.
3. R. Lupas and S. Verdu, "Near-far resistance of multiuser detectors in asynchronous channels," *IEEE Trans. Commun.*, vol. COM-38, pp. 496-508, Apr. 1990.
4. M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communications-Part I: System analysis," *IEEE Trans. Commun.*, vol. COM-25, pp. 795-799, Aug. 1977.
5. M. B. Pursley and H. F. A. Roefs, "Numerical evaluation of correlation parameters for optimal phases of binary shift-register sequences," *IEEE Trans. Commun.*, vol. COM-27, pp. 1597-1604, Oct. 1979.

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