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Parameter Estimation of Mean Field Annealing Technique for Optimal Boundary Smoothing

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최적의 Boundary Smoothing을 위한 Mean Field Annealing 기법의 파라미터 추정에 관한 연구

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ABSTRACT

We propose a method of parameter estimation using order-of-magnitude analysis for optimal boundary smoothing in Mean Field Annealing(MFA) technique in this paper. We previously proposed two boundary smoothing methods for consistent object representation in the previous paper, one is using a constrained regularization(CR) method and the other is using a MFA method. The CR method causes unnecessary smoothing effects at corners. On the other hand, the MFA method smooths out the noise without losing sharpness of corners. The MFA algorithm is influenced by several parameters such as standard deviation of the noise, the relative magnitude of prior term, initial temperature and final temperature. We propose a general parameter estimation method for optimal boundary smoothing using order-of-magnitude analysis to be used for consistent object representation in this paper. In addition, we prove the effectiveness of our parameter estimation and also show the temperature parameter sensitivities of the algorithm.

I. Introduction

It has been noted that the human visual system uses two-dimensional(2-D) boundary information to recognize objects since the shape of the boundary contains the pertinent information about an object. Hence, representing a boundary concisely and consistently is necessary for object recognition. In addition, the boundary-based method has the advantage of using local information which is less subject to problems caused by occlusion in recognizing objects based on shape information in the presence of occlusions.

Corner detection approach is usually used for boundary representation since it is well known that the shape information is concentrated at the points having high curvature[1]. The corners are usually detected in a curvature function space by capturing all the local extrema whose curvature values are

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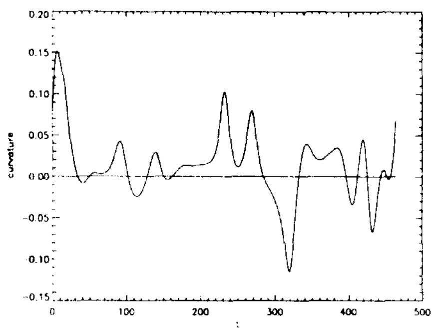
above a certain threshold value. Hence, computing the curvature function properly is very important for consistent boundary representation.

The curvature function obtained naively from a digitized boundary is very ragged due to the discrete nature of a boundary data and noise. Therefore, it must be made smooth to be used for further processing. The computation of curvature involves the first and second derivatives and differentiating discrete and noisy data enhances the noise. It is an ill-posed problem. There are two possible approaches to compute the curvature function on a digitized boundary curve in a wide sense: one is performed in a discrete domain and the other is performed in a continuous domain. The discrete domain has been used to compute the curvature function directly using the concept of k-curvature. We have exploited the continuous domain by investigating several lowpass filtering techniques such as mean, Hanning-windowed FIR lowpass, and Gaussian filters to reduce the effect of quantization. However, a common critical problem of the existing methods is the difficulty in determining a unique smoothing factor. Fig. 1-1 and Fig. 1-2 show examples of the above problem. We had noisy estimates of curvature with $\sigma=1$, i.e., they were undersmoothed as a result of insufficient filtering as shown in Fig. 1-1. On the other hand, the filter with $\sigma=8$ oversmoothed the data as shown in Fig. 1-2. We can see that the curvature extrema are significantly attenuated, and poorly localized. Fig. 1-1 and Fig. 1-2 also show considerable changes in its shape as σ is increased.



(a)

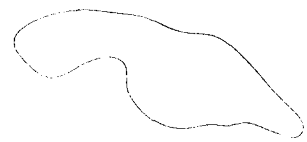


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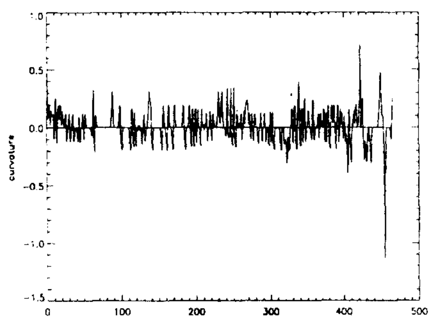
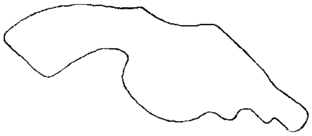
Fig. 1-2 Results of Gaussian filter($\sigma=8$).



(a)



(b)

Fig. 1-1 Results of Gaussian filter($\sigma=1$).

We suggested two boundary smoothing methods to overcome the above problem in the previous paper[2]. First, we used a Constrained Regularization(CR) technique which combines a regularization and a noise constraint. The regularization technique generally transforms ill-posed problems into well-posed problems by introducing constraints such as a smoothness requirement. It results in a consistent, unique and stable solution. The degree of boundary smoothing is determined by a priori noise information. In other words, the noise constraint determines a correct degree of smoothing, i.e., a unique smoothing factor. However, the CR method causes the unnecessary smoothing effects at corners.

Second, we used a Mean Field Annealing(MFA) technique to smooth out the noise without losing sharpness of corners. Simulated Annealing(SA) is an established technique for finding the global minimum of complex nonlinear functions that have several local minima[3]. SA is a random process and converges to a global minimum under special conditions. However, it converges to the global minimum very slowly. MFA is an approximation to SA which replaces the random search with a series of deterministic gradient descents[4]. MFA solves the simultaneous problems of noise removal and preservation of corners.

The MFA algorithm is influenced by several parameters such as standard deviation of the noise(σ), the relative magnitude of prior term(b), initial temperature (T_i) and final temperature (T_f). In this paper, we estimate the parameters for optimal boundary smoothing to be used for consistent boundary representation using order-of-magnitude analysis. In addition, we prove the effectiveness of our parameter estimation with order-of-magnitude analysis and also show the temperature parameter sensitivities of the algorithm in this paper.

This paper is structured as follows: Section 2 gives the theoretical background of MFA as well as a method of optimal boundary smoothing using MFA. In section 3, we propose a method of parameter estimation of MFA using order-of-magnitude analysis for optimal boundary smoothing. We derive it mathematically in detail in this section. In section 4, we show some experimental results and confirm the effectiveness of our proposed method. We also show the sensitivities of the estimated temperature parameters. Finally, a summary of the important results of this paper and concluding remarks are given in section 5.

Ⅱ. Optimal Boundary Smoothing Using Mean Field Annealing Technique

Problems involving the minimization of functions that have many local minima have been prominent in various engineering applications. The gradient descent method is a typical example of a minimization algorithm which may become stuck in a local minimum depending upon the starting point. Kirkpatrick et al. [3] developed the simulated annealing technique to overcome this problem. Geman and Geman[5] used a Bayesian approach for image segmentation. They showed that if the image can be modeled as a Markov Random Field (MRF), there is an equivalence between the MRF and the Gibbs distribution. Hence, the problem of finding the most probable state in a posteriori probability distribution is converted to that of finding a global minimum in the energy function. Once the proper energy function is defined, the next step is to find the global minimum for that function. They proposed a procedure called Stochastic Simulated Annealing(SSA). It converges to a global minimum under certain conditions. However, it is extremely slow in practice. Bilbro et al. [4] developed a mean field annealing(MFA) technique. They sought a way to smooth out the noise without eliminating the edges. They accomplished this by using a global process that combines consistent local measurements to infer global properties. MFA is an approximation to SSA which replaces the random search by deterministic gradient descents. This approximation makes the algorithm converge faster than SSA.

Since a data point is correlated to its neighbors in boundaries, we can model the boundary as an MRF. Thus, we can use the MFA technique for optimal boundary smoothing. We pose the optimal boundary smoothing problem as minimization of the sum of a "noise" term and a "prior" term. Thus, we choose a Hamiltonian which represents both the noise in the boundary and a priori knowledge of the local shape of the boundary data. The noise Hamiltonian (H_n) is:

$$H_n(f_e,f_m)=\sum_k \frac{[f_e(k)-f_m(k)]^2}{2\sigma^2}=\frac{(f_e-f_m)^T(f_e-f_m)}{2\sigma^2} \tag{1}$$

where f_m and f_e are measured boundary and estimated boundary for unknown original boundary f and σ^2 is a noise variance. The prior Hamiltonian (H_p) represents the measure of a certain property. This can be written as

$$H_p(f_e)=-b\sum_k \frac{1}{\sqrt{2\pi T}}\exp\left(-\frac{\Lambda_k^2}{2T}\right). \tag{2}$$

Λ_k is the operator to the neighborhood of the kth element. We use the second derivative for Λ_k because it represents the roughness of data. b is a weighting factor for the prior Hamiltonian against the noise Hamiltonian, and T is a system temperature. As T approaches to zero, Eq. (2) becomes the analytic form of Dirac delta function. In other words, if Λ equals zero everywhere, then H_p will be maximum in absolute value (due to the minus sign in Eq. (2)) and will contribute maximally to making the Hamiltonian (H) small. We use the discrete approximation of the second derivative with three equidistant points. Thus, the digital approximation of the prior term becomes

$$H_p(f_e)=\sum_k \frac{-b}{\sqrt{2\pi T}}$$

$$\exp\left(\frac{-[f_e(k+1)-2f_e(k)+f_e(k-1)]^2}{2T}\right). \tag{3}$$

Minimizing the noise term (H_n) ensures that the unknown boundary should resemble the boundary data f_m, i.e., "closeness of data." The prior term (H_p) encapsulates a priori knowledge of the boundary characteristics, i.e., "smoothness of data."

We choose the mean field Hamiltonian to be

$$H_0(\mu,f_e)=\sum_k \frac{1}{2}[\mu(k)-f_e(k)]^2. \tag{4}$$

With this choice for H_0, the parameter μ becomes the mean of f_e. We already found that the minimization of the mean field error(E_{MF}) is equivalent to the minimization of $\langle H(f_e,f_n)\rangle_\mu$ [2]. In addition, Bilbro and Snyder[6] showed that $\langle H_n\rangle$ is constant and $\langle H_p\rangle$ has the following asymptotic behavior:

$$\langle H_p\rangle \xrightarrow[T\to 0]{} H_p \quad \text{and} \quad \langle H_p\rangle \xrightarrow[T\to\infty]{} 0. \tag{5}$$

Thus, minimizing $\langle H\rangle\mu$ is equivalent to minimizing H. Conclusively, we will minimize

$$H(\mu,f_m)=H(\langle f_e\rangle,f_m)$$

$$=\sum_k \frac{[u(k)-f_m(k)]^2}{2\sigma^2}-\frac{b}{\sqrt{2\pi T}}\sum_k \exp$$

$$\left(-\frac{[\mu(k+1)-2\mu(k)+\mu(k-1)]^2}{2T}\right). \tag{6}$$

We perform the minimization of Eq. (6) by annealing on T. That is, we begin with a large T since $H_p\approx 0$ for large T and minimizing the H of Eq. (6) is equivalent to setting $\mu=f_m$. Then, we start the algorithm with this initial condition, gradually reduce T, and at each new value of T, we minimize H by using gradient descent or other standard minimization methods. The algorithm is summarized as follows:

1. Initialize $\mu=f_m$. Initialize T^0 to some T_i (initial temperature).
2. Lower the temperature ($T^{k+1}=\tau T^k$, τ is the decrement factor).
3. Minimize H at the new T^k using a gradient descent method to reach a local minimum.
4. If $T^k\rangle T_f$ (final temperature), return to step 2.
5. Stop.

Ⅲ. Parameter Estimation

We can see that the algorithm is influenced by the following parameters from Eq. (6) and the algorithm summary of the previous section:(σ:the standard deviation of the noise, b: the relative magnitude of the prior term, T_i; the initial temperature, T_f: the final temperature). The decrement factor τ is also important. However, it was empirically shown that 5-10% decrement of the temperature is usually good enough. Therefore, we use the following annealing schedule:

$$T_{\text{new}} = 0.95\ T_{old}. \tag{7}$$

Since a digitized boundary is represented by 8-neighbor Freeman chain code, the quantization noise is dominant in the digitized boundary. Thus, we can easily estimate σ. We apply an order-of-magnitude analysis to our parameter estimation. For simplicity, we assume we are working in a small region of the boundary. In this region, we have zero-biased the data by subtracting the mean. Therefore, we have $\langle f_m^2(k) \rangle \approx \sigma^2$ in this small region, where $\langle\ \rangle$ is an operator for the expected value. Applying the order-of-magnitude analysis, we have

$$f_e(k) = \zeta f_m(k) + (1-\zeta) f(k), \qquad 0 \le \zeta \le 1. \tag{8}$$

First, consider the noise term,

$$\begin{aligned} &\langle [f_e(k) - f_m(k)]^2 \rangle = \langle [(\zeta-1) f_m(k) + (1-\zeta) f_e(k)]^2 \rangle \\ &= (1-\zeta)^2 \langle [f_e(k) - f_m(k)]^2 \rangle = (1-\zeta)^2 \sigma^2. \end{aligned} \tag{9}$$

Second, we consider the prior term. By using Jensen's inequality, we can examine the exponent of the prior term,

$$\begin{aligned} \langle \Lambda_k^2 \rangle &= \langle [f_e(k+1) - 2f_e(k) + f_e(k-1)]^2 \rangle \\ &= \langle [\zeta(f_m(k+1) - 2f_m(k) + f_m(k-1) \\ &\quad + (1-\zeta)(f(k+1) - 2f(k) + f(k-1))]^2 \rangle \\ &\approx \langle [\zeta(f_m(k+1) - 2f_m(k) + f_m(k-1) + \frac{1-\zeta}{\rho}]^2 \rangle \end{aligned} \tag{10}$$

where ρ is roughly the local radius of curvature of the ideal boundary. Eq. (10) may be further simplified by recalling that we are working in a region where $\langle f_m \rangle \approx 0$,

$$\begin{aligned} \langle \Lambda_k^2 \rangle &\approx \langle \zeta^2 (f_m(k+1) - 2f_m(k) + f_m(k-1))^2 \rangle \frac{(1-\zeta)^2}{\rho^2} \\ &\approx 6\zeta^2 \sigma^2 + \frac{(1-\zeta)^2}{\rho^2} \end{aligned} \tag{11}$$

For a polygon, $\rho \to \infty$. Furthermore, unless σ^2 is zero or ρ impractically small, Λ_k will always be dominated by the first term. For large values of T, the prior term is negligible. The prior term begins to be significant when the exponent is of order one, that is,

$$\left[\frac{6\zeta^2 \sigma^2}{2T}\right] \approx O(1). \tag{12}$$

Since $\zeta = 1$ at high temperature, we start annealing at

$$T_i = O(3\ \sigma^2). \tag{13}$$

Thus, the initial temperature is high enough if we begin annealing at double our estimate, $T_i = 6\ \sigma^2$. The energy corresponding to $f_e(k)$ can be approximated as

$$E[f_e(k)] = \frac{(1-\zeta)^2}{2} - \frac{b}{\sqrt{2\pi T}} \exp\left(\frac{-3\zeta^2 \sigma^2}{T}\right). \tag{14}$$

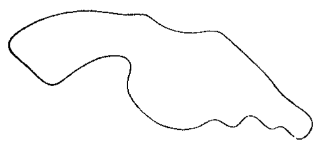
Since the energy is dependent on ζ we find b by choosing the ζ which minimizes $E[f_e(k)]$. Differentiating $E[f_e(k)]$ with respect to ζ yields

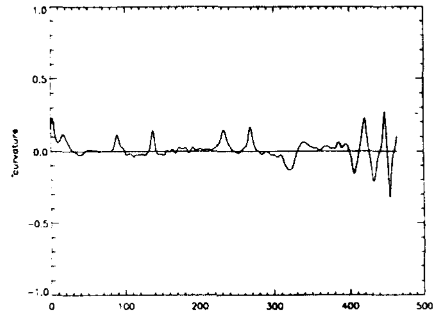
$$(\zeta - 1) + \frac{b}{\sqrt{2\pi T}} \exp\left(\frac{-3\zeta^2 \sigma^2}{T}\right) = 0 \tag{15}$$

We want our estimator to be half way between the original data and the final value at $T = 3\sigma^2$. Hence, we assign $\zeta = \frac{1}{2}$ at $T = 3\sigma^2$. As a result of this approximation, we have

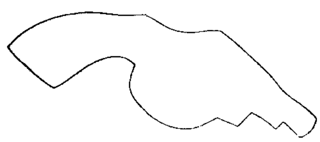
$$b = \frac{\sqrt{6\pi}}{2}\ e^{1/4}\ \sigma \approx 2.8\sigma. \tag{16}$$

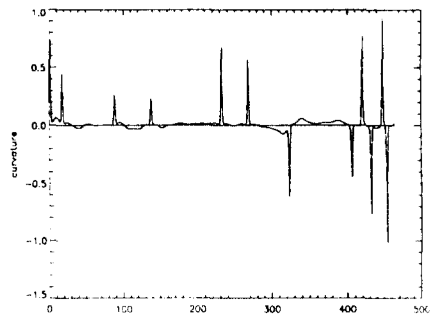
We generally cool down the temperature until the temperature is close to 0. However, as $T \to 0$, the prior term (H_p) dominates the Hamiltonian (Eq. (6)). In the limit, the effect of the prior term will be to





(a) (b)





(c) (d)

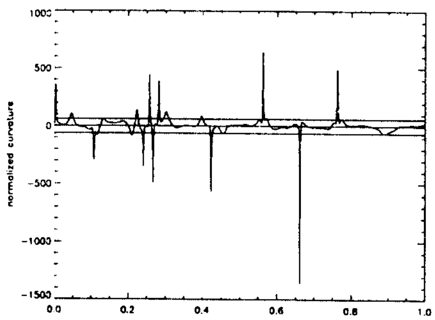
Fig. 4-1 CR vs MFA; (a) Preprocessed boundary(CR) (b) curvature(CR) (c) Preprocessed boundary(MFA) (d) curvature(MFA).

converge to a solution which is locally planar except for a set of discontinuities between segments. Hence, overcooling makes curved segments piecewise linear. Thus, we have too many segments to represent a boundary. It is essential that we predict a final annealing temperature to keep curved parts of the boundary. Bilbro and Snyder[7] discussed this overcooling effect by considering the algorithm as a linear filter. They found an approximate range of final temperature. It works fine at near $T_f=0.01$, but not as cold as $T_f=0.001$. We empirically show that the above T_f works fine in our application.

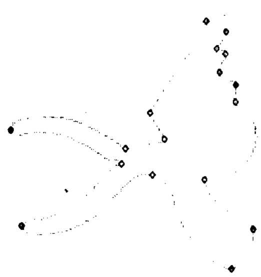
Ⅳ. Experimental Results

We estimated parameters ($T_i \approx 6\sigma$, $T_f=0.01$, $b=2.8\sigma$) in the last section. We tested the MFA algorithm with those estimated parameters for a boundary of a real object. Fig. 4-1(a) is the smooth gun boundary obtained by the CR method and Fig. 4-1(b) is the corresponding curvature function. As shown in the figure, the original boundary was smoothed well enough to be used for further processing, but some corners were not preserved. Fig. 4-1(c) shows the smooth boundary obtained by the MFA method based on the parameters estimated in the previous section. Fig. 4-1(d) is the corresponding curvature function. We can clearly see that the MFA method gives better performance. Fig. 4-1(d), when compared with Fig. 4-1(b), is less noisy and has larger extrema. In other words, corners of Fig. 4-1(c) are sharper than those of Fig. 4-1(a) and curved segments of Fig. 4-1(c) are smoother than those of Fig. 4-1(a).

Considering noise and negligibly small changes on the boundary, it is necessary to give a threshold value to the curvature, based upon which corners may be determined. A human recognize slightly rounded segments as corners as well as sharp corners. Thus, we proposed a method to mimic a human's capability of detecting corners based on the definition of corner sharpness in the previous paper[2]. Sharp corners as well as slightly rounded segments are detected as corners by the definition of corner sharpness. Fig. 4-2(a) shows the normalized curvature function of an

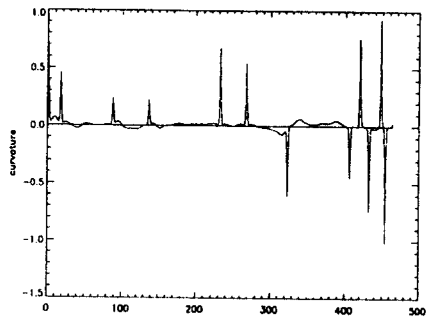


(a)

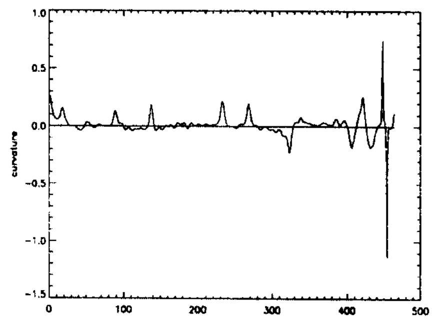


(b)

Fig. 4-2 Results of MFA method(occluded object); (a) normalized curvature with threshold value (b) corners detected by corner sharpness.



(a)



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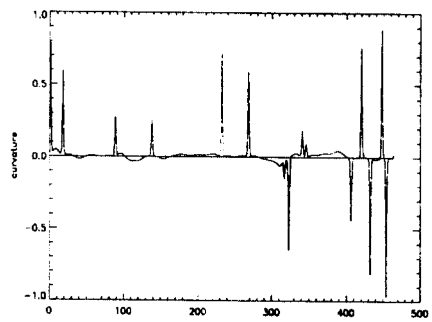

(c)

Fig. 4-3 System temperature sensitivity of MFA method; (a) High initial temperature effect($T_i = 120\ \sigma$) (b) Undercooling effect($T_f = 0.1$) (c) Overcooling effect($T_f = 0.001$).

occluded boundary between a gun and a plier with the threshold value for corner detection in the MFA approach. Fig. 4-2(b) shows the result of corner detection using corner sharpness in the MFA approach. As shown in the figure, all the corners which have very sharp curvature extrema are easily detected.

Fig. 4-3 shows the sensitivities of the initial and final system temperatures. We began annealing at an initial temperature($T_i = 120\sigma$), which is 20 times higher, to see how it affects. As shown in Fig. 4-3(a), we have almost the same results as the original ones (Fig. 4-1(d)). Thus, our estimation of initial temperature is correct. Fig. 4-3(b) shows the undercooling effect($T_f = .1$). As shown in the figures, the resulting curvature function is smooth, but the curvature extrema are not highlighted enough. We can see that it needs more cooling for the preservation of corners and the smoothness of curved parts. Fig. 4-3(b) looks almost the same as the results for the CR method (Fig. 4-1(b)). Fig. 4-3(c) shows the overcooling effect ($T_f = .001$). We can see that we have more curvature extrema. Thus, we conclude that as we gradually cool down the temperature($T_f \rightarrow 0$), the resulting boundary is becoming piecewise linear.

V. Conclusion

We presented the derivation of the MFA algorithm and applied it to the optimal boundary smoothing for curvature estimation in this paper. We found that the MFA algorithm is influenced by some parameters (σ, b, T_i, T_f). Thus, we estimated those parameters using the order-of-magnitude analysis in this paper. We confirmed that those parameter estimates worked well from some experimental results. In addition, we showed that the MFA method worked better than the CR method in a sense of preserving corners. We also showed the sensitivities of the system temperature parameters of the MFA method. We confirmed that our parameter estimation was good enough for optimal boundary smoothing through the system temperature parameter tests.

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