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Performance Evaluation of New Curvature Estimation Approaches

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새로운 곡률 추정 방법들의 성능 분석에 관한 연구

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ABSTRACT

The existing methods for curvature estimation have a common problem in determining a unique smoothing factor. We previously proposed two approaches to overcome that problem: a constrained regularization approach and a mean field annealing approach. We consistently detected corners from the preprocessed smooth boundary obtained by either the constrained egularization approach or the mean field annealing approach. Moreover, we defined corner sharpness to increase the robustness of both approaches. We evaluate the performance of those methods proposed in this paper. In addition, we show some matching results using a two-dimensional Hopfield neural network in the presence of occlusion as a demonstration of the power of our proposed methods.

요 약

기존의 곡률 추정 방법들은 가장 적합한 하나의 스무딩요소를 구하기 어려운 공통의 문제점을 갖고 있다. 그러나, 이전 논문에서 위의 문제점을 극복할 수 있는 두 가지 접근 방법 즉 constrained regularization(CR)을 이용한 방법과 mean field annealing(MFA)을 이용한 방법을 제안하였으며, CR 또는 MFA 접근방법으로 얻은 전처리된 경계선 데이터로부터 코너점을 견고하게 찾을 수 있도록 코너의 날카로운 정도(corner sharpness)를 정의함으로써 코너점을 견고하게 찾을 수 있음을 확인하였다. 본 논문에서는 이전 논문에서 제안하였던 CR과 MFA 접근 방법들의 성능을 기존의 방법들과 비교, 분석해보고 두 방식의 우수성을 입증해 보이기 위해 두 방식으로부터 찾은 코너점을 이용하여 표적이 겹쳐있거나 가려있는 상태에서 이차원 홉필드 신경회로망을 사용하여 얻은 정합 결과를 보여준다.

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Ⅰ. Introduction

Many curvature estimation methods on a digitized boundary have been developed to obtain a smooth boundary both in a discrete domain and in a continuous domain. However, they had a common difficulty in determining a unique smoothing factor. We proposed two boundary smoothing methods to overcome the above problem in the previous paper[1]. First, we applied a constrained regularization(CR) technique which combined a regularization and a noise constraint. We determined the correct degree of regularization for the boundary in the CR approach using a priori noise information. In addition, by using the useful property of a circulant matrix the algorithm was performed in the frequency domain and the computation time was significantly reduced. The CR method worked as well as the generalized cross validation (GCV) method which was proposed by Shahraray and Anderson[2]. The GCV method does not require any knowledge of noise. However, it has higher computational complexity due to many matrix decompositions and inversion of large matrices. Both the CR and the GCV methods for boundary smoothing worked well to smooth boundary. However, they caused unnecessary smoothing at corners.

We proposed a second boundary smoothing method using a mean field annealing(MFA) technique to smooth out the noise without losing local information on the corners. The MFA method solved the simultaneous problems of the noise removal and the preservation of corners. However, it took more time to obtain the smooth boundary due to the annealing process. We established a criterion, called corner sharpness to increase the robustness of corner detection methods. It mimics the human's capability of detecting corners and it compensates for the smoothing effect of the preprocessing in detecting corners in the curvature function space.

We evaluate the performance of those proposed methods in this paper. We compare maximum relative errors for various scale factors of a boundary between the CR, GCV, and MFA methods. In addition, we compare the maximum residual errors between the CR, GCV, and MFA methods. It is well known that mean-squared error techniques generally do not provide a satisfactory measure of noise removal. By calculating residual errors we can see the measure of noise removal as well as preservation of corners. Finally, we show some matching results in the presence of occlusion as a demonstration of the power of our proposed methods. We use a two-dimensional(2-D) hybrid Hopfield neural network to show reliable matching results.

This paper is structured as follows: Our previous work is briefly introduced in section 2. Section 3 gives a comparative study of the performance of the algorithms proposed in the previous paper. Moreover, some matching results using the 2-D Hopfield neural network based on the corners detected by corner sharpness are presented as a demonstration of the power of the proposed algorithms. Finally, concluding remarks are given in section 4.

Ⅱ. New Approaches To Curvature Estimation For Robust Corner Detection

2.1 Constrained regularization approach

We have a knowledge about the quantization noise, which is the dominant noise. In addition, Reinsch[4] suggests that if the noise variance is roughly known, then the regularization parameter should be chosen so that the residual error is equal to the noise variance. By imposing the above noise constraint, we obtain the desirable result which is appropriate for further processing. Thus, we proposed a constrained regularization approach for consistent object representation in the previous paper[1]. The CR approach avoids the difficulty of determining a unique smoothing factor by smoothing the proper amount of noise. In addition, we can significantly reduce the computation time by using the properties of circulant matrices in this method. The mathematical details of the CR approach can be found in the previous paper[1].

2.2 Mean field annealing approach

Bilbro et al. [5] developed an MFA technique. They sought a way to smooth out the noise without eliminating the edges. They accomplished this by using a global process that combines consistent local measurements to infer global properties. MFA is an approximation to stochastic simulated annealing technique which replaces the random search by deterministic gradient descents[6]. This approximation makes the algorithm converge faster than stochastic simulated annealing. Since a data point is correlated to its neighbors in boundaries, we can model the boundary as a Markov random field. Thus, we can use the MFA technique for boundary smoothing. We pose the boundary smoothing problem as minimization of the sum of a "noise" term and a "prior" term. Thus, we choose a Hamiltonian as follows:

$$H(f_e, f_m) = H_n(f_e, f_m) + H_p(f_e) \tag{1}$$

where f_m and f_e are measured boundary and estimated boundary for unknown original boundary f. The noise Hamiltonian(H_n) is

$$H_n(f_e, f_m) = \sum_k \frac{[f_e(k) - f_m(k)]^2}{2\sigma^2} \tag{2}$$

where σ^2 is a noise variance. The prior Hamiltonian (H_p) represents the measure of a certain local property. This can be written as

$$H_p(f_e) = \frac{1}{\sqrt{2\pi T}} \exp\left(-\frac{\Lambda_k^2}{2T}\right) \tag{3}$$

Λ_k is the operator on the neighborhood of the *k-th* element. We use a discrete form for the second derivative for Λ_k as in the CR approach because it represents the roughness of data. b is a weighting factor for the prior Hamiltonian against the noise Hamiltonian, and T is a control parameter known as "temperature".

The Hamiltonian, $H(f_e, f_m)$ may have many local minima and behave poorly in other ways. Hence, instead of minimizing H, we approximate H with a simple convex function H_0 which is easy to minimize. H_0 must depend on a set of parameters to be similar to H. Then, we adjust those parameters in such a way that H_0 will be similar to H. We choose H_0

$$H_0(\mu, f_e) = \sum_{k=0}^{N-1} [\mu(k) - f_e(k)]^2 \tag{4}$$

The above H_0 has only one minimum since it is a paraboloid. Thus, minimizing H_0 is simple. We have a convex form and we can find the μ's which make H_0 similar to H. Mathematical details can be found in the previous paper[1].

2.3 Robust corner detection

The CR method slightly smooths out corners and the resulting curvature at corners are spread over the neighbors. Unless we give a little tolerance when we detect corners in this curvature function space, we may miss some corners. Thus, we detect corners based on the following empirically derived result when we use the CR approach[1]:

Each point on a boundary is considered as a corner if one of the following conditions are satisfied:

$$\int_0^{c\Delta s} \chi \, ds \geq (c+1)\,10\pi \qquad \text{for} \qquad c = 1, 2, 3 \tag{5}$$

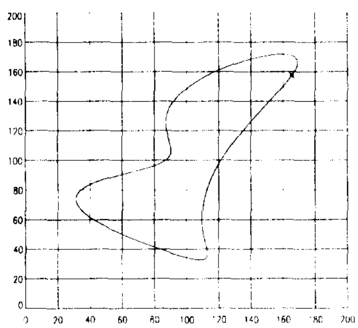
Only the points which satisfy one of the above three conditions are considered as corners in the CR approach. However, we already have shown that the MFA method preserves corners very well, and the resulting curvature function has large and sharp curvature extrema. Thus, we can easily detect corners with the first condition only in the MFA method.

Ⅲ. Performance Evaluation

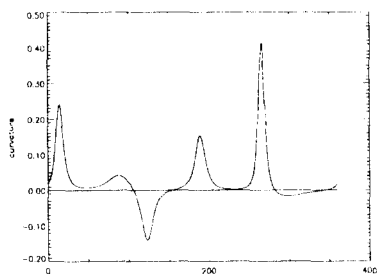
3.1 Curvature estimation

Current curvature estimation methods have a common problem in determining a unique smoothing factor. Shahraray and Anderson[2] solved the above problem with the GCV method based on the smoothing spline approximation. They showed good results for a smooth boundary in Fig.1(a). The continuous boundary was sampled at 360 points corresponding to $t=0$, 1,..., 359 degrees. The starting point is marked with '*'. The direction of tracing is counter-clockwise. The curvature function as a function of t of this noise-free boundary curve is shown in Fig.1(b).

Cross-validation is a data-driven method for estimating the correct degree of regularization. It does not require noise information for curvature estimation. However, the GCV method itself may fail catastrophically in some circumstances, producing either no positive smoothing parameter or a underestimated smoothing parameter[2]. In addition, the GCV method is mathematically intense due to many matrix decompositions and the requirement to obtain the inverse

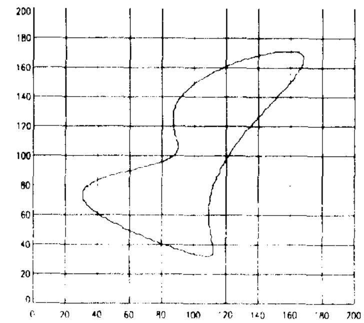


(a)

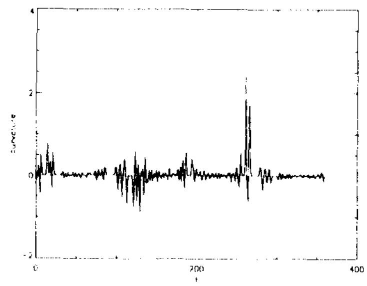


(b)

Fig. 1 Original smooth boundary and curvature ; (a) original boundary (b) curvature.



(a)



(b)

Fig. 2 Quantized boundary and curvature ; (a) quantized boundary (b) curvature.

of large size matrices. We used the CR technique to solve the same problem. We quantized the original smooth boundary to test the performance of the algorithm. Fig.2(a) is the quantized boundary which was interpolated into the nearest integers. Fig.2(b) shows the noisy curvature function directly computed from the quantized boundary. We applied the algorithm to the x-and y-coordinates separately. Then, the preprocessed x-and y-coordinate values compute the curvature function. We had almost the same results as Fig.1. Our boundary smoothing constraint is a little stronger than that used in the GCV method. In other words, we regularize the digitized boundary until the noise constraint is satisfied. We also showed excellent smoothing results with the CR method for the smooth boundary.

Table-1 shows maximum relative errors in the measurement of curvature for the GCV, CR and MFA

Table 1. Maximum relative errors(Fig.1(a)).

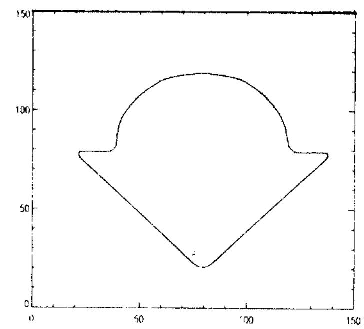
Method	GCV				CR				MFA			
Scale factor(s_f) Rotation(φ)	0.5 0°	2.0 0°	4.0 0°	0.5 150°	0.5 0°	0.2 0°	4.0 0°	0.5 150°	0.5 0°	2.0 0°	4.0 0°	0.5 150°
Maximum relative errors (%)	4.19	4.98	8.30	8.10	6.68	6.70	6.11	7.38	6.50	6.81	7.12	7.30

approaches for Fig.1(a). Maximum relative errors are obtained by:

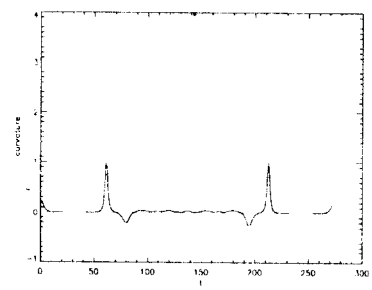
$$\text{maximum relative error}(\%) = \max\left(\frac{|\chi_o - \chi_r|}{\chi_o}\right) \times 100 \quad (6)$$

where κ_o and κ_r are the curvatures of the original bcundary and the smoothed boundary, respectively. The GCV method had better maximum relative errors when scaled by a factor of 0.5 and 2. However, the CR and MFA methods had better maximum relative errors for a scale factor of 4 and with rotation. Hence, we can say that those methods work well in general. However, the boundary in Fig.1 is very smooth. It does not have any discontinuities in the derivatives(i.e., corners). Shahraray[2] showed that the smoothing spline was not an appropriate model for representation of corners. The presence of corners in the boundary results in a considerable amount of error in the prediction of corners. We had almost the same results with the CR method as those achieved with the GCV method. However, the problem caused by the smoothing effect at corners can be overcome by the use of corner sharpness in the GCV method as well as in the CR method.

We proposed another boundary smoothing method using the MFA technique for curvature estnctions and the residual errors of x-and y-coordinate values of the preprocessed boundaries using both the CR method and the MFA method to evaluate the performance of the algorithm. The dotted line of the overlapped boundary in Fig.3(a) is a test boundary (Test-1 boundary) generated with lines, and a circular



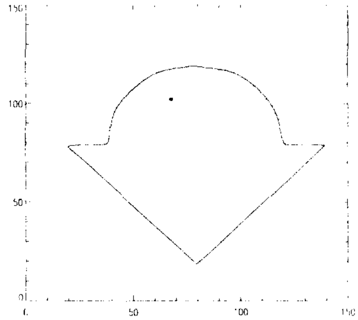
(a)



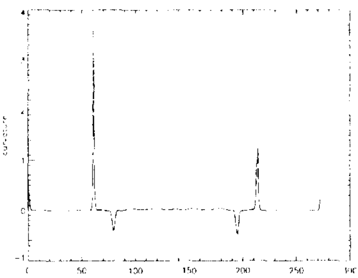
(b)

Fig. 3 Results of the the CR method for Test-1 boundary; (a) overlapped boundary (b) curvarure.

arc. It includes some discontinuities. The boundary is represented by the 8-neighbor Freeman chain code. Fig.3(a) shows the overlapped boundary between the original boundary and the boundary preprocessed by the CR. The CR method works well in a sense of noise removal or smoothing. However, we can clearly see the unnecessary smoothing on corners even though the CR method does not cause oversmoothing as in other current smoothing methods. Fig.4(a) shows the overlapped boundary between the original



(a)



(b)

Fig. 4 Results of the MFA method for Test-1 boundary; (a) overlapped boundary (b) curvature.

boundary and the boundary preprocessed by MFA. We can see that the curved part is smooth and the corners are preserved very well. By computing curvature functions from the smooth boundaries we can see that the MFA method gives better performance in a sense of preservation of corners. The absolute values of curvature extrema in Fig.4(b) are clearly larger than those in Fig.3(b).

By calculating residual errors we can see the measure of noise removal as well as preservation of corners. We can clearly see from Table-2 that the maximum residual errors in both the CR method and the GCV method are larger than those in the MFA method. Maximum residual errors are obtained by:

$$\text{maximum residual error} = \max |f_e - f_o| \qquad (7)$$

where f_e and f_o are the smoothed boundary and the

Table 2. Maximum residual errors(Test-1 Boundary).

Method	GCV	CR	MFA
x-residual errors	1.40	1.50	0.90
y-residual errors	1.81	1.60	0.57

original boundary, respectively. It therefore implies that the CR method and the GCV method have distorted the corners. Such distortion is less evident in the MFA method.

3.2 Corner detection

We established the criterion called corner sharpness. It mimics the human's capability of detecting corners and it compensates for the smoothing effect of the preprocessing in detecting corners in the curvature function space. We applied this criterion to the curvature functions obtained by the various methods. Table-3 shows the number of corners detected in the various methods for three model boundaries(model-1 : gun, model-2 : hammer, model-3 : plier) and two input boundaries(input-1 : gun + hammer, input-2 : gun + plier). They will also be used to show matching results in the next section. We obtained the results of human observers in the table by asking several people to choose corners in the given boundaries. As shown in Table-3, we detected almost the same number of corners using corner sharpness and the CR approach as human observers did. Our results using corner sharpness and MFA gave equally good results.

3.3 Matching results

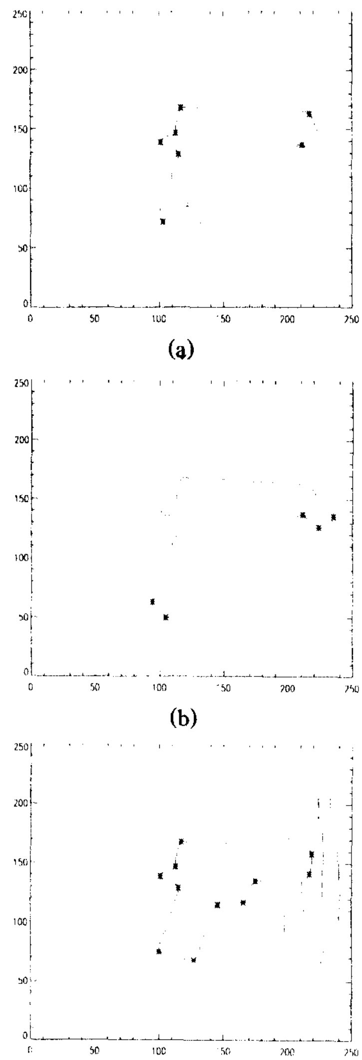
As a demonstration of the power of the corner detection algorithm, we consider an example of matching the silhouette of a partially occluded object to a model. We formulate object recognition as matching a model graph with an input image graph. We pose this graph matching problem as an optimization problem where an energy function is minimized. This optimization problem can be mapped into the Hopfield network[7]. Li and Nasrabadi[8] applied a discrete Hopfield network to image matching, using

Table 3. Number of corner points in various methods.

	Gaussian smoothing			CR method	MFA method	Human
	$\sigma=2$	$\sigma=4$	$\sigma=8$			
Model-1	13	9	3	11	11	11
Model-2	10	8	4	8	8	8
Model-3	9	5	4	6	6	6
Input-1	17	11	3	15	12	13
Input-2	29	21	16	21	21	21

sub-graph matching (isomorphism) to recognize objects. The discrete Hopfield network is formulated as a 2-D array. Rows correspond to an input image and columns correspond to an object model. The output of neurons after convergence shows the measure of similarity between two images. Lin et al. [9] applied the same structure of Hopfield network to 3-D object recognition. However, they used a continuous type of Hopfield network for better matching results, which takes more time to arrive at stable states.

We proposed a hybrid Hopfield neural network by combining the advantage of continuous Hopfield network and the advantage of discrete Hopfield network which gives excellent matching results very fast. The details of the hybrid Hopfield network can be found in [10]. Thus, we use the 2-D hybrid Hopfield network for matching in this paper. A graph is constructed to create a model for each object using unique corner points as nodes of the graph. Each node has local features(angle) as well as relational features(distances between nodes) with other nodes. Not only relations between neighboring corners but also relations between all other corners are used as constraints to increase the robustness of the algorithm. During the matching procedure, we construct a similar graph for the input image which may consist of one or several occluded objects. Each model graph is then matched against the input image graph to find the best matching subgraph. Fig.5(a-c) show matching results between Model-1 and Input-1, between Model-2 and Input-1, and between Model-1 and Input-2, respect-



(a)

(b)

(c)

Fig. 5 Matching results; (a) model-1 vs input-1 (b) model-2 vs input-1 (c) model-1 vs input-2.

ively. The nodes marked '*' in the figures represent the matched nodes. As shown in Fig.5, they show excellent matching results between model objects and input objects in the presence of occlusion.

Ⅳ. Conclusion

We compared the performance of our proposed curvature estimation and corner detection methods with current methods in this paper. The CR method resulted in slight unnecessary smoothing at corners. However, it did not cause any serious problem in detecting corners since corner sharpness compensated for the unnecessary smoothing effect at corners. In addition, the computation time of the CR method was small since it was performed in the frequency domain with the useful property of a circulant matrix. On the other hand, the MFA method preserved corners very well. Hence, we detected corners easier in this approach than in the CR approach. However, it took more time to obtain the smooth boundary due to the annealing process. The performance of the CR method and the MFA method in detecting corners were as good as that of the human observers. Finally, we showed some matching results based on the corners detected by our proposed approaches to support the excellence of our methods even in the presence of occlusion. We used the 2-D hybrid Hopfield neural network for matching. It converged to the stable states fast and showed excellent matching results.

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