論文 98-23-8-12

Performance of the Adaptive LMAT Algorithm for Various Noise Densities in a System Identification Mode

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ABSTRACT

Convergence properties of the stochastic gradient adaptive algorithm based on the least mean absolute third (LMAT) error criterion is presented. In particular, the performance of the algorithm is examined and compared with least mean square (LMS) algorithm for several different probability densities of the measurement noise in a system identification mode. It is observed that the LMAT algorithm outperforms the LMS algorithm for most of the noise probability densities, except for the case of the exponentially distributed noise.

I. Introduction

The adaptive LMS algorithm [1] has received a great deal of attention during the last two decades and is now widely used in variety of applications due to its simplicity and relatively robust performance. The algorithm attempts to minimize the mean-squared estimation errors at each iteration. Meanwhile, the adaptive filtering algorithms that are based on high order error power (HOEP) conditions [2]-[6] have been proposed and their performances have been investigated. Despite their potential advantages, these HOEP algorithms are much less popular comparing to the LMS and sign algorithms in practice since they can be very sensitive to the stability.

The paper by Walach and Widrow [2] seems to be the first one dealing with the HOEP conditions in the stochastic gradient adaptive signal processing. They presented convergence analyses of the adaptive least mean fourth (LMF) algorithm and its family. The performance of the LMF algorithm is then compared with that of the LMS algorithm for different plant noise densities in a system identification mode. By evaluating the ratio between the misadjustment of the LMS algorithm and that of the LMF algorithm, it was shown that the LMF algorithm has substantially less noises in the filter coefficients than the conventional LMS algorithm for the same speed of convergence, except the case when the plant measurement noise of the unknown system has a Gaussian distribution. The necessary condition for the convergence of the mean and mean-squared behavior of the LMF algorithm was also derived. The results in [2] were, however, somewhat restrictive due to the employment of the wild assumption that the filter coefficients are already close to the optimal values.

Douglas and Meng [3] examined a family of adaptive algorithms based on general error criteria (or non-mean-square error criteria) for which the error

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function to be minimized was modeled as an arbitrary memoryless odd-symmetric nonlinear function. It was shown that, in the system identification mode, using an error criterion optimized for the plant noise density can significantly improve the overall performance and reduce the fluctuations in the coefficient estimates. Pei and Tseng [4] also investigated the performances of the HOEP criteria for adaptive FIR filters, and several important observations were made. They showed that the HOEP criteria yield the same optimum solution for any high order power when the input signals are Gaussian processes. It was also shown that the sign algorithm is preferred when the signals are corrupted by an impulsive noise.

Kim, et al. [5] presented a convergence analysis of the LMAT algorithm by deriving equations to characterize the statistical mean and mean-squared behavior of the algorithm. They also investigated the steady-state responses of the LMAT algorithm and compared the performance of the LMAT algorithm with that of the LMS algorithm [6]. It was observed that the LMAT algorithm often converges faster than the LMS algorithm.

In this paper, the convergence of the LMAT algorithm is further examined. In particular, the misadjustment performance of the LMAT algorithm is simulated and compared with that of the LMS algorithm when the two algorithms are employed in the system identification mode for an unknown plant noise with several different probability densities.

II. Problem Statement

Consider the problem of adaptively identifying an unknown linear and time-invariant (LTI) system as illustrated in Figure 1. Here, $d(n)$ and $x(n)$ denote the primary and reference inputs, respectively, $H(n)$ is the adaptive filter weight vector of size N, and $e(n)$ is the estimation error at time n. The measurement noise of the unknown plant is denoted by $\eta(n)$.

Define the reference input vector $X(n)$ as

$$X(x) = [x(n), x(n-1), \cdots x(n-N=1)]^T, \qquad (1)$$

where denotes the transpose of . The LMAT algorithm tries to minimize the mean of the absolute error value to the third power. The error function is the perfect convex function with respect to the adaptive filter coefficients, and therefore does not have local minima [4]. The LMAT algorithm updates H(n) using

$$H(n+1) = H(n) + \mu X(n) e^2(n) sign[e(n)], \qquad (2)$$

where is the adaptive step-size of positive value,

$$\text{sign}\{e(n)\} = \begin{cases} 1 & \text{if } e(n) \geq 0 \\ -1 & \text{otherwise}, \end{cases} \qquad (3)$$

and the estimation error is given by

$$e(n) = d(n) - H^T(n) X(n). \qquad (4)$$

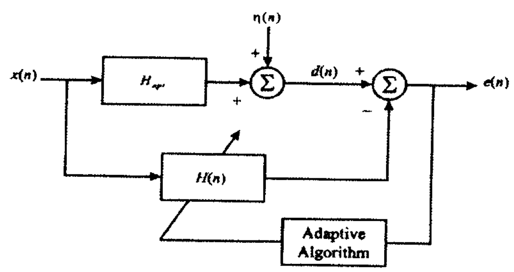

Fig. 1 A block diagram for the adaptive system identification.

Let H_{opt} denote the optimal coefficient vector given by

$$H_{opt} = R_{XX}^{-1} \; R_{dX}, \qquad (5)$$

where R_{XX} and R_{dX} denote the autocorrelation matrix of $X(n)$ and the crosscorrelation vector of $d(n)$ and $X(n)$, respectively, such that

$$R_{XX} = E\{X(n)X^T(n)\}, \tag{6}$$

and

$$R_{dX} = E\{d(n)X(n)\}. \tag{7}$$

Also, define the coefficient misalignment vector V(n) as

$$V(n) = H(n) - H_{opt}, \tag{8}$$

and its autocorrelation matrix K(n) as

$$K(n) = E\{V(n)V^T(n)\}. \tag{9}$$

Even though the convergence properties of the LMAT algorithm have already been analytically investigated [5], [6], it will be still interesting to examine the misadjustment performance of the LMAT algorithm from a different viewpoint. In the next section, we present some simulation results to compare the performance of the LMAT algorithm with that of the LMS algorithm when the two algorithms are employed in a system identification mode for an unknown plant noise with several different probability densities.

III. Simulation Results

In order to make interesting comparisons between the performance of the LMAT algorithm and that of the LMS algorithm for various noise environments, we select the following probability distributions for the unknown plant noise $\eta(n)$ that are particularly of practical importance: 1) Gaussian, 2) uniform, 3) sinusoidal with uniformly distributed random phase, 4) rectangular (i.e., binary random square wave), 5) exponential, and 6) Rayleigh distributions. The reference input x(n) to the adaptive filter is selected as a third-order autoregressive signal described by

$$x(n) = \zeta(n) + 0.9x(n) - 0.1x(n-1) - 0.2x(n-2), \tag{10}$$

where $\zeta(n)$ is a zero-mean, white Gaussian process with variance such that the variance of $x(n)$ is one. The corresponding primary input $d(n)$ is generated by processing $x(n)$ through the LTI system consisting of seven coefficients given by

$$H_{opt} = [0.1, 0.3, 0.5, 0.7, 0.5, 0.3, 0.1]^T, \tag{11}$$

and then corrupting the system output with the unknown plant noise having the above six probability densities. The variances of the noise densities are all set to be one. Note that the eigenvalue spread ratio of the input $x(n)$ is approximately 25.3.

Table 1. The values for used in the simulations.

Noise density	μ for LMS	μ for LMAT
Gaussian	0.0044	0.00267
Uniform	0.004	0.004
Sinusoidal	0.003	0.004
Rectangular	0.003	0.0067
Exponential	0.008	0.00167
Rayleigh	0.004	0.00167

We now compare the misadjustment performances of the LMAT algorithm with that of the LMS algorithm by computing the trace value of the covariance matrix $K(n)$. The convergence parameters are selected differently for each noise probability density in such a way that both the algorithms produce the same trace value of $K(n)$ in the steady-state. The values of the convergence parameters used in the simulations are tabulated in Table 1.

Figures 2 through 7 illustrate the trace curves of the matrix $K(n)$ for various noise probability densities. These curves are obtained by taking the ensemble averages over 30 independent runs. As can be seen in the figures, the LMAT algorithm outperforms the

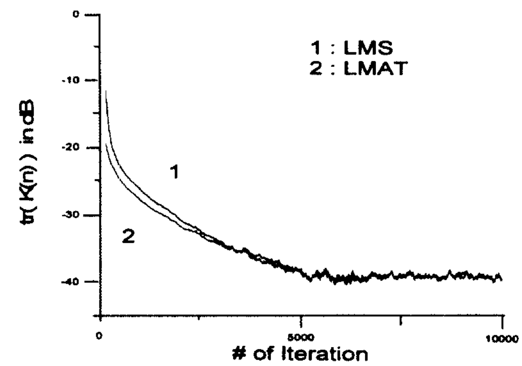

Fig.2 The trace values of the matrix K(n) for the plant noise having a Gaussian density.

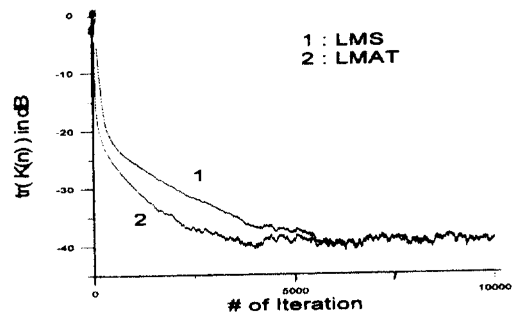

Fig.3 The trace values of the matrix K(n) for the plant noise having a uniform density.

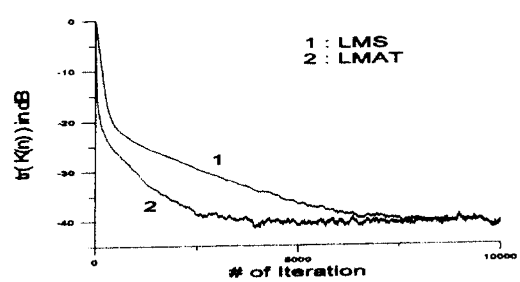

Fig.4 The trace values of the matrix K(n) for a sinusoidal plant noise with random phase.

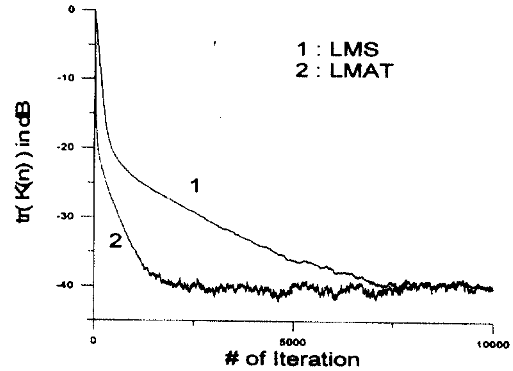

Fig.5 The trace values of the matrix K(n)for a rectangular (i.e., square wave) plant noise.

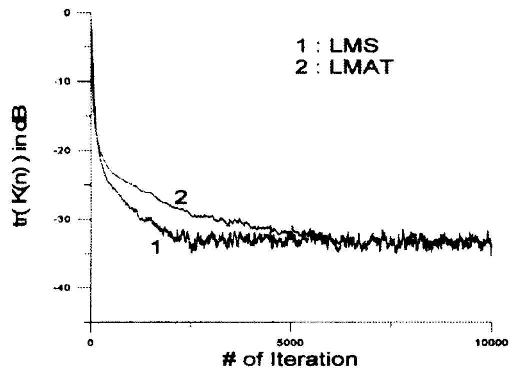

Fig.6 The trace values of the matrix K(n)for the plant noise having a exponential density.

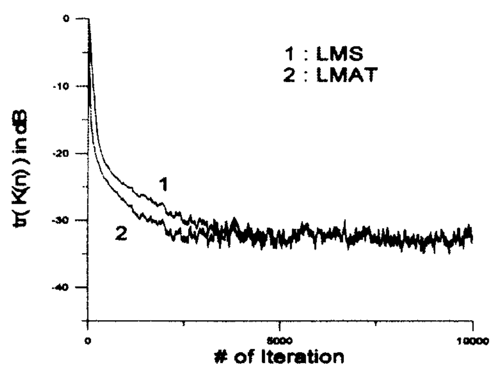

Fig. 7. The trace values of the matrix K(n) for the plant noise having a Rayleigh density.

LMS algorithm for most noise probability densities, except for the case of the exponentially distributed noise. In particular, the performance of the LMAT algorithm is much better than the LMS algorithm for the cases of the sinusoidal noise with uniformly distributed random phase and the rectangular (i.e., binary random square wave) noise.

IV. Conclusion

In this paper, the performance of the LMAT algorithm is further examined by extending the previous works in [5] and [6]. In particular, the misadjustment performance of the LMAT algorithm is compared with that of the LMS algorithm in the system identification mode for an unknown plant noise with several different probability densities. We have observed that the LMAT algorithm outperforms the LMS algorithm for most noise densities, except for the case of the exponential distribution. It was also seen that the performance of the LMAT algorithm is much better than the LMS algorithm for the cases of the sinusoidal noise with random phase and the rectangular noise. We are currently working on finding analytical reasons for our results.

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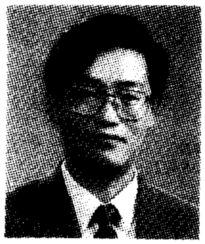

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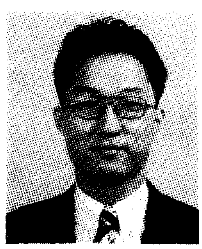

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