

# ATM망 분석을 위한 D-BMAP/Geo/1/K 큐의 Departure 프로세스의 Markov 변조 특성화

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## A Markov Modulated Characterization of the Departure Process of a D-BMAP/Geo/1/K Queue in ATM Networks Analysis

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### 요 약

본 논문에서는 ATM망의 모델링 시 발생하는 D-BMAP/Geo/1/K 큐의 departure 프로세스를 구하고 이 프로세스를  $k$ -state MMBP로 특성화하는 방법을 제시하였다. 그리고 제시된 특성화 방법의 정확도를 여러 가지 방법을 사용하여 테스트하였다. 또한, 본 논문에서 제안한 특성화 방법은 이 departure 프로세스가 가질 수 있는 burstiness 뿐만 아니라 correlation에 대한 특성화가 가능하며, 따라서, 단순히 decomposition 알고리즘을 적용하므로써 셀 손실을 갖는 tandem구조의 이산시간 한정용량 큐잉시스템의 분석이 가능해진다.

### ABSTRACT

We first obtain the departure process of a D-BMAP/Geo/1/K queue. The departure process of this queue is characterized by a  $k$ -state MMBP in order to capture both the burstiness and correlation of the departure process. The tractable fitting model for characterizing the departure process of the queue by a  $k$ -state MMBP is proposed and its accuracy was examined through extensive validation tests. The fitting model is then used in a simple decomposition algorithm to analyze a tandem configuration of discrete-time finite capacity queues with cell loss

### I. Introduction

In recent years there has been a lot of interest in the development of high-speed communication networks. The most promising design for high-speed networks is the Asynchronous Transfer Mode(ATM). The need for performance evaluation of ATM networks has given rise to a widespread interest for the analysis of discrete-time queueing systems. Discrete-time single server queues with or without finite capacity have been extensively

analyzed. For a review of relevant results see Pujolle and Perros<sup>[1]</sup>. However, little has been done for the analysis of networks of discrete-time finite capacity queues. A network of discrete-time finite capacity queues can be used to model the queueing within an ATM switch, or the queueing within a network of ATM switches. The external arrival process to the network is assumed to be bursty and correlated. Markov Modulated Poisson Processes(MMPP)<sup>[2,3]</sup>, and Markov Modulated Bernoulli Processes(MMBP) are used to model a bursty arrival stream since they capture the

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논문번호 : 98147-0310, 접수일자 : 1998년 3월 10일

※ 본 연구는 배재대학교 학술연구 연구비 일부 지원에 의함

randomly varying arrival rate. The MMPP and MMBP capture the notion of burstiness and correlation of successive interarrival times. In this paper, we assume that the arrival process to the queue is a Discrete-time Batch Markovian Arrival Process (D-BMAP) which belongs to a class of versatile point processes discussed in <sup>[4,5]</sup>. A D-BMAP is the proposed model for a single variable bit rate source. Also, it can be used to model the superposition of several such sources <sup>[6]</sup>. The MMBP or IBP is a special case of the D-BMAP, with all arrival having a batch of size 1.

In this paper, we consider discrete-time finite capacity queues with cell loss. The service time at the queue is assumed to be geometrically distributed. The choice of the geometric distribution was motivated by ATM networks<sup>[7]</sup>. In general, a service time represents a transmission time. In an ATM networks the size of a cell is constant, and therefore, the transmission time is constant as well. However, in some ATM switch architectures a cell may be re-transmitted several times due to possible collisions with other cells. In this case, the total transmission time is typically modeled by a geometric distribution.

In general, discrete-time queueing networks as they arise in ATM do not lend themselves to an exact analysis. They can be analyzed, however, approximately using the notion of decomposition. That is, the network is decomposed into individual queues, and each queue is then analyzed separately. The most important aspect of such a decomposition is the characterization of the arrival process to an intermediate queue. In continuous-time queueing networks, typically such as the departure process is characterized approximately by a phased-type distribution, or by a general distribution defined by the mean and squared coefficient of variation. Although there has been some work regarding the departure process<sup>[8-13]</sup>, most of this work bears some limitations which seriously undermine their applicability on network-wide traffic analysis. Most of these studies only provide results on the stationary

distribution of the interdeparture time. Although this is a very important piece of information, it is by no means sufficient for characterizing the non-renewal departure process: the lengths of successive interdeparture times are highly correlated and such correlation will have significant impact on downstream queueing performance. As a result, details about the dynamic behaviour of the departing stream, e. g., burstiness and correlation, have to be studied. In this paper, the departure process of the D-BMAP/Geo/1/K queue has been studied.

Blondia and Casals<sup>[6]</sup> showed that the output process of a D-BMAP/G/1/K queue is a D-BMAP. Park and Perros<sup>[14,15]</sup> derived the generating function of the interdeparture time distribution and correlation of the departure process of an MMBP/Geo/1/K queue. They also obtained an approximation model for characterizing the departure process by an MMBP in order to capture the correlation and burstiness of the departure process of the queue.

This paper is organized as follows. In section II,\* we give a brief description of the D-BMAP. The generating function of the interdeparture time of a D-BMAP/Geo/1/K queue and the correlation coefficients for the departure process are obtained in section III. In section IV, we present a tractable model for characterizing approximately the departure process as a *k*-MMBP and we examine its accuracy.

## II. The Discrete-time Batch Markov Arrival Process

### 2.1 The Generating Function of the Interarrival Time of the D-BMAP

A D-BMAP can be represented by a 2-dimensional discrete-time Markov process  $\{(J(k), N(k)) : k \geq 0\}$  on the state space  $\{(i, j) : 1 \leq i \leq m, j \geq 0\}$ , where *i* indicates the state of the arrival process, and *j* indicates the number of arrivals. The transition matrix *T* of the counting process has the following structure:

$$T = \begin{bmatrix} P_0 & P_1 & P_2 & P_3 & \dots \\ & P_0 & P_1 & P_2 & \dots \\ & & P_0 & P_1 & \dots \\ & & & P_0 & P_1 & \dots \\ & & & & \ddots & \ddots \end{bmatrix}$$

where  $P_k, k \geq 0$ , are  $m \times m$  matrices. Let  $P = \sum_{k=0}^{\infty} P_k$  be the transition matrix of the underlying Markov process. If  $J(k)$  represents a phase variable and  $N(k)$  a counting variable then the above Markov process defines a batch arrival process where transitions from a state  $(i, j)$  to state  $(1, j+n)$ , corresponding to batch arrivals of size  $n$ .

Consider a discrete-time Markov chain with transition probability matrix  $P$ . Assume the underlying Markov process is in some state  $i, 1 \leq i \leq m$  at time  $k$ . At the next time instant  $k+1$ , the process may transit to another state or it may stay in the same state, and a batch arrival may or may not occur. Let  $p_{(n,i,j)}, n \geq 0, 1 \leq i, j \leq m$ , be the probability that there is a transition to state  $j$  from state  $i$  with a batch arrival of size  $n$ . Then, with probability  $p_{(0,i,j)}, n \geq 1, 1 \leq i, j \leq m$ , a transition to state  $j$  will take place without an arrival, and with probability  $p_{(n,i,j)}, n \geq 1, 1 \leq i, j \leq m$ , there will be a transition to state  $j$  with a batch arrival of size  $n$ . We have

$$\sum_{j=1}^m p_{(0,i,j)} + \sum_{n=1}^{\infty} \sum_{j=1}^m p_{(n,i,j)} = 1.$$

Using this notation, it is clear now that matrices  $P_0 = [p_{(0,i,j)}]_{m \times m}$  and  $P_k = [p_{(k,i,j)}]_{m \times m}$ , govern transitions that correspond to no arrival and arrival of batch of size  $k$  where  $k \neq 0$ , respectively. A D-MAP is a special case of the D-BMAP, with all arrivals having a batch of size 1.

Through this paper, we consider an arrival process to the queue which is a D-BMAP characterized by the transition probability matrix

$P$  of the Markov process,  $A, m \times m$  diagonal matrix with elements  $a_1, \dots, a_m$  and  $B$ , defined by

$$P = \begin{bmatrix} p_{11} & & p_{1m} \\ & \ddots & \\ p_{m1} & & p_{mm} \end{bmatrix}, A = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_m \end{bmatrix}, \text{ and } B = \begin{bmatrix} b_{11} & b_{12} \\ & \ddots \\ b_{m1} & b_{m2} & \dots \end{bmatrix}$$

where  $p_{ij}, 1 \leq i, j \leq m$  is the transition probability that the process changes from state  $i$  to state  $j, \sum_{j=1}^m p_{ij} = 1, a_i$  is the probability that a batch arrival occurs when the D-BMAP shifts to state  $i$ , and  $b_{ik}$  is the probability that the arriving batch size is equal to  $k, k \geq 1, \sum_{k=1}^{\infty} b_{ik} = 1$ . The D-BMAP satisfies following equations: For  $1 \leq i, j \leq m, n \geq 1$

$$p_{(0,i,i)} = p_{ii}(1 - a_i)$$

$$p_{(n,i,i)} = p_{ii} a_j b_{jn}$$

$$p_{ij} = \sum_{k=0}^{\infty} p_{(k,i,j)}$$

This process can be also referred to as a Markov Modulated Batch Bernoulli Process (MMBBP). In general, a D-BMAP becomes an MMBP if the following relation are satisfied: For  $1 \leq i, j \leq m,$

$$b_{ii} = 1$$

$$p_{ij} = p_{(0,i,j)} + p_{(1,i,j)} \tag{1}$$

$$p_{ij}(1 - a_j) = p_{(0,i,j)}$$

$$p_{ij} a_j = p_{(1,i,j)} \tag{2}$$

A D-BMAP has been proposed as a model from single variable bit rate source and its superposition [6]. Therefore, we assume that the batch size of a batch is bounded. Let  $N$  be the maximum batch size. Let  $T$  be the interarrival time between two successive batch arrivals. Also let  $\vec{\pi} = [\pi_1, \dots, \pi_m]^T$  be the stationary probability vector satisfying  $\vec{\pi} = \vec{\pi} P$ , where  $\pi_i, 1 \leq i \leq m$ , is the probability that the process is in state  $i$ . The generating function of batch interarrival time  $T(z)$  is

$$T(z) = \vec{p}_a \vec{T}(z) = z \vec{p}_a (I - zM)^{-1} P \vec{\lambda}$$

where  $\vec{p}_a = \frac{\vec{\pi} A}{\vec{\pi} \vec{\lambda}}, \vec{T}(z) = z(I - zM)^{-1} P \vec{\lambda}$ ,

$$M = P(I - A) \text{ and } \vec{\lambda} = [a_1, \dots, a_m]^T,$$

The average batch arrival rate  $\rho_b$ , the average

cell arrival rate  $\rho_c$ , and the squared coefficient of variation of the interarrival time between two successive arrival of batch,  $C_b^2$  are as follows:

$$\rho_b = \vec{\pi} \vec{\lambda}, \quad \rho_c = \sum_{i=1}^M i \vec{\pi} \Lambda \vec{b}_i, \text{ and}$$

$$C_b^2 = \frac{T^{(2)}(1)}{[T^{(1)}(1)]^2} + \rho_b - 1,$$

where  $\vec{b}_i = [b_{1i}, \dots, b_{mi}]^T$  and  $T^{(n)}(1) = \left. \frac{d^n T(z)}{dz^n} \right|_{z=1}$ .

### 2.2 The Autocorrelation of the D-BMAP

In this section, we obtain the autocorrelation of the interarrival time of batches, and the autocorrelation of the number of arrivals per slot. Let  $t_n$  be the time interval between the  $(n-1)$ st and  $n$ th arrival of a batch. Also, let  $t_{ij}^n, 1 \leq i, j \leq m$ , be the time interval to the moment that the D-BMAP is in state  $j$  and  $n$ th arrival occurs given that the D-BMAP is in state  $i$ , and  $t_i^n, 1 \leq i \leq m$ , be the time interval to the  $n$ th arrival given that the D-BMAP is in state  $i$ . Define

$$A(z) = \begin{bmatrix} A_{11}(z) & & A_{1m}(z) \\ & \ddots & \\ A_{m1}(z) & & A_{mm}(z) \end{bmatrix} \text{ and}$$

$$\vec{A}(z) = \begin{bmatrix} A_1(z) \\ \vdots \\ A_m(z) \end{bmatrix}$$

where  $A_{ij}(z)$  and  $A_i(z)$  are  $z$ -transforms of  $t_{ij}^n$  and  $t_i^n$ , respectively. From the definition of  $A_{ij}(z)$  and  $A_i(z)$  for  $1 \leq i, j \leq m$ , we have following equations:

$$A(z) = zPA + zMA(z) \text{ and } \vec{A}(z) = \vec{T}(z).$$

Therefore, we can obtain

$$A(z) = z(I - zM)^{-1}PA \text{ and}$$

$$\vec{A}(z) = z(I - zM)^{-1}P\vec{\lambda}. \tag{3}$$

Using equation (3), we have

$$G_a(z_1, z_2) =$$

$$E\{z_1^{t_n} z_2^{t_{n+k}}\} = \vec{p}_a A(z_1) T^{k-1} \vec{A}(z_2)$$

$$= \vec{p}_a z_1 (I - z_1 M)^{-1} P \Lambda T^{k-1} z_2 (I - z_2 M)^{-1} P \vec{\lambda}$$

where  $T = [I - M]^{-1} P \Lambda$ .

By differentiating equation (4) with respect to  $z_1$  and  $z_2$ , we have

$$E\{t_n t_{n+k}\} = \left. \frac{\partial^2 G_a(z_1, z_2)}{\partial z_1 \partial z_2} \right|_{z_1=1, z_2=1}$$

$$= \vec{p}_a (I - M)^{-1} P \Lambda T^{k-1} (I - M)^{-2} P \vec{\lambda}.$$

The autocorrelation coefficient of the interarrival time of batches of a D-BMAP for lag  $k, \phi_b(k)$ , is given by

$$\phi_b(k) = \frac{E\{t_n t_{n+k}\} - E^2\{t_n\}}{Var\{t_n\}}. \tag{5}$$

Let  $X_n$  be the random variable representing the number of arrivals at  $n$ th slot, where  $X_n = 0, 1, \dots, N$ . Then, we have

$$E\{X_n\} = \rho_c.$$

$$E\{X_n^2\} = \sum_{i=1}^M i^2 \vec{\pi} \Lambda \vec{b}_i,$$

$$E\{X_n X_{n+k}\} = \sum_{i,j=1}^M i j \vec{\pi} \Lambda B_i P^k \Lambda \vec{b}_j,$$

$$Var\{X_n\} = E\{X_n^2\} - E^2\{X_n\}$$

where  $B_i$  is a diagonal matrix with elements  $b_{1i}, \dots, b_{mi}$ .

Of interest is the autocorrelation coefficient of the number of arrival per slot of a D-BMAP for lag  $k, \phi_c(k)$ , given by

$$\phi_c(k) = \frac{E\{X_n X_{n+k}\} - E^2\{X_n\}}{Var\{X_n\}}. \tag{6}$$

### III. The Departure Process of a D-BMAP/Geo/1/K Queue

We consider a D-BMAP/Geo/1/K queue, where the service time is defined over a slotted time axis. A service starts at the beginning of a

service slot, and service completion is assumed to take place just before the end of the service slot. The arrival process is also defined over a slotted time axis with the same slot size, and it is assumed to be a D-BMAP. The parameters of the arrival process are:  $p_{ij}^A$ ,  $\alpha_i^A$ , and  $b_{ij}^A$ , where  $p_{ij}^A$  is the  $(i, j)$ th element of the transition probability matrix  $P$ ,  $\alpha_i^A$  is the  $(i, i)$ th element of the diagonal matrix  $\Lambda$ , and  $b_{ij}^A$  is the  $(i, j)$ th element of the matrix  $B$ . We define the state of the queue by the variable  $(s, n)$ . Variable  $s$  represents the state of the arrival process at the end of a slot and it takes the values:  $i$ ,  $1 \leq i \leq m$ , if the arrival process is in the state  $i$ . Variable  $n$  indicates the number of cells in the system at the end of a slot. We have  $n=0, 1, \dots, K$ , where  $K$  is the capacity of the system including the cell in service. Let  $P_d$  be the transition probability matrix of the queue. Define  $P_{wd}$  and  $P_{wod}$  as follows:

$$P_{wd} = (1-\sigma) \begin{pmatrix} 0 & 0 & 0 & 0 \\ M & LB_1 & LB_2 & LB_3 \\ & M & LB_1 & LB_2 \\ & & M & LB_1 \\ & & & \ddots \\ & & & & LB_1 & LB_2 & L\overline{B}_2 & 0 \\ & & & & M & LB_1 & L\overline{B}_1 & 0 \\ & & & & & M & L & 0 \\ & & & & & & P & 0 \end{pmatrix}$$

and

$$P_{wod} = \begin{pmatrix} M & LB_1 & LB_2 & LB_3 \\ M\sigma & LB_1\sigma & LB_2\sigma & \\ & M\sigma & LB_1\sigma & \\ & & M\sigma & \\ & & & \ddots \\ & & & & M\sigma & LB_1\sigma & LB_2\sigma & L\overline{B}_2\sigma \\ & & & & M\sigma & LB_1\sigma & L\overline{B}_1\sigma & \\ & & & & & M\sigma & L\sigma & \\ & & & & & & P\sigma & \end{pmatrix}$$

where  $\overline{B}_i = \sum_{n=i+1}^m B_n$  and  $L = PA$ .

We can see that the transition probability matrix,  $P_d$  can be decomposed into two matrices,

$P_{wd}$  and  $P_{wod}$ , where  $P_{wd}$ ,  $P_{wod}$  is a matrix that contains transitions with a departure respectively without a departure. Therefore,

$P_d = P_{wd} + P_{wod}$ . We compute the generating function of the probability distribution of the interdeparture time, and then we obtain the

autocorrelation of the interdeparture time and the autocorrelation of the number departure per slot.

### 3.1 The Generating Function of the Interdeparture Time Distribution

Let  $t_n$  be the time interval between the  $(n-1)$ st and the  $n$ th departure. Also, let  $t_{ij}^n$ ,  $1 \leq i, j \leq L$  where  $L = m(K+1)$ , be the time interval to the moment that the state of the queue is  $j$  and the  $n$ th departure occurs given the queue is in state  $i$ , and  $t_i^n$ ,  $1 \leq i \leq L$ , be the time interval to the  $n$ th departure given that the queue is in state  $i$ . Define

$$D(z) = \begin{bmatrix} D_{1,1}(z) & & D_{1,L}(z) \\ & \ddots & \\ D_{L,1}(z) & & D_{L,L}(z) \end{bmatrix} \text{ and}$$

$$\overrightarrow{D}(z) = \begin{bmatrix} D_1(z) \\ \vdots \\ D_L(z) \end{bmatrix}$$

where  $D_{i,j}(z)$  and  $D_i(z)$  are the  $z$ -transforms of  $t_{ij}^n$  and  $t_i^n$ , respectively. Also, let  $P^+(s, n)$  be the probability that immediately after a departure the system is in state  $(s, n)$ . From the definition of  $D_{i,j}(z)$  and  $D_i(z)$ , we have following equations:

$$D(z) = z(I - zP_{wod})^{-1}P_{wd} \text{ and}$$

$$\overrightarrow{D}(z) = z(I - zP_{wod})^{-1}P_{wd}\vec{e}$$

where  $\vec{e} = [1, 1, \dots, 1]^T$ . Then, the generating function of the interdeparture time distribution  $D(z)$  can be obtained from as follows:

$$D(z) = \overline{P}^+ \overrightarrow{D}(z) = z \overline{P}^+ (I - zP_{wod})^{-1} P_{wd} \vec{e}$$

where

$$\overline{P}^+ = [P^+(1, 0), \dots, P^+(m, 0), P^+(1, 1),$$

$$P^+(2, 1), \dots, P^+(m, K)] = \frac{\vec{x} P_{wd}}{\vec{x} \vec{\lambda}_d}$$

From the generating function, we can obtain the moments of the time between successive departures, the squared coefficient of variation of the interdeparture time  $C_d^2$ , and throughput  $\rho_d$ .

### 3.2 The Autocorrelation of the Departure Process

In this section, we obtain the autocorrelation of the interdeparture time, and the autocorrelation of the number of departure per slot. In order to obtain the autocorrelation of the interdeparture time, we have

$$G_d(z_1 z_2) = E\{z_1^k z_2^{k+t}\} = \vec{P}^+ D(z_1) R^{k-1} \vec{D}(z_2) \vec{P}^+ z_1 (I - z_1 P_{wd})^{-1} P_{wd} R^{k-1} z_2 (I - z_2 P_{wd})^{-1} P_{wd} \vec{e} \quad (7)$$

where  $R = (I - P_{wd})^{-1} P_{wd}$ .

By differentiating equation (7) with respect to  $z_1$  and  $z_2$  and substituting  $z_1=1$  and  $z_2=1$  into equation (7), we have

$$E\{t_n t_{n+k}\} = \vec{P}^+ (I - P_{wd})^{-2} P_{wd} R^{k-1} (I - P_{wd})^{-2} P_{wd} \vec{e}.$$

The autocorrelation coefficient of the interdeparture time of an D-BMAP/Geo/1/K queue for lag  $k$ ,  $\varphi_d(k)$ , can now be obtained using expression (5)

Let  $X_n$  be the random variable representing the number of departures in the  $n$ th slot, where  $X_n=0,1$ . We have

$$E\{X_n\} = E\{X_n^2\} = \rho_d \quad \text{and}$$

$$E\{X_n X_{n+k}\} = \vec{x} P_{wd} P_d^{k-1} \vec{\lambda}_d$$

where  $\vec{\lambda}_d = [0, \dots, 0, 1 - \sigma, \dots, 1 - \sigma]^T$  and  $\vec{x}$  is the steady-state probability vector satisfying  $\vec{x} P_d = \vec{x}$ . The autocorrelation coefficient of the number of departures of the queue for lag  $k$ ,  $\varphi_d(k)$ , can now be obtained from (6).

Let us consider the autocorrelation of the interdeparture time of the queue. One of the most

interesting facts that we have observed is that the autocorrelation coefficients of the interdeparture time (correlogram) may fluctuate quite a lot<sup>[16]</sup>. As an example, consider the case where

$$P = \begin{bmatrix} 0.98 & 0.01 & 0.01 \\ 0.01 & 0.98 & 0.01 \\ 0.01 & 0.01 & 0.98 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0.9 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 0.1 \end{bmatrix}.$$

The correlogram for

$$B_A = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \\ 1.0 & 0.0 & 0.0 \end{bmatrix} \quad \text{and} \quad B_B = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

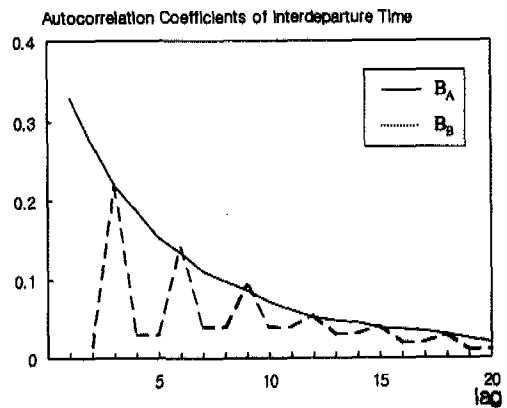


Fig. 1 Interdeparture correlation,  $\varphi_d(k)$

is shown in Figure 1. We note that for  $B_A$ , we have a smooth curve, whereas for  $B_B$ , we have an oscillating curve. This oscillation seems to be due to the variability of the number of arrivals per slot within the same state of the arrival process. Let us consider the example given in Figure 1 assuming that the batch size distributions are given by  $B_B$ . We note that when the arrival process is in state 3, the rate of arrivals  $a_3$  is very low. Also,  $b_{33}$  is quite large in relation to  $b_{31}$  and  $b_{32}$ . When the arrival process is in state 3, there may be long interarrival periods and the queue may empty out between successive batch arrivals. In this case, the pattern of the interdeparture times consists of one long interval followed by small intervals. This pattern causes the autocorrelation of the interdeparture time to fluctuate.

### IV. Characterization of the Departure Process

In this section, we obtain an approximation model for characterizing the departure process by a  $k$ -MMBP. This model captures the correlation and burstiness of the departure process of the queue. It can be shown that the output process of a D-BMAP/G/1/K queue is a D-MAP<sup>[8]</sup> and the

MMBP is a special case of the D-BMAP. Note that the fitted  $k$ -MMBP is characterized by the transition probability matrix  $P_{est}$  of the Markov process and  $\Lambda_{est}$  given by

$$P_{est} = \begin{bmatrix} p_{11}^{est} & p_{1k}^{est} \\ p_{k1}^{est} & p_{kk}^{est} \end{bmatrix} \text{ and } \Lambda_{est} = \begin{bmatrix} a_1^{est} & 0 \\ 0 & a_k^{est} \end{bmatrix}$$

where  $p_{ij}^{est}$ ,  $1 \leq i, j \leq k$ , is the transition probability that the fitted MMBP changes from state  $i$  to state  $j$ ,  $\sum_{j=1}^k p_{ij}^{est} = 1$  for  $1 \leq i \leq k$ , and  $a_i^{est}$ ,  $1 \leq i \leq k$ , is the probability that a slot contains a cell during the time that the MMBP is in state  $i$ . Therefore, a  $k$ -MMBP is characterized by  $k^2$  parameters. It is practically impossible to obtain these parameters using the method of moments, particularly when  $k$  is large. Other fitting techniques, such as minimum distance estimation and least squared estimation, can be used, but they are time consuming.

Unlike the case of the  $m$ -MMBP/Geo/1/K queue, we can see that the autocorrelation of coefficients of the interdeparture time of the queue can fluctuate as shown in Figure 1. Due to the characteristic of the departure process, the model proposed in the previous works<sup>[14-15]</sup> is not suitable for characterizing the departure process of a D-BMAP/Geo/1/K queue. The method estimates poorly the autocorrelation coefficients and the interdeparture time distribution. In this section, we present a simple method for fitting a  $k$ -MMBP to the departure process of a D-BMAP/Geo/1/K

queue. We note that we do not address the problem of how many stages the fitted MMBP should consist of.

#### 4.1 Model

The departure process of a queue is governed by the states of the queue. Therefore, we can obtain valuable information regarding the departure process from the states of the queue. By letting each state  $(s, n)$  be a separate state in the departure process, we can easily characterize the departure process as a D-MAP with  $P_0 = P_{wod}$  and  $P_1 = P_{wd}$ . Note that this D-MAP does not satisfy equations (1) and (2), and therefore, it is not an MMBP. However, we can have an exact MMBP characterization of the departure process of the  $m$ -MMBP/Geo/1/K queue only when  $\sigma = 0$ . In order to characterize the departure process by an MMBP, we have to obtain  $p_{ij}^{est}$  and  $a_i^{est}$  for  $1 \leq i, j \leq k$ , so that they satisfy equations (1) and (2). Given a state, then in the next slot a transition will occur with a departure or without a departure. Let  $(s, n)_{wod}$  and  $(s, n)_{wd}$  be the two states of the queue representing that the system shifted to  $(s, n)$  without a departure and with a departure, respectively. Then, we can separate all states  $(s, n)$  into  $(s, n)_{wod}$  and  $(s, n)_{wd}$ . Note that  $P(s, n) = P(s, n)_{wod} + P(s, n)_{wd}$  and  $P(s, K) = 0$  for all  $s$ . We can now consider states  $(s, n)_{wod}$  and  $(s, n)_{wd}$  for  $1 \leq s \leq m$ ,  $0 \leq n \leq K$  as a separate state of the fitted MMBP. The total number of states of the fitted MMBP is  $k = 2m(K+1)$ . Then, the departure process of the queue can be exactly characterized by the  $k$ -MMBP with matrices

$$P_{est} = P_0 + P_1 \text{ and } \Lambda_{est} = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}$$

where

$$P_0 = \begin{bmatrix} P_{wod} & 0 \\ P_{wod} & 0 \end{bmatrix}, \quad P_1 = \begin{bmatrix} 0 & P_{wd} \\ 0 & P_{wd} \end{bmatrix},$$

and  $I$  is a  $m(K+1) \times m(K+1)$  identity matrix.

We can see that the number of states of the

fitted MMBP is very large when  $m$  and  $K$  is large. That is, the computational complexity is directly proportional to the buffer capacity and the number of states of the Markov chain of the arrival process. We can significantly reduce the number of states by simply aggregating the states of the fitted MMBP. By only matching the interdeparture time distribution, the number of states of the fitted MMBP can be reduced to 2. In this case, however, we will ignore the autocorrelation which has a significant impact on the accuracy of the fitted MMBP. There is a trade-off between the number of states of the fitted MMBP and the accuracy of the estimated autocorrelation of the interdeparture time. In general, we can see that there is a large variation in the number of customers in the queue given that the arrival process is in a state which has a large variation in the number of arrivals. This variation can cause the autocorrelation coefficients of the interdeparture time to fluctuate. Due to this fact, a different grouping of the states than the one used in the previous works<sup>[14-15]</sup> has to be considered which gives a tractable number of states of the fitted MMBP and a satisfactory accuracy. It is, however, difficult to determine such a grouping. In this section, we introduce an intuitive grouping method for a  $k$ -MMBP characterization of the departure process.

Let us consider a state of the queue  $(s, n)$ . Note that the state  $(s, n)$  can be seen as the aggregate state of  $(s, n)_{wd}$  and  $(s, n)_{ud}$ . Let  $N_Q^i$  be the number of customers in the queue given that the arrival process is in state  $i$ . Also, let  $C_{N_Q^i}^2$  be the squared coefficient of variation of  $N_Q^i$ . Then,  $C_{N_Q^i}^2$  is defined by

$$C_{N_Q^i}^2 = \frac{Var\{N_Q^i\}}{E^2\{N_Q^i\}}.$$

The state classification is done based on the following two arguments. First, we consider state  $i$  which has a large variation in  $N_Q^i$  (for instance, state 3 in the example given in Figure 1). Intuitively speaking, during the time that the

process is in this state,  $N_Q^i=0$  for a long period. Then,  $N_Q^i$  can be suddenly changed to  $B_i$  with a batch arrival. Subsequently,  $N_Q^i$  is reduced gradually due to successive departures, and it finally becomes 0. That is, during state  $i$ , the pattern of the successive interdeparture time is as follows: one long interval followed by several consecutive short intervals. This pattern can create fluctuation in the autocorrelation coefficients of the interdeparture time. Also, we can argue that  $P(i, n), n=0, \dots, B_i$  is significantly larger than  $P(i, n)$  for  $B_i < n \leq K$ . In view of this, we group all states  $(i, n)$  which have insignificant values of  $P(i, n)$  into a single state of the fitted MMBP, and each state  $(i, n)$  which has a significant value of  $P(i, n)$  is considered as a separate state of the fitted MMBP. The second argument that we can use for the state classification is the following: Let us now consider state  $j$  which has a small variation in  $N_Q^j$  (for instance, state 2 in Figure 1). This state has a small effect on the fluctuation of the correlation coefficients of the interdeparture time. Note that the departure rate depends on the states of the queue. When the queue is not empty, the departure rate in a slot depends only on the parameter of the service time distribution  $\sigma$ . When the queue is empty, the departure rate in a slot depends on the state of the arrival process.

Using the above two arguments we can classify the states as follows. For state  $i$  of the arrival process which has  $C_{N_Q^i}^2 \geq q$ , each state  $(i, n), n=0, \dots, B_i+1$ , is considered as a separate state of the fitted MMBP. For a state  $j$  of the arrival process which has  $C_{N_Q^j}^2 < q$ , state  $(j, 0)$  is also considered as a separate state of the fitted MMBP. Note that states  $i$  and  $j$  are not the states which have the highest peak arrival rate. In order to simplify the presentation below, state 1 of the arrival process is assumed to be the state which has the highest peak arrival rate, i.e.  $a_1^A = \max(a_i^A)$ . All remaining states are grouped into a state of the fitted MMBP. We define  $S_i, 1 \leq i \leq k$  to be the set of all states of the queue



which belong to state  $i$  of the  $k$ -MMBP departure process. We have the following grouping of the states:

$$S_i = \{(s, 0): s \neq 1, C_{N_0}^2 < q\} \text{ for } 1 \leq i \leq k^*$$

$$S_j = \{(s, n): s \neq 1, C_{N_0}^2 \geq q, \text{ and } 0 \leq n \leq B_s + 1\}$$

for  $k^* < i < k$

$$S_k = \{\text{all remaining state}\}$$

where  $k^*$  is the total number of states which have  $C_{N_0}^2 < q$  and  $B_s$  is the maximum size of a batch during state  $s$ . Note that  $q$  is empirically set to 1.

Now, we can obtain  $P_{est}$  and  $\Lambda_{est}$  based on the above grouping of the states. The parameters of the fitted MMBP,  $p_{ij}^{est}$  and  $a_i^{est}$ ,  $1 \leq i, j \leq m$ , can be calculated as follows:

$$p_{ij}^{est} = \frac{\sum_{(s, n) \in S_i} P(s, n) \left[ \sum_{(s, n) \in S_j} tr[(s, n) \rightarrow (\bar{s}, \bar{n})] \right]}{\sum_{(s, n) \in S_i} P(s, n)}$$

$$a_i^{est} = \frac{(1 - \sigma) \left[ \sum_{(s, n) \in S_i} P(s, n) \right]}{\sum_{(s, n) \in S_i} P(s, n)}$$

where  $tr[(s, n) \rightarrow (\bar{s}, \bar{n})]$  is the transition probability the process changes from a state  $(s, n)$  to state  $(\bar{s}, \bar{n})$ .

This method always gives a feasible set of parameters which satisfy the basic conditions,  $0 < p_{ij}^{est} < 1$  and  $0 \leq a_i^{est} \leq 1$  for  $1 \leq i, j \leq k$ .

### 4.2 Validation

Extensive tests were carried out in order to establish the accuracy of the estimated MMBP. In particular, we considered a D-BMAP/Geo/1/ $K$  queue with  $K=8$ ,  $\sigma=0.1$ , and  $m=2, 4, 6$ . The parameters of the arrival process were varied so that the departure process corresponded to different values for  $\rho_d$  and  $C_d^2$  and different patterns of fluctuation in the autocorrelation coefficients of the interdeparture time. 24 different test cases were thus created.

Table 1. Validation result

Ex	m	k	$\rho_d$	$C_d^2 / C_m^2$	$\epsilon_d(n)$	$\epsilon_a(n)$	$\epsilon_p(n)$	n
1*	2	5	4.603e-1	4.409e+1/4.450e+1	4.897e+1	4.892e+1	1.956e-3	2736
2	2	5	4.531e-1	4.086e+2/4.096e+2	2.225e-2	1.306e+0	2.045e-4	5000
3*	2	2	8.940e-1	1.103e+1/1.102e-1	2.115e-2	1.805e-2	6.008e-5	40
4	2	2	5.851e-1	3.182e+0/3.267e+0	6.622e-1	9.789e-1	1.324e-2	262
5*	2	5	6.363e-1	1.039e+1/1.049e+1	4.134e+2	1.382e+2	2.537e-2	3015
6	2	5	1.200e-1	1.099e+2/1.098e+2	1.981e-2	2.437e-1	2.605e-1	5000
7*	2	5	4.838e-1	3.732e+0/3.796e+0	3.077e-1	1.113e-1	6.197e-2	1621
8	2	5	1.833e-1	1.133e+0/1.185e+0	1.827e-2	3.997e-2	4.436e-2	215
9*	4	9	6.577e-1	7.797e+1/7.799e+1	8.586e+1	4.453e+2	3.352e-4	5000
10	4	14	7.350e-1	5.533e+1/5.539e+1	1.593e-1	3.163e-1	2.406e-4	5000
11*	4	9	8.320e-1	3.517e-1/3.634e-1	1.360e+0	2.625e+0	7.390e-3	69
12	4	4	8.995e-1	1.008e-1/1.009e-1	1.626e-3	1.242e-3	8.425e-5	37
13*	4	19	1.388e-1	9.073e+1/9.075e+1	3.556e+1	1.838e+2	3.210e-2	5000
14	4	19	3.864e-1	3.538e+2/3.537e+2	7.491e-1	1.810e+2	3.024e-2	5000
15*	4	9	1.591e-1	7.906e+0/7.976e+0	3.121e+1	2.127e+2	2.344e-2	1043
16	4	19	2.318e-1	2.962e+0/2.744e+0	4.819e-2	1.825e-1	4.402e-2	346
17*	6	20	6.129e-1	6.575e+1/6.579e+1	3.293e+0	2.287e-1	6.902e-4	5000
18	6	13	6.401e-1	6.148e+1/6.160e+1	5.503e-1	8.639e+1	4.865e-4	4302
19*	6	6	8.997e-1	1.004e-1/1.005e-1	1.330e-3	1.491e-3	3.623e-5	26
20	6	41	8.704e-1	6.157e+1/6.159e+1	4.157e+1	2.793e+1	3.365e-3	5000
21*	6	27	2.531e-1	1.133e+1/1.139e+1	2.775e+1	1.216e+2	7.121e-2	1085
22	6	41	2.441e-1	2.064e+2/2.063e+2	6.807e-2	3.670e+0	1.746e-1	5000
23*	6	27	2.327e-1	4.741e+0/4.824e+0	7.337e-1	3.110e+0	4.503e-2	509
24	6	41	3.131e-1	3.774e+0/3.667e+0	4.803e-2	5.134e-2	2.399e-2	343

\*  $\phi_d(i)$  oscillates

The validation results are given in Table 1. One of the measure of accuracy employed was  $\epsilon_D(n)$  given by

$$\epsilon_D(n) = \sum_{i=1}^n |P(D=i) - P_{est}(D=i)|$$

where  $P_{est}(D=i)$  is the estimated probability that the interdeparture time is equal to  $i$  slot(s) and  $n$  is the number of distribution points that were compared.  $n$  was selected so that

$$\sum_{i=1}^n P(D=i) \approx 1.$$

The value for  $n$  for each test case is also reported in table 1. Also, we give the number of states of the fitted  $k$ -MMBP. For each case, we also give errors computed using the expressions

$$\epsilon_\phi(n) = \sum_{i=1}^n |\phi_d(i) - \phi_{est}(i)| \text{ and}$$

$$\epsilon_\varphi(n) = \sum_{i=1}^n |\varphi_d(i) - \varphi_{est}(i)|$$

where  $\psi_{est}(i)$  is the estimated autocorrelation coefficient of the interdeparture time for lag  $i$  and  $\varphi_{est}(i)$  is the estimated autocorrelation coefficient of the number of departures for lag  $i$ . We also give the values for the squared coefficient of variation of the interdeparture time of the fitted MMBP,  $C_{est}^2$ .

The estimated autocorrelation coefficients of the interdeparture time using the model can follow only the pattern of fluctuation but not each value of the exact  $\psi_d(i)$ . Note that we can have large  $\varepsilon_D(n)$  when  $C_{N_b}^2 \geq 1$  as in example 6 and 22 of Table 1. We can have a large number of states of the fitted MMBP when the number of states of the arrival process  $m$  and the maximum size of a batch  $B_s$  are large.

Table 2. The characteristics of the arrival processes

Example	m	$\rho_c$	$C_s^2$	$\phi_c(1)$	$\phi_c(1)$
1	4	9.055e-1	3.61-e+1	2.842e-1	2.586e-1
2	4	8.139e-1	9.662e+1	2.922e-2	7.011e-1
3	2	6.750e-1	3.693e+0	3.883e-1	4.217e-1

Table 3.  $\rho_d$ ,  $C_d^2$ , and cell loss probability for node 2

Example	fitting model	simulation/exact analysis	
1	$\rho_d$	6.4676e-1	6.4320e-1 ± 2.8864e-3
	$C_d^2$	4.4570e+1	4.4349e+1 ± 8.6560e-1
	cell loss	7.1830e-3	8.7225e-3 ± 1.1850e-4
2	$\rho_d$	3.9804e-1	3.9922e-1 ± 1.7183e-3
	$C_d^2$	8.9489e+1	8.8868e+1 ± 1.0232e+0
	cell loss	8.2549e-3	8.2808e-3 ± 1.1034e-4
3	$\rho_d$	5.3299e-1	5.3280e-1 (exact analysis)
	$C_d^2$	3.5382e+0	3.4283e+0 (exact analysis)
	cell loss	2.0537e-2	2.0887e-2 (exact analysis)

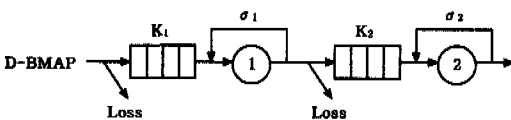


Fig. 2 A two-node tandem queueing network

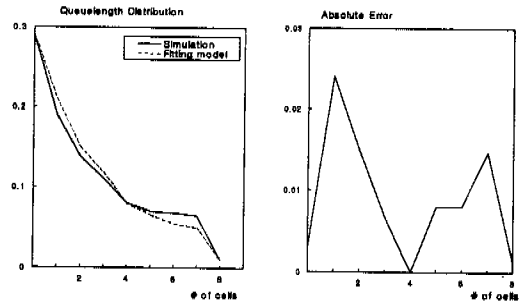


Fig. 3 QLD of node 2 and absolute error (Example 1)

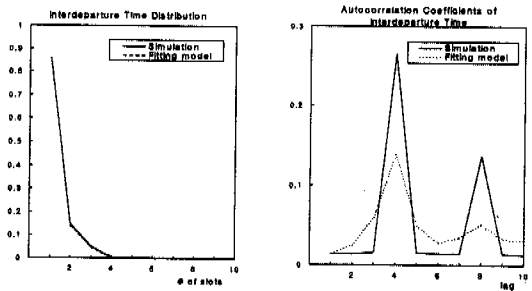


Fig. 4  $P(D=i)$  and  $\psi_d(i)$ , for  $1 \leq i \leq 10$ , of node 2 (Example 1)

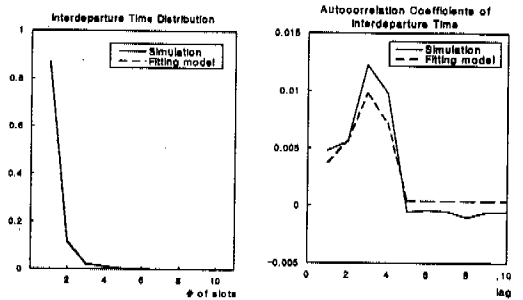


Fig. 5 QLD of node 2 and absolute error (Example 2)

We further validate the fitting model by using it to analyze approximately a two-node tandem configuration of discrete-time finite capacity queues. Let us consider an open queueing network consisting of two nodes linked in tandem as shown in Figure 2. 3 different examples were considered, the first two corresponding to a case of a 4-state D-BMAP as input traffic to the first node and the other to a case of a 2-state D-BMAP. Note that the autocorrelation coefficients of the interdeparture time of the output traffic from the first node fluctuate more

in example 1 than in example 2. The output traffic from the first node in examples 1 and 2 is bursty. The characteristics of the arrival process to the first node for the three examples are given in Table 2. The values of  $K_i$  and  $\sigma_i$  for examples 1 and 2 are:  $K_i=8$  and  $\sigma_i=0.1$  for  $i=1,2$ . The values of  $K_i$  and  $\sigma_i$  for example 3 are:  $K_i=4$  and  $\sigma_i=0.1$  for  $i=1,2$ . The approximation results for examples 1 and 2 were compared against simulation data in Figures 3 to 6 and in Table 3. The approximation results for example 3 were compared against exact values in Figures 7 to 8 and in Table 3. The exact values were obtained by fitting an exact MMBP to the departure process of node 1. This MMBP was obtained using state classification of  $(s, n)_{wd}$  and  $(s, n)_{wd}$ . In particular, figures 3 to 4 are for example 1, figures 5 to 6 for example 2, and figures 7 to 8 for example 3. Note that for the exact analysis the total number of states of fitted MMBP is 72 for examples 1 and 2. The exact analysis is time-consuming and computationally complex procedure. Therefore, the approximation results for examples 1 and 2 were compared against simulation results.

In Figure 3 we give the queue length distribution and corresponding absolute errors node 2. In Figure 4 we give the interdeparture time distribution  $P_{est}\{D=i\}$  and the autocorrelation coefficients of the interdeparture time  $\psi_{est}(i)$ ,  $i=1, \dots, 10$ , for node 2. We give the throughput, the squared coefficient of variation of the interdeparture time, and cell loss probability for node 2 in Table 3. We note that the confidence intervals were not plotted in certain graphs as they were extremely small. The approximate results for examples 2 and 3, given in Figures 5 to 6 and in Figures 7 to 8, respectively, are presented in the same way as in example 1. We can see that even though  $\psi_{est}(i)$  of the fitted MMBP follows only the pattern rather than the values of  $\psi_{est}(i)$ , as shown in Figure 8, the model gives a satisfactory accuracy of performance analysis on the downstream node.

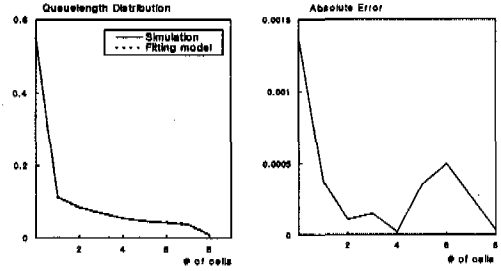


Fig. 6  $P(D=i)$  and  $\psi_{est}(i)$ , for  $1 \leq i \leq 10$ , of node 2 (Example 2)

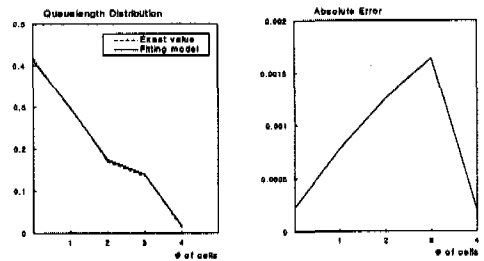


Fig. 7 QLD of node 2 and absolute error (Example 3)

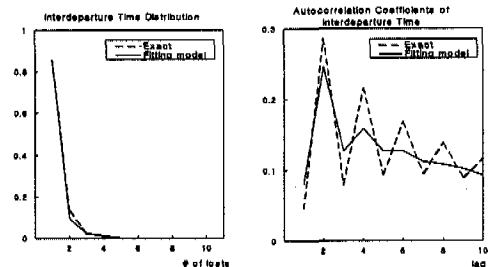


Fig. 8  $P(D=i)$  and  $\psi_{est}(i)$ , for  $1 \leq i \leq 10$  of node 2 (Example 3)

## V. Conclusion

In this paper, we obtained the generating function of the interdeparture time distribution and the autocorrelation of the departure process of a D-BMAP/Geo/1/K queue. The departure process of this queue was characterized approximately by an MMBP in order to capture both the burstiness and correlation of the departure process. The tractable fitting model for characterizing the departure process of the queue by a  $k$ -MMBP is proposed and its accuracy was examined through extensive validation tests.

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