RBF 신경망 등화기 운용에 있어서의 센터수 축소화 알고리즘

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A New Algorithm for Reducing the Number of Centers in Operating the RBF Neural Net Equalizer

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요 약

이 논문의 목적은 기존의 RBF 등화기에 사용되는 RBF 센터수를 현격히 감소화시켜 그 효율성을 향상시키는데 있다. 감소화의 기본적인 아이디어는 서로 다른 결정 클래스 사이의 경계선에 근접한 센터들만을 선택하여 사용하는 것이다. RBF 등화기 시스템을 줄이는데 있어서, 가장 중요한 요소는 2^d 로서 여기서 d는 채널 지연을 나타낸다. 이러한 센터수의 감소화는 RBF 등화기 트레이닝에 요구되는 계산의 부담을 현저히 감소화 시킴으로서 RBF 등화기의 운용을 소프트웨어 및 하드웨어적으로 가능케한다. 현격히 감소화된 센터수를 사용한 새로운 RBF 등화기의 오률 (error rate) 특성이 기존의 RBF 등화기 시스템과 필적한다는 것을 시뮬레이션 결과를 통하여 보여주고 있다.

ABSTRACT

This paper concerns with improving the previously developed RBF equalizer by greatly reducing the number of centers. The basic idea is to select only centers close to the boundary between the different decision classes. The first factor of reducing the network is 2^d where d is the channel delay. The number of centers was further reduced by representing several centers by a single point. This reduction of centers greatly reduces the burden of computation in training, and makes the hardware implementation of RBF equalizers realistic. Simulation studies show that the error rate performance of an RBF equalizer with the proposed reduction in the number of centers compares favorably with the RBF equalizer having the conventional number of centers.

I. 서 론

1. Preliminaries

In digital communication systems, data symbols are transmitted at regular intervals, but time dispersion caused by the imperfect channel frequency response or multipath transmission creates overlapping of the received symbols, or intersymbol interference (ISI). The most widely known equalizer is an adaptive linear transversal equalizer, in which the output signal is compared to the expected signal and the tap (FIR filter) coefficients are updated in accordance with the error between the desired and actual filter output. After an initial convergence has been obtained during a training period, the output decision can be substituted for the desired signal (decision-directed learning)[1].

Recently, some researchers developed radial basis function (RBF) equalizers [2]-[5] which are simpler

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and faster to train because of their structural simplicity. The RBF equalizer systems usually require a large number of centers which increases the computational complexity, and thus makes its hardware/software implementation impractical.

In[5], it is shown that an RBF equalizer provides maximum likelihood decisions. Autocorrelation techniques were employed to determine the channel order, which is required to specify the number of centers, and the supervised K-means clustering algorithm and least mean square (LMS) algorithm were used for estimating the desired RBF centers and updating the output layer weights respectively. However, a higher channel order requires a correspondingly larger equalizer order, and may lead to the requirement for a very large number of RBF centers; this results in exponential increase of computational complexity. Thus it becomes necessary to consider a method for selecting a reduced number of centers, rather than using all possible centers. Thus many researchers have been focusing on reducing the computation burden in operating the RBF equalizer. Recently, there has been a study related to the reduction of computational burden by replacing the RBF centers with scalar centers[11]. This method, however, still use all the possible number of centers, as required In[5].

This paper provides a simple solution which uses completely unique method in reducing the number of RBF centers by using channel delay. The main purpose of this paper is to solve the problem of requiring a large number of centers in the RBF equalizer system when channel order is high. The basic idea is to select the lesser number of centers without degrading the error rate performance. The main factor in reducing the number of centers is to consider the channel delay where the dominant impulse response exists.

2. Radial Basis Function Networks

The RBF network[6] is a three-layer network whose output is a linear combination of the basis function outputs, as depicted in Fig. 1. Each unit of the hidden layer produces a function of the normed distance (usually Euclidean) between its own reference center and the network input. A common choice of basis functions for the hidden nodes in the network is the Gaussian basis functions. The output response of an RBF network is a mapping F

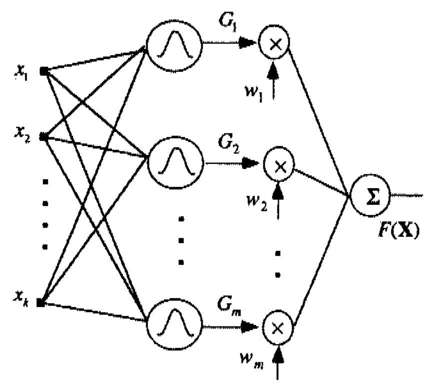


Fig. 1 Schematic diagram of an RBF network

$$F(X)=\sum_{i=1}^{m} w_i G_i, \tag{1}$$

where

$$X=[x_1, x_2, \cdots, x_m]^T \tag{2}$$

$$G_i=\exp\left(\frac{-\|X-C_i\|^2}{2\sigma_i^2}\right) \quad i=1,2,\cdots,m \tag{3}$$

X, G_i, w_i, C_i, and σ_i denote input vector, Gaussian basis function output, output layer weight, center vector, and center spread parameter respectively. The output response of the Gaussian basis function depends only on the Euclidean norm of the difference between the centers and input vectors. In that way, the Gaussian basis function gives a strong response to the inputs for which the difference between the center and input vectors is small. On the other hand, if the difference of these two is large, the response is weak.

There are different approaches to training an RBF networks, depending on how the centers are specified and how the center spread parameters are specified[7]-[9]. Conventional training of the previously developed RBF equalizer consists of two stages; the first stage is to determine (estimate) the number and location of center points, and the second stage is to update the output layer weights. As will be seen in Section II, the proposed algorithm for reducing the number of desired centers, based on the supervised K-means clustering [5], was developed by considering the channel delay and distance criterion.

3. Outline of the Paper

Section II describes a new algorithm for reducing the number of RBF centers. In Section III, error rate performance of the proposed RBF equalizer is compared to a linear equalizer and an RBF equalizer having the conventional number of centers. Section IV provides the conclusion of the paper.

II. Reduction of the Number of Centers

1. Maximum Number of RBF Centers

We consider the RBF equalizer system, as shown in Fig. 2. A consecutive symbol sequence a_k is transmitted into a dispersive channel which has the transfer function

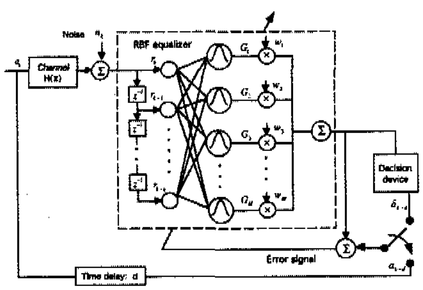


Fig. 2 The structure of the RBF equalizer system

$$H(z) = h_0 + h_1 z^{-1} + \cdots + h_p z^{-p} \tag{4}$$

where p denotes the channel order, and the transmitted symbol sequences are assumed to be binary equiprobable. Then the received signal, corrupted by additive noise, is represented as

$$r_k = h_0 a_k + h_1 a_{k-1} + \cdots + h_j a_{k-j} + \cdots + h_p a_{k-p} + n_k. \tag{5}$$

The equalizer input vector is

$$\boldsymbol{R}_k = [\, r_k, r_{k-1}, \cdots, r_{k-q}]^T \tag{6}$$

where q denotes the order of the comparable linear equalizer (the number of units in the input layer is $q+1$). Because the channel input vector corresponding to $\boldsymbol{R}_k$ is

$$\boldsymbol{A}_k = [\, a_k, a_{k-1}, \cdots, a_{k-p-q}]^T, \tag{7}$$

the number of possible equalizer input vectors is

$$M = 2^{p+q+1}. \tag{8}$$

Thus, there exist M possible candidates for the RBF centers used in training. In the previous work [5], the locations of these M centers were estimated using the supervised K-means clustering, and used for training the RBF equalizer. In this case channel order estimation was required to determine the number of centers, M.

2. Reduction Process

As shown in (8), the number of centers exponentially increases with the increase of channel order and equalizer order. This results in considerable computational complexity, and thus makes the hardware implementation of the RBF equalizer impractical.

The approach of reducing the number of centers relies on the fact that when only the desired centers close to the boundary between two different classses are used, the error rate performance is almost the same as when all the centers are used. The first step is to reduce the maximum number of RBF centers, $M(=2^{p+q+1})$, by a factor of 2^d, where d is the channel delay. Then the number of centers is further reduced by representing several centers by a single point. This simplifies the equalizer for the channel whose order is high enough to make the center distribution very dense.

The proposed method of reducing the number of centers first considers a channel input vector only to length d. Then

$$A^{sub}_{k-1} = [a_{k-1}, a_{k-2}, \cdots, a_{k-d}]^T \quad (9)$$

which allows A_k to be divided into 2^d sub groups. In order to set up the learning algorithm, we denote the M combinations of A_k as A_j, $1 \le j \le M$, and the 2^d combinations of A^{sub}_{k-1} as sub_n, $1 \le n \le 2^d$ respectively. The following is the algorithm for the best choice for the reduced number of centers.

Algorithm

Step 1 : The supervised K-means clustering algorithm operates by considering the A^{sub}_{k-1}

```
if( A_k = A_j){
if( A^sub_{k-1} = sub_n && a_k = -1 ||
    A^sub_{k-1} = sub_{2^d+1-n} && a_k = 1){
    counter_j = counter_j + 1;
    C^{k,n}_j = ((counter_j - 1) · C^{k-1,n}_j + R_k) / counter_j ;
    }
}                                                   (10)
```

where $C^{k,n}_j$ denotes the jth center in category n

$$C^{k,n}_j = [C^{k,n}_{j0}, C^{k,n}_{j1}, \cdots, C^{k,n}_{jq}]^T \quad (11)$$

Step 2 : Find the center category J, for which the maximum value of $C^{k,n}_j$ is smaller than that of other categories:

$$\left.\begin{array}{l} D_n = \max\{C^{k,n}_{j0}, n = 1, 2, \cdots, 2^d\} \\ J = \arg[\min\{D_n\}], j = 1, 2, \cdots, M \end{array}\right\} \quad (12)$$

where D_n represents the maximum value of $c^{k,n}_{j0}$ among all the centers in category n.

Step 3 : Find all the centers in category J.

Step 4 : Sort the selected centers in ascending order of index j of $C^{k,J}_j$, and set C^k_l, $1 \le l \le L$ as a set of ordered centers, where $L = M/2^d$.

Step 5 : Find the average distance (AD) between the +1 centers and -1 centers; here +1 stands for the centers whose corresponding received symbol, $a_{k-d} = 1$, while -1 stands for the centers corresponding to a received symbol $a_{k-d} = -1$.

```
for(l = 1; l ≤ L/2; ++l){
AD = AD + fabs( c^k_{l0} - c^k_{(l+L/2)0});
}                                                   (13)
AD = AD / (L/2)
```

Step 6 : Check if $(AD \ge \rho)$, where ρ is a distance parameter in the range $0.5 \le \rho \le 1.0$ used to keep the centers properly separated without overlapping (severe intersymbol interference causes the regions containing +1 and -1 centers to overlap)

If NO, then the current candidate category is rejected (inhibited); return to **Step 2**

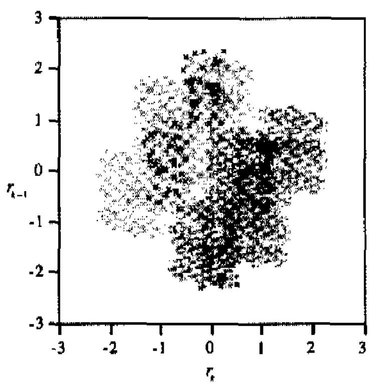
If YES, stop (all the centers in category J will be used to train the proposed RBF equalizer).

The main difference between the above algorithm and the conventional algorithm in [5] is that the conventioanal algorithm in [5] is just doing process to select all the possible centers $M = 2^{p+q+1}$ through step 1 (supervised k-means clustering) only, but the proposed method is to select the reduced number of centers from M through step 1-6, with considering the channel delay and average distance (AD). In other words, step 1 is for categorizing all the centers with one of the 2^d types of center categories. Step 2-3 describes the process of selecting the centers with selected category J. Step 4-6 checks if the selected centers with category J are properly separated by considering AD.

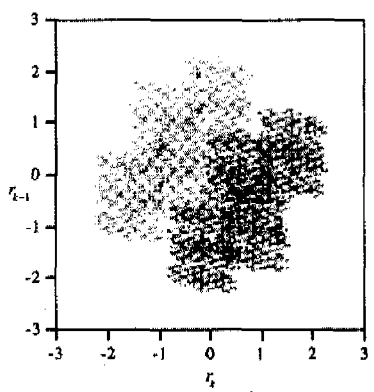
Finally, the above method shows that the number of centers in operating RBF equalizer has been reduced from $M = 2^{p+q+1}$, required in conventional method [5], to $M/2^d$ by considering channel delay d. Furthermore, for the channel whose order is high enough to make the distribution of channel output states very dense, the reduced number of consecutive centers, C^k_l, $1 \le l \le L$ in step 4, is further reduced by averaging each group of 2^k centers where $L/2^k$ is the final number of centers. The simulation results with various channel models are presented in the next section.

3. Results of Reducing the Number of Centers

Simulation studies were performed over several types of channel models. For the high-order channel, a typical telephone channel impulse response was used[10]. Fig. 3(a) shows that the proposed algorithm reduced the number of centers to 128 from 4096 ($M=2^{p+q+1}=4096$, $p=10, q=1$). For the second stage, groups of 16 consecutive centers, ordered according to their binary value, were replaced by one center located at the average of the 16, as illustrated in Fig. 3(b).



(a)



(b)

Fig. 3 Distribution of the selected centers, ρ =1.0:
(a) distribution of 128 selected centers,
(b) distribution of 8 selected centers
$H(z)=0.04-0.05z^{-1}+0.07z^{-2}-0.21z^{-3}-0.5z^{-4}+0.72z^{-5}+0.36z^{-6}+0.0z^{-7}+0.21z^{-8}+0.03z^{-9}+0.07z^{-10}$

Error rate performance of RBF equalizers with reduced number of centers is presented in Section III, and compared with both the RBF equalizer with the full number of centers and with the linear equalizer.

Ⅲ. Equalizer Performance Comparison

The error rate was measured for the typical telephone channel models. The comparison of error rate performance between the RBF equalizers with and without reducing the number of RBF centers, based on the conventional LMS training method.

1. Simulation Results

For the high order channel case, the reduction of the number of centers was notable, as illustrated in Fig. 4. The number of centers was first reduced to 2048 from 65,536 ($M/2^d=65536/32=2048$), and further reduced to 32 by averaging groups of 64 consecutive centers from the first selection. Simulation results show that the error rate performance of the RBF equalizer with the reduced number of centers is approximately the same as with the full number of centers.

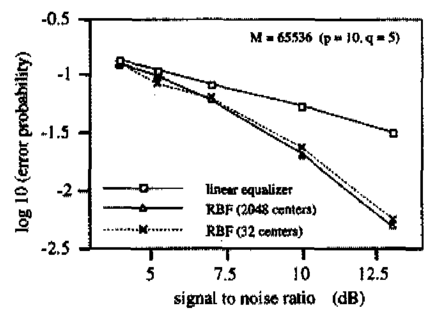


Fig. 4 Error rate performance with reduced number of centers
$H(z)=0.04-0.05z^{-1}+0.07z^{-2}-0.21z^{-3}-0.5z^{-4}+0.72z^{-5}+0.36z^{-6}+0.0z^{-7}+0.21z^{-8}+0.03z^{-9}+0.07z^{-10}$

2. Concluding Remarks

As shown in all the results above, the RBF equalizer with the proposed algorithm for reducing the number of centers performed as well as with the full number of centers. The step of averaging the first selected centers further reduces the number of centers, while maintaining approximately the same error rate performance.

Ⅵ. Conclusions

This paper described a new RBF equalizer system that was developed mainly for overcoming the obstacle of the large number of centers required for the previously developed RBF equalizers. The design procedure of the proposed RBF equalizer system is as follows:

1) Estimate the channel order, and channel delay using the autocorrelation techniques.

2) Determine the RBF centers used in training, using the proposed center reduction algorithm. The number of RBF centers after the first reduction is $M/2^d = 2^{p+q+1-d}$, where M is the maximum number of centers. When channel order is high enough to make the center distribution dense, the number of RBF centers can be reduced further by averaging the selected centers.

3) Update the output layer weights using the selected centers.

Throughout the simulation studies, it was found that the error rate performance of an RBF equalizer with the reduced number of centers compared favorably with the RBF equalizer having the conventional number of centers, and both performed better than the linear equalizer. This improvement in the RBF equalizer makes its hardware implementation practical.

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