Performance Analysis of Dual-Mode Constant Modulus Algorithm

Jai-Chul Song*, Byung-Lok Cho**, Gordon Stuber***, ***Regular Members***

ABSTRACT

In this paper, a class of Stop-and-Go Dual-mode blind Algorithms for QAM signal set is analyzed and its performance evaluated. The concept of these algorithms is based on the Dual-Mode algorithm, the Godard algorithm and the Stop-and-Go algorithm. In order to evaluate the Performance of these algorithms, computer simulation are performed for the 32QAM signal constellations. Form the simulation results, we can verify that these algorithms converges very fast compare to conventional Dual-Mode algorithm.

I. INTRODUCTION

Blind Equalizers of self-recovering equalizers have recently received a freat deal of attention. Several papers have appeared in the paper[1][2][3][4]. Recently, among the various blind equalization algorithms, the Constant Modulus Algorithm (CMA), which originated from the Godard Algorithm have been widely used for blind channel equalization for complex data systems[2]. The advantage of CMA is that the equalizer convergence does not depend on carrier recovery since the cost function depends only on the absolute value of output of the equalizer.

This allows the equalizer and the carrier recovery loops to be decoupled which results in simpler design and implementation. In practice, the CMA is probably the most widely adopted algorithm for blind channel equalization. However, a major drawback of the CMA is that it converges very slowly for QAM signal constellations. In addition, the residual error after convergence is unacceptably large even when the equalizer tap weights have converged to their optimal values. Therefore, a relatively small step size is required for the CMA to achieve an acceptable steady-state error. The small step size leads to slow convergence rate, one may use gear-shift the adaptation step size to smaller values at appropriate time intervals. This allows the use of a large adaptation step size after convergence is achieved and a smaller adaptation step size after convergence for a reduced residual error. When the adaptation error can be controlled to a level low enough for the carrier recovery circuit to achieve stable lock, the usual practice is to switch to Decision Directed(DD) mode for coefficient updates. However, it is very difficult to determine the proper timing for making the switch.

In order to solve this problem, a dual-mode CMA was developed[5]. The main idea of this method is that the transitions between the two modes are automatically done by evaluating an error function determined by the radius defined as [5] of the equalizer output. If the equalizer output has a relatively large error level, the equalizer considers itself far from optimum and thus adapts the CMA to adjust its tap weights. However, if the error level of the equalizer output comes within a predetermined range, the equalizer considers itself close to optimum and thus updates its tap weights.

Even though the Dual-mode algorithm switches

* Induk Institute of Technology, Department of Information & Communication
** Sunchon National University, Department of Electronics Engineering
*** Georgia Institute of Technology, School of Electrical & Computer Engineering
논문번호 : 98371-0826, 접수일자 : 1998년 8월 26일

in two modes, this algorithm's problem still remains.

Even when the adjustment is in the wrong direction, it never stops adjusting the equalizer tap weights. In order to solve this problem, we can apply the Stop-and-Go concept to dual-mode algorithms proposed by Pittch and Prati [4].

In this algorithms for blind equalization for QAM signal set is proposed and their performance evaluated under the multipath fading channel environment.

Ⅱ. DESCRIPTION OF DUAL-MODE BLIND EQUALIZER ALGORITHMS

Dual-mode algorithm for blind equalization was proposed by Weerackody and Kassma[5]. In general, because of stability considerations, the value of step size that can be used in the blind equalization algorithms are smaller than the corresponding values used in the Least Mean Square(LMS) algorithms.

Since the step size of these type of algorithms, blind equalizers have very slow convergence rates. After converging, residual error term does not go to zero.

Therefore, for the same steady state Mean Square Error(MSE), the value of step size that is used in the blind equalization algorithms has to be significantly smaller than its permissible value for the LMS algorithm. Because of above reasons, convergency of blind equalization algorithms becomes slow.

In order to increase the convergence rate of blind equalization algorithms, dual-mode algorithm which makes the value of steady state residual error very small without changing its transient values significantly. With this modification, it is possible to use a larger value for step size in blind equalization algorithms. So we can expect faster convergence characteristics for the same steady state MSE.

Suppose the channel impulse response, input, output, and noise are denoted by h(n), a(n), x(n) and n'(n) respectively.

$$x(n) = a(n) * h(n) + n'(n) \tag{1}$$

where, * denotes linear convolution. x(n) is sent to a tap-delay line equalizer with a tap length N=2L+1. Denoting the input data vector of the equalizer by [x(n)==[x(n+L) …… x(n-L)] and the equalizer tap weight vector W, we may express the equalizer output z(n) can be expressed as

$$z(n) = \mathbf{W}(n)^T \cdot \mathbf{x}(n) \tag{2}$$

DMCMA(Dual Mode Constant Modulus Algorithm) which is proposed in [5] can be expressed as

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \mu(|z(n)^2| - R_k^2) \cdot \mathbf{x}^*(n)$$

$$z(n) \in D_k, \quad k = 1, 2, 3, \cdots \tag{3}$$

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \mu(|z(n)^2| - R^2) \cdot z(n) \cdot \mathbf{x}^*(n)$$

$$z(n) \notin \bigcup D_k, \quad k = 1, 2, 3, \cdots \tag{4}$$

where, μ denotes the step size, $\bigcup D_k$ denotes the union of the annular regions D_k, $k=1,2,3,\cdots$ D_k contains the QAM signal points which possess a common amplitude R_k. In the case of 32QAM, R_k is equal to $\sqrt{2}$, $\sqrt{10}$, $\sqrt{18}$, $\sqrt{26}$, $\sqrt{34}$, $\sqrt{50}$ respectively. D_k is also bounded by its inner radius $R_k - d$ and outer radius $R_k + d$, where d is a predetermined constant. where, R^2 which is already defined in [2] is expressed as follows:

$$R^2 = E\left[\frac{|a(n)|^4}{|a(n)|^2}\right] \tag{5}$$

The DMCMA does not make judgement on whether a particular adjustment is correct or not. Since the equalizer is in a blind equalization mode, many adjustments could actually be in wrong direction. If we can somehow avoid some of those incorrect adjustments, the convergence behavior of the equalizer will be improved. Such an idea was first proposed in[4],[5], where a blind "stop-and-go" decision directed equalizer decided whether adaptation of the current iteration should "stop" or "go" by observing a simple flag. The flag suggests "go" by if the self-determined output error is sufficiently reliable for adaptation, and suggests "stop" otherwise.

In order to derive a class of new Stop-and-Go Dual-Mode Blind Equalization Algorithms. We first consider the following possible cost functions[6].

$$J_1 = E\,[(\,|\,Zn\,|^{\,2} - R^2)] \quad (6)$$

$$J_2 = E\,[(\,|\,Zn\,| - R^2)] \quad (7)$$

$$J_3 = E\,[(\,|\,Zn\,|^{\,2} - R)] \quad (8)$$

It is possible that applying the LMS stochastic gradient algorithm for coefficient update on the cost functions in Eq.(6)(7)(8) derive the following error signal leading to algorithm 1,2,3,and 4 respectively.

$$\begin{aligned} e_1(n) &= z(n)\cdot(\,|\,Z(n)\,|^{\,2} - R_k^2\,) \\ e_2(n) &= sgn(z(n))\cdot(\,|\,Z(n)\,|^{\,2} - R_k^2\,) \\ e_3(n) &= sgn(z(n))\cdot(\,|\,Z(n)\,| - R_k\,) \\ e_4(n) &= sgn(z(n))\cdot sgn(\,|\,Z(n)\,| - R_k\,) \end{aligned} \quad (9)$$

Define the two error term ê(n) and e(n) as follows;

$$\begin{aligned} \hat{e}_1(n) &= z(n)\cdot(\,|\,Z(n)\,|^{\,2} - R_k^2\,) \\ \hat{e}_2(n) &= sgn(z(n)\cdot(\,|\,Z(n)\,|^{\,2} - R_k^2\,) \\ \hat{e}_3(n) &= sgn(z(n))\cdot(\,|\,Z(n)\,| - R_k) \\ \hat{e}_4(n) &= sgn(z(n))\cdot sgn(\,|\,Z(n)\,| - R_k) \end{aligned} \quad (10)$$

$$\begin{aligned} \tilde{e}_1(n) &= z(n)\cdot(\,|\,Z(n)\,|^{\,2} - R^2\,) \\ \tilde{e}_2(n) &= sgn(z(n)\cdot(\,|\,Z(n)\,|^{\,2} - R^2\,) \\ \tilde{e}_3(n) &= sgn(z(n))\cdot(\,|\,Z(n)\,| - R\,) \\ \tilde{e}_4(n) &= sgn(z(n))\cdot sgn(\,|\,Z(n)\,| - R\,) \end{aligned} \quad (11)$$

The coefficient update equation is given by

$$W(n+1) = W(n) - \mu\cdot f(n)\cdot \hat{e}(n)\cdot x^*(n)$$

$$z(n) \in D_k, \quad k = 1,2,3,\cdots \quad (12)$$

$$W(n+1) = W(n) - \mu\cdot f(n)\cdot \tilde{e}(n)\cdot x^*(n)$$

$$z(n) \notin \bigcup D_k, \quad k = 1,2,3,\cdots \quad (13)$$

where, μ denotes step size, D_k denotes the union of the annular regions, and

$$f(n) = \begin{cases} 1, & \text{if } sgn(|z(n)|^2 - R^2) \\ & \quad = sgn(|z(n)|^2 - R_k^2) \\ 0, & otherwise \end{cases} \quad (14)$$

Especially, Algorithm 1 was already proposed in[7].

Ⅲ. Simulation Result

We present the simulation results for the Godard, Dual-Mode Godard and the four new algorithms proposed in the paper. A rectangular 32-QAM random symbol signal Equalizers are usually realized in the form of a transversal filter with variable tap gains and tap spacing equal to the symbol spacing T. A 223-tap equalizer is assumed with an initial tap weight setting of (1.0,0.0) for the center tap and (0.0,0.0)for all other taps. For pulse shaping, raised-cosine filter with roll-off factor of 0.5 is used. Signal to Noise Ratio(SNR) is set to 60dB. The adaptation step size μ for the algorithm was chosen so as to minimize convergence time with a reasonable residual error. In the simulation, μ is used 5×10^{-7}, $5\times10^{-7}, 2.5\times10^{-6}$, 2.5×10^{-6}, and 7.5×10^{-6} for Dual Mode CMA, Algorithm 1,2,3 and 4 respectively. In order to compare the performance of the different algorithms, we compute the ensemble pseudo MSE denoted, MSE(n) defined follows,

$$MSE(n) = \frac{1}{m}\sum_{k=1}^{m}(z_{n(k)} - \hat{a}_n(k))^2 \quad (15)$$

where m is the number of iteration symbol, In simulation, m is used 200, 400, 1000, respectively. A multi-path channel can be represented as a

TDL (Tapped Delayed Line) with time-varying coefficients and T fixed tap spacings. The output signal can be written

$$y(t)=\sum_n a_n(t)\cdot S(t-\tau_n(t)) \qquad (16)$$

S(t) is the bandpass input signal, $a_n(t)$ is the attenuation factor for the signal received on the n-th path, and $\tau_n(t)$ is the corresponding propagation delay. If we express S(t) as

$$S(t)=Re\{\tilde{S}(t)\cdot e^{j2\pi f_c t}\} \qquad (17)$$

where, $\tilde{S}(t)$ is complex envelope then we can express the channel output as

$$y(t)=Re\left[\left(\sum_n a_n(t)\cdot e^{-j2\pi f_c t}\cdot \tilde{S}(t-\tau_n(t))\right)\cdot e^{-j2\pi f_c t}\right] \qquad (18)$$

and it is clear that the complex envelope of the output is

$$\begin{aligned}\tilde{y}(t) &= \sum_n a_n(t)\cdot e^{-j2\pi f_c \tau_n(t)}\cdot \tilde{S}(t-\tau_n(t)) \\ &= \sum_n a_n(t)\cdot e^{-j2\varphi_n}\cdot \tilde{S}(t-\tau_n(t))\end{aligned} \qquad (19)$$

where $\tilde{a}_n(t)$ is the complex envelope of $a_n(t)$, $\tau_n(t)$ is the complex envelope of $\tau_n(t)$, and φ_n is the variation in phase. In the simulation. the number of multi-path components excluding the main path is 3. In simulation program, sampling frequency is set to 172.16Mhz. The characteristics of Multi -path channels shown in Table 1, which is the Multi-path characteristic used in [8].

Table 1. The characteristics of Multi-path channel

Delay	Attenation Factor	Phase
-1.153 μs	0.1	-24.7 °
2.203 μs	0.3	151.2 °
5.046 μs	0.2	-63.8 °

In the Fig. 4, the convergence characteristics for various d is shown. We can see that as the value of d increases, the steady state MSE gradually decreases and then starts to increase, because too many erroneously detected symbols are being used to update the equalizer[5]. When d is greater then 0.4, we can see that MSE was not converged. To get the results of Fig. 4, algorithm 2 was used. Fig.5,6,7,8,9 shows the simulation results of Dual-Mode algorithm, algorithm 1,2,3 and algorithm 4, respectively. Fig. 10 shows the MSE curves for the various blind equalization algorithms. From Fig.10, algorithm 2 offers best overall performance among the various proposed algorithms. Especially, it is apparent that the convergence rate of Algorithm 1 and 2 is faster than that of conventional Dual Mode CMA. To get the results of Fig.5, d set to 0.3.

Ⅳ. Conclusion

In this paper, we have proposed a new class of Stop-and-Go Dual-Mode blind algorithms for QAM modulation scheme and evaluated its performance. From the simulation results, we can verify that all the proposed algorithms converge. In the simulations, multi-path fading channel is modeled and used. One or two algorithms among the proposed algorithms in the paper shows better convergency characteristics than conventional Dual-Mode blind algorithms. Therefore, these algorithms is expected to operate well in practical blind equalizer.

REFERENCES

[1] Y.Sato, "A method of self-recovering equalization for multilevel amplitude modulation system," IEEE Trans. Commun., vol. Com-23, pp.679-682, June 1975.

[2] D.N Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication system." IEEE Trans. Commun., vol. COM-28, pp.1867-1875, Nov. 1980

[3] A. Benbeniste and M. Goursat, "Blind Equalizer," IEEE Trans. Commun., vol. COM-32, pp.871-883, Aug.1984.

[4] Picchi and G.prati, "Blind equalization and carrier recovery using a stop-and-go decision-directed algorithm," IEEE Trans. Commun.,vol COM-35, pp.877-887,Sept. 1987.

[5] V. Weerackody and S.A. Kassam, "Dual-Mode type algorithm for blind equalization," IEEE tans. Commun., vol COM-42, pp.22-28, Jan. 1994.

[6] Simon Haykin, Adaptice Filter Theory, Prentice Hall Inc. 1996.

[7] Ching-Hsiang Tseng and Cheng-Bin Lin, "A Stop-and-Go Dual-Mode Algorithm for Blind Equalization," Proceeding Globecom '96, pp.1427-pp.1431.

[8] Y.S. Choi, H.Hwang, and D.I. Song, " Adaptive blind equalization coupled with carrier recovery for HDTV modem, "IEEE Trans. on Conumer Electronics. ,vol.39 , No.3, Aug. 1993.

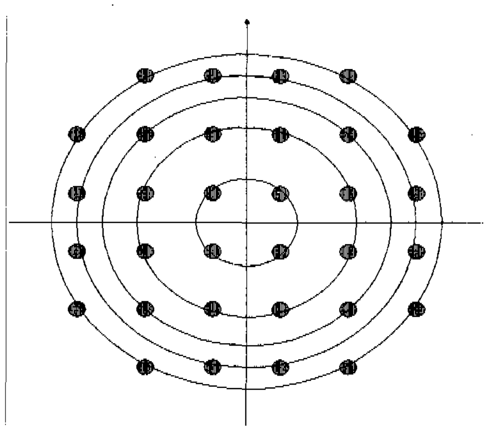


Fig. 1 Signal Constellation for 32QAM

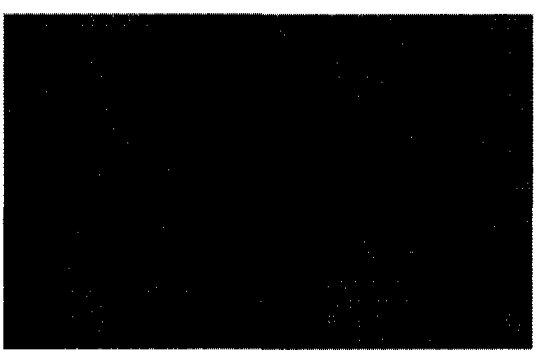


Fig. 2 Scatter Diagram before Equalization

Fig. 3 Scatter Diagram After Equalization

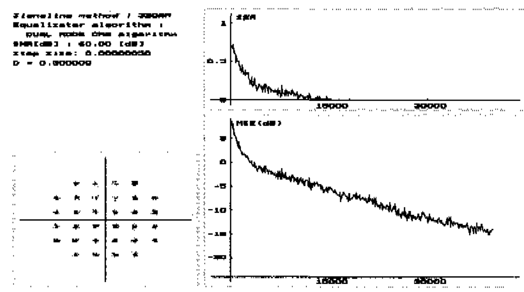


Fig. 4 Simulation Result of Dual-Mode CMA

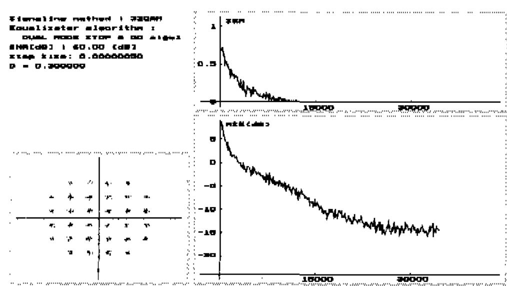


Fig. 5 Simulation Result of Algorithm 1

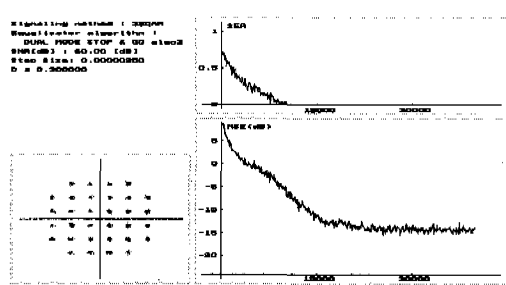


Fig. 6 Simulation Result of Algorithm 2

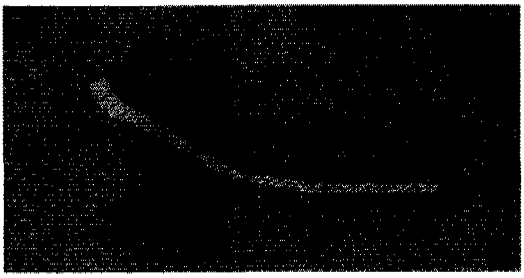


Fig. 7 MSE Characteristics for d

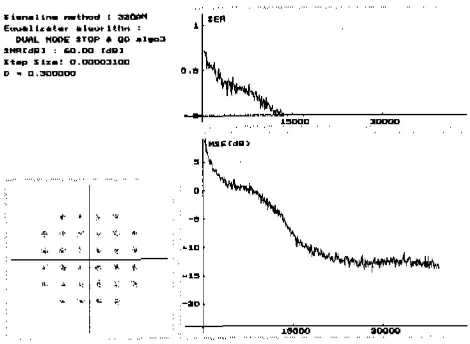


Fig. 8 The Simulation Result of Algorithm 3

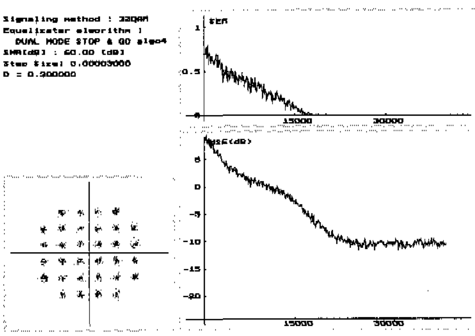


Fig. 9 The Simulation Result of Algorithm 4

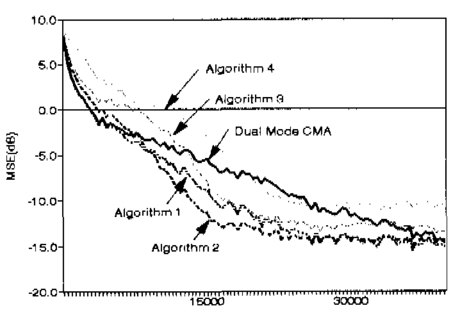


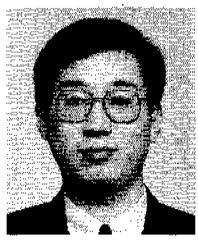
Fig. 10. The Characteristics of Convergency of Various Algorithms

Acknowledgment

The first author wish to thanks to Prof. Gordon Stuber for his helpful guidence during the Post Doctoral period.

본 연구는 인덕대학 해외연수 지원정책에 의해 일부 수행되었음

송 재 철(Jai-Chul Song) 정회원



1985년 2월 성균관대학교 전자공학과(공학사)
1989년 2월 성균관대학교 전자공학과(공학석사)
1994년 2월 성균관대학교 전자공학과(통신공학박사)
1996년 8월~1997년 8월 : Geogia Institute of Technology 전기전자공학과 Post Doctor 과정
1991년 9월~현재 : 인덕대학교 정보통신과 부교수
<주관심 분야> 디지털통신시스템, 디지털 모뎀설계 및 통신신호처리, 통신 ASIC 설계, 동기화이론을 포함한 모뎀설계

조 병 록(Byung-Lok Cho) 정회원

1987년 2월 : 성균관대학교 전자공학과(공학사)
1990년 2월 : 성균관대학교 대학원 전자공학과(공학석사)
1994년 2월 : 성균관대학교 대학원 전자공학과(공학박사)
1996년 10월~1997년 2월 : ETRC 초빙 연구원
1987년 1월~1988년 3월 : 삼성전자 (주)종합연구소
1994년 3월~현재 : 순천향대학교 전자공학과 조교수
<주관심 분야> 디지털 통신이론, 디지털 통신시스템 ASIC 설계, 무선멀티미디어용 고속 모뎀 설계, 무선망 성능분석

Dr. Gordon L. Stuber received the B.A.Sc. and Ph. D. degrees in Electrical Engineering from the University of

Waterloo, Ontario, Canada, in 1982 and 1986 respectively. Since 1986, he has been with the School of Electical and Computer Engineering, Georigia Institute of Technology, where he is currently a Professor.

Dr. Stuber served as Technical Program Chair for the 1996 IEEE Vehicular Technology Conference (VTC '96), and Technical Program Chair for the 1998 IEEE International Conference on Communications, (ICC '96) He is a past Editor for Spread Spectrum with the IEEE Transactions on Communication (1993~1998). He is an IEEE FELLOW.