

Individual Cell Loss Analysis in an ATM Multiplexer with Heterogeneous Input Sources

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ABSTRACT

An Asynchronous Transfer Mode multiplexer having a shared buffer is considered, which is loaded with discrete-time heterogeneous ON-OFF input traffic sources. An asymptotic behavior of the joint steady-state probability distribution of input process and queue length is analyzed using the asymptotic decay rate when the buffer capacity is sufficiently large. We prove that under heavy traffic the joint steady-state probability distribution approximately equals to the product of the two marginal distributions of queue length and input state. We also propose an approximation method of obtaining the joint steady-state probability distribution for a finite buffer case which will lead to the approximate cell loss probability of the individual input source. A numerical example together with the computer simulation results will be provided to validate this approximation.

I. 서론

Evaluating cell loss probability in an ATM (Asynchronous Transfer Mode) multiplexer loaded with possibly heterogeneous input traffic sources may be one of the most important tasks for an effective traffic control. The individual traffic source or traffic class may have different qualities of service. Therefore, it is essential to predict whether an ATM multiplexer can provide the required quality of service for each traffic class. This paper provides a method for obtaining the approximate cell loss probability of the individual input source in an ATM multiplexer with finite buffer capacity and heterogeneous input sources. Only a few studies are available to deal with the exact analysis of the ATM multiplexer with heterogeneous bursty input sources. Among them Bae et. al.[1] analyzed to obtain the individual cell loss probability using a Markov chain. But the number of Markov chain's states to be considered increases exponentially with the number of input sources. In general the exact analysis may not be feasible practically due to the computational complexity and therefore some approximation approach

are often adopted.

This study is based on the asymptotic decay rate of queue length distribution in an infinite buffer. As stated in [16], it is known that for a wide range of queueing systems including GI/G/c, the distribution of the queue length (or buffer contents) has a geometric form, i.e., for sufficiently large s and certain positive constants η and γ ,

$$\Pr\{\text{queue length} = S\} \approx \eta\gamma^S. \quad (1)$$

The constant γ is said to be the asymptotic decay rate of queue length distribution. We relate this asymptotic decay rate of queue length distribution with that of the joint steady-state probability distribution of input process state and queue length. We prove under heavy traffic assumptions that the joint steady-state probability distribution approximately equals to the product of the two marginal distributions of queue length and input state. Using this finding, we propose an approximation method of obtaining the joint steady-state probability distribution for a finite buffer case which will lead to the approximate cell loss probability of the individual input source.

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The complexity of the method proposed in this paper will be $O(N^3)$ regardless of the shared buffer size when N is the number of input sources. The previous researches dealing with the asymptotic decay rate of queue length distribution like in Eq. (1) can be also found in [9,10,14]. Many papers ([4-8,15]) proposed the exponential form of the overall cell loss probability. There is few research using the asymptotic behavior to obtain the individual cell loss probability in the literature known to us.

This paper is organized as follows. In the next section, the queueing model of an ATM multiplexer is described. Section III is devoted to the approximation method to obtain the joint steady-state probability distribution of input process and queue length. In Section IV, we provide the approximate cell loss probability of the individual input source when a buffering discipline is employed. We validate the result by using a numerical example in Section V. The concluding remarks are given in Section VI.

II. Queueing Model of an ATM Multiplexer

We consider the shared buffer multiplexer as a queueing system as shown in Fig.1. An ATM multiplexer transmits incoming cells from each of N bursty input sources onto the outgoing link. All incoming cells are stored in a shared buffer, whose size will be denoted by K . A cell will be lost if it arrives to find the shared buffer full.

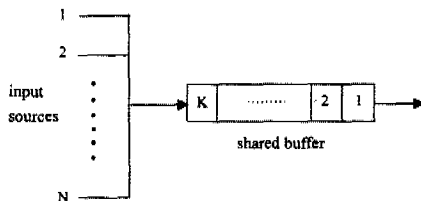


Fig. 1 Our queueing system of for shared buffer ATM multiplexer

Let us assume that cells arrive at each input source according to a heterogeneous interrupted Bernoulli process. The time needed to transmit a

cell onto the outgoing link is chosen as a time slot. An input source in a time slot has two states, ON and OFF. When the i -th input source is in ON state in a time slot, one cell is generated with the probability of λ_i from the input source. When it is in OFF state, no cell is generated. Suppose that the i -th input source is in ON (or OFF) state in time slot t . Then, in the next time slot $t+1$, it will move to the OFF (or ON) state with probability α_i (or β_i), or it will remain in the ON (or OFF) state with probability $1-\alpha_i$ (or $1-\beta_i$). The transitions between the ON and OFF states for the i -th input source are shown in Fig.2. Let $X_i(t)$ be 1 (or 0) if the i -th input source is in ON (or OFF) state in time slot t . Also let $Y_i(t)$ be 1 if a cell is generated from the i -th input source in time slot t and zero, otherwise.

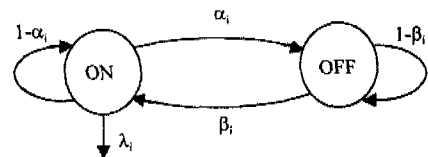


Fig. 2 Transition between ON and OFF states for the i -th input source

All new cells are assumed to arrive at the beginning of a time slot and to be immediately available for transmission in the same time slot. The cell in the shared buffer, if any, departs at the end of a time slot one at a time.

Let $Q(t)$ be the random variable denoting the queue length in the infinite buffer at the beginning of time slot t before cell arrivals and let Q be the steady-state version of $Q(t)$. Let $\mathbf{X} = (X_1(t), X_2(t), \dots, X_N(t))$ be the vector of random variables representing the states of input sources in time slot t and $\mathbf{X} = (X_1, X_2, \dots, X_N)$ be the steady-state version of $\mathbf{X}(t)$. Define

$$P_Q(s) = \text{pr}\{Q = s\}$$

$$P_{\mathbf{X}}(\mathbf{X}) = \text{pr}\{\mathbf{X} = \mathbf{x}\}$$

$$P_{\mathbf{X}|Q}(\mathbf{X}|s) = \text{Pr}\{\mathbf{X} = \mathbf{x} | Q = s\}$$

$$P_{(x,s)} = \text{Pr}\{\mathbf{X} = \mathbf{x}, Q = s\}$$

III. Joint Steady-State Probability under Heavy Traffic

With infinite buffer assumption, the asymptotic decay rate of queue length distribution, γ , can be defined as follows.

$$\gamma = \lim_{s \rightarrow \infty} \frac{\Pr\{\text{queue length} = s + 1\}}{\Pr\{\text{queue length} = s\}} \quad (2)$$

In M/M/1 queue for example, γ corresponds to the utilization ρ . From other researches dealing with the multiplexer having homogeneous ([11]) or heterogeneous bursty input sources ([12,13,16]), we can expect that the asymptotic decay rate γ grows to be 1 as the cell arrival rate increases. Let us assume that the asymptotic decay rate γ of queue length distribution is sufficiently close to 1. As mentioned earlier this assumption corresponds to the heavy traffic assumption which says that the utilization is sufficiently close to 1. Then the following theorem holds.

Theorem 1

If the asymptotic decay rate γ of queue length distribution is close to 1 and $\delta(\underline{x}) = \lim_{s \rightarrow \infty} P_{\underline{x}|Q}(\underline{x} | s)$ exists, then $P(\underline{X}, s)$ can be approximated as follows for sufficiently large s and all input source state \underline{X} .

$$P(\underline{x}, s) \approx P_{\underline{x}}(\underline{X}) \eta \gamma^s \quad (3)$$

Proof) By the definition of conditional probability, it follows that $P(\underline{x}, s) = P_{\underline{x}|Q}(\underline{X} | S) P_Q(s)$. If $\delta(\underline{x}) = \lim_{s \rightarrow \infty} P_{\underline{x}|Q}(\underline{X} | s)$ exists, then we have for sufficiently large s

$$P(\underline{x}, s) \approx \delta(\underline{x}) \eta \gamma^s \quad (4)$$

Conditioning on the state of the input sources at the t -th time slot and letting t go to infinity, the following equalities can be derived.

$$\begin{aligned} \delta(\underline{x}) &= \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \Pr\{\underline{X}(t+1) = \underline{x} | Q(t+1) = s\} \\ &= \sum_{\underline{y}} \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \Pr\{\underline{X}(t) = \underline{y} | Q(t+1) = s\} R_{\underline{y}, \underline{x}} \end{aligned} \quad (5)$$

where $R_{\underline{y}, \underline{x}}$ is the one-step transition probability from state \underline{y} to state \underline{x} and it depends only on the transition probabilities (α_i, β_i) ($i = 1, 2, \dots, N$). The first term inside the summation of Eq. (5) reduces to:

$$\begin{aligned} &\lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \Pr\{\underline{X}(t) = \underline{x} | Q(t+1) = s\} \\ &= \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \frac{\Pr\{\underline{X}(t) = \underline{x}, Q(t+1) = s\}}{\Pr\{Q(t+1) = s\}} \\ &= \lim_{s \rightarrow \infty} \lim_{t \rightarrow \infty} \frac{\sum_{s'=s-N+1}^{s+1} \delta(\underline{x}) \eta \gamma^{s'} \Pr\{Q(t+1) = s | \underline{X}(t) = \underline{x}, Q(t) = s'\}}{\eta \gamma^s} \\ &= \delta(\underline{x}) \lim_{t \rightarrow \infty} \sum_{n=0}^N \gamma^{1-n} \Pr\left[\sum_{i=1}^N Y_i(t) = n | \underline{X}(t) = \underline{x}\right] \\ &= \delta(\underline{x}) f(\gamma, \underline{x}) \end{aligned} \quad (6)$$

where

$$f(\gamma, \underline{x}) = \lim_{t \rightarrow \infty} \sum_{n=0}^N \gamma^{1-n} \Pr\left[\sum_{i=1}^N Y_i(t) = n | \underline{X}(t) = \underline{x}\right]. \quad (7)$$

Therefore, from Eq. (5) and (6) we have

$$\delta(\underline{x}) = \sum_{\underline{y}} \delta(\underline{y}) f(\gamma, \underline{y}) R_{\underline{y}, \underline{x}} \quad (8)$$

We see from Eq. (7) that $f(\gamma, \underline{y})$ goes to 1 as the asymptotic decay rate γ goes to 1. So, we can conclude that $\delta(\underline{x})$ becomes nothing but $P_{\underline{x}}(\underline{X})$ since Eq. (8) for $f(\gamma, \underline{y})$ represents the condition for steady-state distribution of input source state \underline{x} . Note that $\delta(\underline{x})$ satisfying Eq. (8) is unique when α_i and β_i are any positive real number and $f(\gamma, \underline{y})$ is 1 for all \underline{y} . By replacing $\delta(\underline{x})$ with $P_{\underline{x}}(\underline{X})$ in Eq. (4), the proof is completed.

Theorem 1 implies that the input source states and the buffer states behave independently under heavy traffic when the queue length is sufficiently large. Let $P_{(k)}(\underline{x}, s)$ denote the joint steady-state probability of input process and queue length for finite buffer of size K , which is the probability of our main interest. To approximate $P_{(k)}(\underline{x}, s)$ by using $P(\underline{x}, s)$, the following Lemma 2 and Theorem 3 may be useful.

Lemma 2

$$\sum_{i=0}^{\infty} P(\underline{x}, i) \geq \sum_{i=0}^{K-1} P_{(K)}(\underline{x}, i) \text{ for all } s \leq K-1$$

proof) For any state of input sources, the queue length for infinite buffer is larger than or equal to the queue length for finite buffer. So, the result follows.

Theorem 3

The joint distribution for finite buffer of size K can be approximated by the joint distribution for infinite buffer for all \underline{x} and sufficiently large K, i.e.,

$$\lim_{K \rightarrow \infty} P_{(K)}(\underline{x}, s) = P(\underline{x}, s), s < K-1.$$

proof) Under the infinite buffer assumption, consider a renewal cycle that begins at the time slot where all the input sources are in OFF state and buffer is empty. Let us denote by $Z(\underline{x}, s)$ the number of time slots where the state vector of input sources is \underline{x} and queue length is s in a renewal cycle. Analogously, with the finite shared buffer of size K, let us define $Z_{(K)}(\underline{x}, s)$. Then, using the renewal theory, we can prove the following for $s < K-1$:

$$\begin{aligned} P(\underline{x}, s) &= \frac{E[Z(\underline{x}, s)]}{E[\text{renewal cycle length}]} \\ &= \lim_{K \rightarrow \infty} \frac{E[Z(\underline{x}, s) \mid \text{queue length is always less than } K]}{E[\text{renewal cycle length}]} \\ &= \lim_{K \rightarrow \infty} \frac{E[Z_{(K)}(\underline{x}, s) \mid \text{queue length is always less than } K]}{E[\text{renewal cycle length}]} \\ &= \lim_{K \rightarrow \infty} P_{(K)}(\underline{x}, s). \end{aligned}$$

In above we use the fact that $\Pr\{\text{queue length is always less than } K \text{ in a renewal cycle}\}$ converges to 1 as K becomes larger. So, the proof is completed.

By Theorem 3, the joint probability distribution for finite buffer of size K can be approximated for sufficiently large K as follows.

$$\begin{aligned} P_{(K)}(\underline{x}, s) &\approx P(\underline{x}, s), s < K-1 \\ P_{(K)}(\underline{x}, K-1) &\approx \sum_{s=K-1}^{\infty} P(\underline{x}, s) \end{aligned} \tag{9}$$

By Lemma 2, the approximate distribution in Eq. (9) has the larger tail probability for all \underline{x} than the exact distribution does. In the next section, we propose the method for obtaining the approximate individual cell loss probability using Eq. (3) and (9).

IV. Approximation of Individual Cell Loss Probability

Consider the following buffering discipline. When many cells arrive in a time slot from N input sources, the highest level priority for being stored in the shared buffer is given to the cell generated from the first input source. And the second highest level priority is given to the cell generated from the second input source and so on. Generally, the i-th highest level priority is given to the cell generated from the i-th input source ($i=1, 2, \dots, N$). Let $L(i, K)$ denote the cell loss probability of the i-th input source when the shared buffer capacity is K. Then, $L(1, K)=0$ since every cell generated from the first input source is stored in a shared buffer with the first level priority.

Let Y_i denote the steady-state version of $Y_i(t)$ ($i=1, 2, \dots, N$). Similarly to Eq. (3), the steady-state probability for the infinite buffer,

$\Pr\left\{\sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s\right\}$ (than n cells from the first (i-1) input sources are generated, one cell is generated from the i-th input source and queue length at the beginning of a time slot before the cell arrivals is s) for $2 \leq i \leq N$ and $0 \leq i-1$ can be approximated as follows.

$$\begin{aligned} &\Pr\left\{\sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s\right\} \\ &\approx \Pr\left\{\sum_{(j=1)}^{(i-1)} Y_j = n, Y_i = 1\right\} \eta^s \end{aligned} \tag{10}$$

Then, using Eq. (9) and (10), when the buffer capacity is K, the corresponding joint steady-state probability $\Pr_{(K)}\left\{\sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s\right\}$ for $2 \leq i \leq N$ and $0 \leq n \leq i-1$ can be approximated as follows.

$$\Pr_{(K)} \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s \right\}$$

$$\begin{cases} \Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\} \eta \gamma^s, & s < K-1 \\ \Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\} \eta \gamma^{K-1} \\ \hline 1 - \gamma \end{cases}, \quad s = K-1 \quad (11)$$

The next lemma shows that

$\Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\}$ in Eq. (11) for $2 \leq i \leq N$ and $0 \leq n \leq i-1$ can be calculated in $O(N^2)$.

Lemma 4

$\Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\}$ for $2 \leq i \leq N$ and $0 \leq n \leq i-1$ can be calculated in $O(N^2)$.

proof) The generating function $G_j(z)$ of Y_j is

$$G_j(z) = \left(\frac{\beta_j}{\alpha_j + \beta_j} \lambda_j \right) z + \left(1 - \frac{\beta_j}{\alpha_j + \beta_j} \lambda_j \right), \quad j = 1, 2, \dots, N.$$

So, the generating function $G^{i-1}(z)$ of $\sum_{j=1}^{i-1} Y_j$ is

$$G^{i-1}(z) = \prod_{j=1}^{i-1} G_j(z)$$

$$= \prod_{j=1}^{i-1} \left[\left(\frac{\beta_j}{\alpha_j + \beta_j} \lambda_j \right) z + \left(1 - \frac{\beta_j}{\alpha_j + \beta_j} \lambda_j \right) \right], \quad 2 \leq i \leq N.$$

When $G^{i-1}(z)$ is arranged in the ascending order of z , $\Pr \left\{ \sum_{j=1}^{i-1} Y_j = n \right\}$ is the coefficient of z^n of $G^{i-1}(z)$ And,

$$\Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\} = \Pr \left\{ \sum_{j=1}^{i-1} Y_j = n \right\} \Pr \{ Y_i = 1 \}$$

$$= \Pr \left\{ \sum_{j=1}^{i-1} Y_j = n \right\} \frac{\beta_j}{\alpha_j + \beta_j} \lambda_j.$$

So, the complexity of obtaining

$\Pr \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1 \right\}$ for $0 \leq n \leq i-1$ would be $O(i^2)$. Since $G^i(z) = G^{i-1}(z)G_i(z)$ for $i=2, 3, \dots, N$, for $2 \leq i \leq N$ and $0 \leq n \leq i-1$ can be obtained in $O(N^2)$.

Using $\Pr_{(K)} \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s \right\}$ for finite buffer of size K , we can calculate $L(i,K)$ for $2 \leq i \leq N$ as follows.

$$L(i,K) = \frac{E[\text{number of the } i\text{-th source cells lost in a time slot}]}{E[\text{number of the } i\text{-th source cells generated in a time slot}]}$$

$$= \frac{\sum_{n=1}^{i-1} \sum_{s=K-n}^{K-1} \Pr_{(K)} \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s \right\}}{\frac{\beta_j}{\alpha_j + \beta_j} \lambda_j} \quad (12)$$

If $\Pr_{(K)} \left\{ \sum_{j=1}^{i-1} Y_j = n, Y_i = 1, Q = s \right\}$ is approximated by Eq. (11) and h is replaced by h_u which is suggested by [16], then Eq. (12) is expected to give a good approximate individual cell loss probability when buffer size is sufficiently large. Using Eq. (12), the approximate individual cell loss probabilities can be calculated in $O(N^3)$. Overall cell loss probability, if required, can be also calculated as follows.

$$\text{overall cell loss probability} = \frac{\sum_{n=2}^N \sum_{s=K-n+1}^{K-1} (n-K+s) \Pr_{(K)} \left\{ \sum_{j=1}^{i-1} Y_j = n, Q = s \right\}}{\sum_{i=1}^N \frac{\beta_j}{\alpha_j + \beta_j} \lambda_j} \quad (13)$$

V. Numerical Example

In this section, the approximate individual cell loss probability described in Section IV will be validated by comparing with the computer simulation results. Fifteen very different input sources ($N=15$) are selected as shown in Table 1. Each source has been chosen to have similar ON state probability equal to about 0.093 but to have different burstiness and different mean cell arrival rate. The buffering discipline described in Section IV was used for computer simulation. An input source with smaller burstiness and larger cell arrival rate has lower level priority for being

stored in the shared buffer. Computer simulation using regenerative process was done during 10^7 renewal cycles equal to about 6×10^9 time slots for each buffer size K . It is believed that simulation results provide the accurate cell loss probabilities of the individual input source.

Table. 1 Parameters of 15 input sources selected for computer simulation

input source (i)	α_i	β_i	γ_i
1	0.00193	0.00020	0.4
2	0.00207	0.00021	0.4
3	0.00223	0.00023	0.4
4	0.00242	0.00025	0.5
5	0.00264	0.00028	0.5
6	0.00290	0.00030	0.5
7	0.00322	0.00033	0.6
8	0.00363	0.00038	0.6
9	0.00414	0.00043	0.6
10	0.00483	0.00050	0.7
11	0.00580	0.00060	0.7
12	0.00725	0.00075	0.7
13	0.00967	0.00100	0.8
14	0.01450	0.00150	0.8
15	0.02900	0.00300	0.8

Using the method in [16], γ of 0.9986 and γ_u of 0.0009349 are obtained. In Fig. 3, we compare the approximated individual cell loss probability using Eq. (12) with the individual cell loss probability obtained by computer simulation for input sources 4, 9, 13, 15 and various buffer sizes. The ratio of the approximated individual cell loss probability obtained by Eq. (12) to the simulation result ranges from 0.52 to 3.37. We can see that the approximate individual cell loss probability using Eq. (12) becomes more accurate when buffer size is larger.

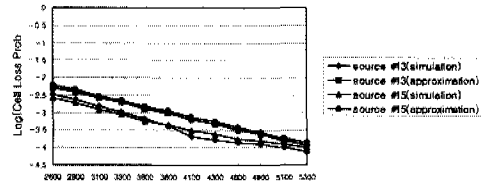
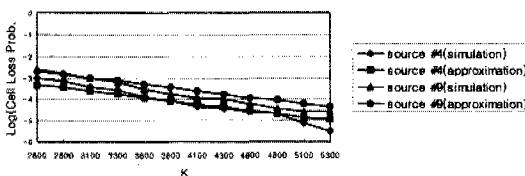


Fig. 3 Comparisons of approximation with simulation results of individual cell loss probability

In Fig. 4, we compare the approximated overall cell loss probability using Eq. (13) and simulation result for various buffer sizes. The overall cell loss probability calculated by the heuristic method in [3] is also given for the reference. The ratio of the approximate overall cell loss probability in Eq. (13) to the simulation result ranges from 0.85 to 1.32. It can be also seen that the approximated overall cell loss probability using Eq. (13) is closer to the exact one as the buffer size increases. Moreover, as shown in Fig. 4, the approximate overall cell loss probability is more accurate than the result of the heuristic developed in [3].

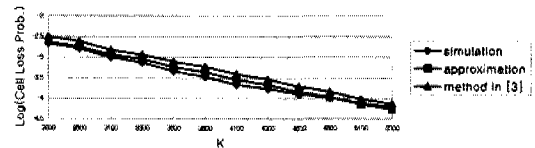


Fig. 4 Comparisons of approximation with simulation results of overall cell loss probability

VI. CONCLUDING REMARKS

When the size of shared buffer is relatively small, the approximation based on Eq. (3) may give an inaccurate joint steady-state probability distribution. The log-scaled cell loss probability is convex function versus the buffer size as shown in previous study [2]. This implies that the asymptotic decay rate of the cell loss probability increases with the buffer size. So, it is expected that Eq. (12) and Eq. (13) rather give the lower bounds of the cell loss probability when the buffer size is relatively small.

We consider the time for transmitting an

arriving cell onto the outgoing link as a time slot and assume that cells arrive at each input traffic source according to a heterogeneous interrupted Bernoulli process. If the actual service rate of the multiplexer is $M (\geq 1)$ cells per unit time where "unit time" is arbitrarily defined, we can set a time slot to $1/M$ unit time. Carefully deciding some parameters of arriving cells for a time slot, we can approximately describe the actual input process as an interrupted Bernoulli process.

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