Mode matching방법을 사용한 유전체 구형도파로의 해석

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Analysis of dielectric rectangular waveguide structures with mode matching methods

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요 약

ISDN을 비롯한 대용량 고속 전송망을 비롯하여 모든 통신 시스템에서 광통신로의 사용이 점차 증가하고 있다. 그러나 아직도 단말기에서는 광 도파로를 사용하지 못하고 광-전기 변환 회로를 경유하여 전기 신호로서 처리하고 있다. 앞으로 광신호의 사용이 증가하면 광신호를 변환하지 않고 광형태로 교환 접속, 증폭 또는 변복조할 필요성이 생기게 된다. 이때 전기 회로에서 프린트 배선이나 스트립 선로(strip line)에 필적하는 광 도파로가 필요하게 된다. 광도파로는 전자파 이론에서의 도파관 이론으로 해석이 가능하다. 이 때의 광도파로는 전자장의 유전체 도파로에 해당된다. 따라서 광처리 회로에서 도파로 부분을 설계하는 데는 유전체 도파로의 이론이 필수적이다. 구형 유전체 도파관은 광 도파관의 전형적인 형태에 대한 모델 일뿐 아니라 실제적으로 집적화된 광 회로에서 빛을 도파 하는데 사용하는 것이다.

파동 방정식에 그 매질 사이의 경계 조건을 도입하여 구형 유전체 도파관의 외부와 내부에서의 전장 E와 자장 H에 대해 수치 해석적 근사화 없이 이 문제를 실질적으로 좀더 정확히 구형 도파관에 대한 분석을 하였다. 이 전자파 식들은 간단한 파동 방정식에 의해서 유사 TE모드와 유사 TM모드를 각 영역 별로 분석되었다. 그러므로 수치 해석에 의해 분석 할 때 발생하는 존재 할 수 없는 모드 (spurious mode)를 제거 할 수 있었다. Marcatili의 근사식과 비교하면 β^y_{pq}와 β^x_{pq} 값은 Marcatili의 결과와 동일하고 β_{TE} 와 β_{TM} 은 Marcatili가 예측하지 못한 값이다. 그러나 이논문의 결과에 의하면 β^y_{pq}와 β^x_{pq}는 존재할 수 없고 β_{TE} , β_{TM} 만이 존재함을 알 수 있다. Marcatili의 근사식은 $\frac{b}{d}=1$ 일 때 오차가 가장 심하며, $\frac{b}{d}$가 점점 커짐에 따라서 그 오차는 작아 지는 것을 알 수 있다.

여기서 얻은 결과를 이용하면 인접하고 있는 두 유전체 구형 도파관 사이에서의 결합 계수를 구하는데 용이하다고 생각된다.

ABSTRACT

A dielectric waveguide structure using rectangular dielectric strip is analyzed directly in terms of the wave equation for quasi TE and quasi TM modes. This problem can be solved, with no approximation in the wave equation for the electric field $\vec{E}$ and magnetic field $\vec{H}$ inside and outside the dielectric rectangular waveguide matching the boundary conditions between interfaces. This leads to an eigenvalue problem where spurious modes do not appear. Dispersion characteristic examples are presented for square and rectangular waveguides. The formulation is general and can be used for comparison with other methods such as FDM or FEM in various structures.

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I. Introduction

Dielectric rectangular waveguides are the most useful structures that are used to confine and guide light in the guided-wave devices and circuits of integrated optics [1],[2]. A well-known dielectric waveguide is, of course, the optical fiber which usually has a circular cross-section. In contrast the guides of interest to integrated optics are usually planar structures such as planar films or strips.

The study of dielectric rectangular waveguides and their properties is often useful in gaining an understanding of the waveguiding properties of more complicated dielectric waveguides. Dielectric rectangular waveguides are not only useful as models for more general types of optical waveguides, but they are actually employed for light guidance in integrated optics circuits [1],[2].

This problem can be solved, with no approximation in the wave equation for the electric field $\vec{E}$ and magnetic field $\vec{H}$ inside and outside the dielectric rectangular waveguide matching the boundary conditions between interfaces. The ability to generate, guide, modulate, and detect light in such thin film configurations opens up new possibilities for monolithic "optical circuits"

II. The slab waveguide

Dielectric slabs are the simplest optical waveguides. A dielectric slab waveguide is shown schematically in Fig 2.1.

The figure shows a slab waveguide as it would be used in a typical integrated optics application. The core region of the waveguide is assumed to have refractive index n_1 and is deposited on a substrate with refractive index n_2. The refractive index of the medium above the core is denoted by n_3. The typical film thickness is from 0.2 μm to 1 μm. Light is confined by total internal reflection at the film-substrate and film-cover interfaces. In order to achieve true mode guidance, it is necessary that n_1 be larger than n_2 and n_3. We will assume that $n_3 \leq n_2 < n_1$ [1].

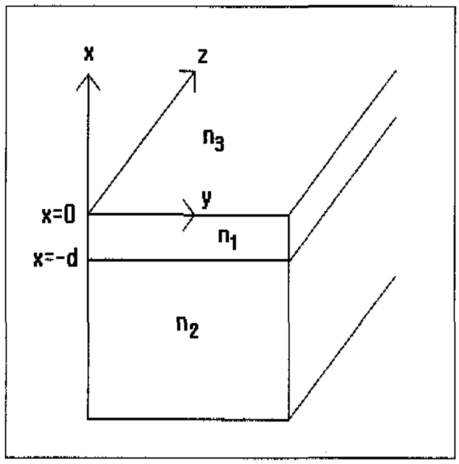

Fig. 2.1 A dielectric slab waveguide

In case $n_2 \neq n_3$, the slab waveguide is asymmetric. Modes of asymmetric slabs become cutoff if the frequency of operation is sufficiently low. Like all dielectric waveguides the asymmetric slab supports a finite number of guided modes.

We start with Maxwell's equations (in MKS units). With $\rho = 0$, $\vec{J} = 0$ and substituting $i\omega$ for $\frac{\partial}{\partial t}$, Maxwell's equations can be written in the form

$$\nabla \cdot \vec{D} = 0, \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -i\omega\mu\vec{H}, \quad \nabla \times \vec{H} = i\omega\varepsilon\vec{E} \tag{1}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$ is the dielectric permittivity and $\mu = \mu_0 \mu_r$ is magnetic permeability of the materials. We do not consider lossy and magnetic materials in this paper so that the use of constant μ_0 is sufficient. The index of refraction of the medium will be denoted by $n = \sqrt{\varepsilon_r \mu_r} = \sqrt{\varepsilon_r}$. Let $\vec{k} = (k_x, k_y, k_z)$ and $\beta = k_z$. A mode of a dielectric waveguide at a frequency ω is a solution of the wave equation

$$\nabla^2 H_z + k^2 H_z = 0$$

where $k^2 = n^2 k_0^2 = \omega^2 \mu\varepsilon$. It is possible to express the transverse field components in terms of the longitudinal components [1].

$$E_y = -\frac{i}{k^2-\beta^2}\left(\beta\frac{\partial E_z}{\partial y} - \omega\mu\frac{\partial H_z}{\partial x}\right)$$

$$H_x = -\frac{i}{k^2-\beta^2}\left(-\omega\varepsilon\frac{\partial E_z}{\partial y} + \beta\frac{\partial H_z}{\partial x}\right)$$

$$E_x = -\frac{i}{k^2-\beta^2}\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)$$

$$H_y = -\frac{i}{k^2-\beta^2}\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right) \qquad (2)$$

We simplify the description of the slab waveguide by assuming that there is no variation in y direction, which we express symbolically by the equation $\frac{\partial}{\partial y} = 0$ or $k_y = 0$. The modes of a slab waveguide can be classified as TE or TM modes.

2.1 Guided TE modes

We start with the analysis of a transverse electric (TE) wave with the electric field polarized along $\hat{y}$, transverse to the direction of propagation, the $z-$ direction. The wave equation becomes

$$\frac{\partial^2 H_z}{\partial x^2} + (k^2 - \beta^2) H_z = 0$$

TE modes have only three field components : E_y, H_x and H_z. It is assumed that the slab is infinitely extended in the yz plane and omit the $e^{-i\beta z}$ term which describes a wave traveling in the positive $z-$direction with phase velocity $v_p = \frac{\omega}{\beta}$. For $k_3, k_2 < \beta < k_1$, the solution is sinusoidal in region 1, but is exponential in region 2 and 3. This makes it possible to have a solution H_z that satisfies the boundary conditions while decaying exponentially in region 2 and 3. The energy carried by these modes is confined to the vicinity of the guiding layer 1 and we will refer to them as confined, or guided, modes [1],[2]. For $0 < \beta < k_2, k_3$ the solution becomes sinusoidal in all three regions. These are the so-called radiation modes of the waveguides.

The problem of finding the TE modes of the slab waveguide has thus become very simple. We only need to find solutions of the one-dimensional reduced wave equation. The only remaining complication is the requirement that the solutions should satisfy the boundary conditions at the two interfaces at $x=0$ and $x=-d$. The boundary conditions require that the tangential E and H fields be continuous at the dielectric discontinuities. With $E_z = 0$, $E_x = 0$ and $H_y = 0$, we obtain at $-d \le x \le 0$

$$H_z = A\sin(k_x x + \theta_1)$$

$$E_y = iA\frac{\omega\mu}{k_x}\cos(k_x x + \theta_1)$$

$$H_x = -iA\frac{\beta}{k_x}\cos(k_x x + \theta_1) \qquad (3)$$

where $k_1^2 = \omega^2\mu\varepsilon_1 = k_x^2 + \beta^2$ and note that

$$Z_{TE} = \frac{E_y}{-H_x} = \frac{\omega\mu}{\beta}.$$

At $x \le -d$, $k_2^2 = \omega^2\mu\varepsilon_2 = -\alpha_2^2 + \beta^2$

$$H_z = -A\sin(k_x d - \theta_1)e^{\alpha_2(x+d)}.$$

The E_y and H_x can be derived by Eq (2). At $0 \le x$, $k_3^2 = \omega^2\mu\varepsilon_3 = -\alpha_3^2 + \beta^2$

$$H_z = A\sin\theta_1 e^{-\alpha_3 x}.$$

The other components can be derived by Eq (2). We must require that components E_y and H_z are continuous at $x=0$ and $x=-d$ and vanish at $x = \pm\infty$. Then the other component H_x is automatically matched at $x=0$ and $x=-d$. From these boundary conditions

$$\tan\theta_1 = \frac{\alpha_3}{k_x} \qquad (4)$$

$$\tan k_x d = \frac{k_x(\alpha_2 + \alpha_3)}{k_x^2 - \alpha_2\alpha_3} \qquad (5)$$ [1].

With $\beta^2 = k_1^2 - k_x^2$, we obtain

$$\alpha_2^2 = -k_2^2 + \beta^2 = k_1^2 - k_2^2 - k_x^2 \quad (6)$$

$$\alpha_3^2 = -k_3^2 + \beta^2 = k_1^2 - k_3^2 - k_x^2 \quad (7)$$

By substituting α_2 and α_3 into the eigenvalue equation (5), we have [1]

$$\tan k_x d = \frac{k_x(\alpha_2 + \alpha_3)}{k_x^2 - \alpha_2\alpha_3} = F(k_x d) = \frac{k_x d(\alpha_2 d + \alpha_3 d)}{(k_x d)^2 - (\alpha_2 d \cdot \alpha_3 d)} \quad (8)$$

This equation is used to obtain the eigenvalue β for the confined TE modes. An example of such a solution is shown in Figure 2.2

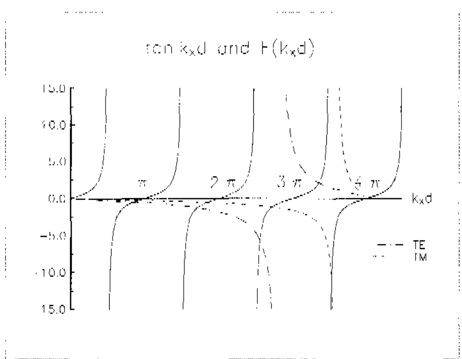

Fig 2.2 Graphical solution of the eigenvalue. The crossing points of the solid and dashed lines correspond to solutions where $n_1^2 = 2.1$ and $n_2^2 = n_3^2 = 1.0$.

The parameter α_2 becomes imaginary as β becomes smaller than k_2. At the cutoff point where $\alpha_2 = 0$ (or $\beta = k_2$) we see from the field expression that the field extends undiminished to infinite distances below the waveguide core, if $\alpha_2 = 0$. When α_2 becomes imaginary the evanescent field in the substrate region turns into a radiation field, and the wave is no longer guided by the dielectric waveguide. The cutoff condition can easily be shown to be identical to the condition for the loss of total internal reflection from the dielectric interfaces at $x = 0$ and $x = -d$ which means that the critical angle $\theta_c = \cos^{-1}(\frac{\beta}{k_1}) = \cos^{-1}(\frac{n_2}{n_1})$ [1],[2]. The constant, A, is arbitrary. It is advantageous to define A in such a way that it is simply related to total power in the mode. We choose A so that the field $E_y(x)$ corresponds to a power flow in the mode. For the slab with its infinite extension in y direction, P is actually the power per unit length. The power is obtained by integrating the z component of the Poynting vector :

$$S_z = \frac{1}{2} Re(\vec{E} \times \vec{H}^*) \cdot \hat{z}$$ [3],[4].

over the infinite transverse cross section of the waveguide.

$$P = -\frac{1}{2}\int_{-\infty}^{\infty} E_y H_x^* dx = \frac{|\beta|}{2\omega\mu_0}\int_{-\infty}^{\infty} |E_y|^2 dx$$

and

$$A^2 = \frac{4k_x^2\omega\mu_0 P}{|\beta|\,[d + \frac{1}{\alpha_2} + \frac{1}{\alpha_3}][k_x^2 + \alpha_3^2]} \quad (9)$$

2.2 Guided TM modes

The derivation of the confined TM modes is similar in principle to that of the TE modes. The wave equation becomes

$$\frac{\partial^2 E_z}{\partial x^2} + (k_i^2 - \beta^2)E_z = 0$$

TM modes have three field components : E_z, E_x and H_y. With $H_z = 0$, $H_x = 0$ and $E_y = 0$, we obtain at $-d \le x \le 0$,

$$E_z = A\sin(k_x x + \theta_2)$$

$$E_x = -iA\frac{\beta}{k_x}\cos(k_x x + \theta_2)$$

$$H_y = -iA\frac{\omega\varepsilon_1}{k_x}\cos(k_x x + \theta_2) \quad (10)$$

where $k_1^2 = \omega^2\mu\varepsilon_1 = k_x^2 + \beta^2$ and $\tan\theta_2 = \frac{\varepsilon_1}{\varepsilon_3}\frac{\alpha_3}{k_x}$.

Note that

$$Z_{TM} = \frac{E_x}{H_y} = \frac{\beta}{\omega\varepsilon_1}$$

The field components E_z, E_x and H_y are continuous at $x=0$ and $x=-d$. The eigenvalue equation can be written :

$$\tan k_x d = \frac{k_x \varepsilon_1(\alpha_3\varepsilon_2 + \alpha_2\varepsilon_3)}{k_x^2\varepsilon_2\varepsilon_3 - \alpha_2\alpha_3\varepsilon_1^2} \quad (11)$$ [1]

Ⅲ. The dielectric rectangular waveguide

The dielectric slab waveguide is a useful model for complicated waveguide structures. However, in most practical applications more complicated waveguides are used [2]. The waveguides used in integrated optics are usually rectangular strips of dielectric material that are embedded in other dielectrics. The rectangular strip is embedded in the material of the substrate of the integrated optics. We analyze a structure that is more general. Instead of assuming that the waveguide core is embedded in th material of a substrate, we allow the materials on all four sides of the rectangular core to be different [9],[10]. This geometry is shown in Fig. 3.1

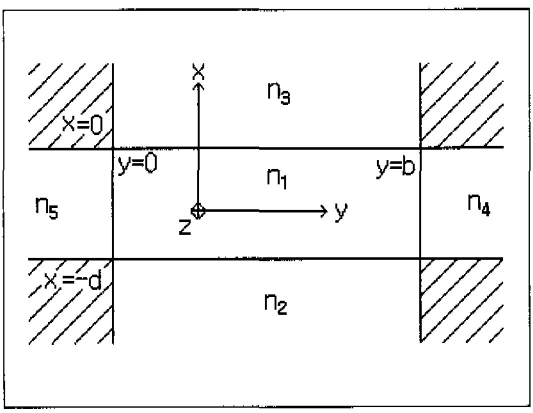

Fig 3.1 The five dielectric regions of a dielectric rectangular waveguide. The field in the shaded regions is ignored in this paper.

An exact analytical treatment of this problem has not yet been proposed [1],[2],[5]. Only approximate analytical approach which was developed by Marcatili [5] was reported. The Marcatili's method assumed $E_y \approx 0$ and $H_x = 0$ for the case of E^x_{pq} modes and $E_x \approx 0$ and $H_y = 0$ for the case of E^y_{pq} modes. Other approximate solutions by numerical methods have been obtained that can be made as accurate as desired [1],[6],[7],[8]. If the mode is not very close to cutoff, its field is confined almost exclusively to the region of the core, and only very little field energy is carried in the surrounding media. The ray angle θ is small for well-guided modes, so that the propagation constant $\beta \approx n_1 k$. Because k is a large quantity, thus we have $k_x \ll \alpha_2$ and $k_x \ll \alpha_3$. From Maxwell's equation, we have

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + (k_i^2 - \beta^2)\psi = 0 \quad (12)$$ [3],[4].

where $\psi = E_z$ or H_z. The oscillatory solutions in the core must be matched to the exponential solutions outside at the boundary of the dielectric waveguide. The boundary conditions are the continuity of normal $\vec{B}$ and $\vec{D}$ and tangential $\vec{E}$ and $\vec{H}$, rather than the vanishing of normal $\vec{B}$ and tangential $\vec{E}$ appropriate for hollow conductors [3],[4]. Because of the more involved boundary conditions the type of fields do not separate into TE and TM modes. In general, axial components of both $\vec{E}$ and $\vec{H}$ exist. Such waves are sometimes designated as HE modes or quasi TE or TM modes [3].

3.1 The Asymmetric waveguide

Let's assume $n_3 \neq n_4 \neq n_5 \neq n_2 < n_1$. The following set of field components satisfies the reduced wave equation (2) in region 1 with

$$k_1^2 = \omega^2\mu\varepsilon_1 = k_x^2 + k_y^2 + \beta^2$$

$$E_z = A_e e^{-ik_x x - ik_y y} + B_e e^{-ik_x x + ik_y y} + C_e e^{ik_x x - ik_y y} + D_e e^{ik_x x + ik_y y}$$

$$H_z = A_m e^{-ik_x x - ik_y y} + B_m e^{-ik_x x + ik_y y} + C_m e^{ik_x x - ik_y y} + D_m e^{ik_x x + ik_y y}$$

$$E_x = \frac{-1}{k_1^2 - \beta^2}[(\beta k_x A_e + \omega\mu k_y A_m)e^{-ik_x x - ik_y y} + (\beta k_x B_e - \omega\mu k_y B_m)e^{-ik_x x + ik_y y} + (-\beta k_x C_e + \omega\mu k_y C_m)e^{ik_x x - ik_y y} + (-\beta k_x D_e - \omega\mu k_y D_m)e^{ik_x x + ik_y y}]$$

$$H_y = \frac{-1}{k_1^2 - \beta^2}[(\omega\varepsilon_1 k_x A_e + \beta k_y A_m)e^{-ik_x x - ik_y y} + (\omega\varepsilon_1 k_x B_e - \beta k_y B_m)e^{ik_x x + ik_y y} + (-\omega\varepsilon_1 k_x C_e + \beta k_y C_m)e^{ik_x x - ik_y y} + (-\omega\varepsilon_1 k_x D_e - \beta k_y D_m)e^{ik_x x + ik_y y}]$$

$$E_y = \frac{-1}{k_1^2 - \beta^2}[(\beta k_y A_e - \omega\mu k_x A_m)e^{-ik_x x - ik_y y} - (\beta k_y B_e + \omega\mu k_x B_m)e^{-ik_x x + ik_y y} + (\beta k_y C_e + \omega\mu k_x C_m)e^{ik_x x - ik_y y} + (-\beta k_y D_e + \omega\mu k_x D_m)e^{ik_x x + ik_y y}]$$

$$H_x = \frac{-1}{k_1^2 - \beta^2}[(-\omega\varepsilon_1 k_y A_e + \beta k_x A_m)e^{-ik_x x - ik_y y} + (\omega\varepsilon_1 k_y B_e + \beta k_x B_m)e^{-ik_x x + ik_y y} - (\omega\varepsilon_1 k_y C_e + \beta k_x C_m)e^{ik_x x - ik_y y} + (\omega\varepsilon_1 k_y D_e - \beta k_x D_m)e^{ik_x x + ik_y y}]$$

On the outside we must require all field components in region 2-5 to vanish at infinite distance from the core. We are thus limited to use decaying exponential functions that lead away from the region 1. The field components in these regions are chosen similarly to the field in the core, with the additional requirement that the electric field components E_z and the magnetic field components H_z are continuous at the boundary. We thus have in region 2 with $k_2^2 = \omega^2 \mu\varepsilon_2 = -\alpha_2^2 + k_y^2 + \beta^2$

$$E_z = [(A_e e^{ik_x d} + C_e e^{-ik_x d})e^{-ik_y y} + (B_e e^{ik_x d} + D_e e^{-ik_x d})e^{ik_y y}]e^{\alpha_2(x+d)}$$

$$H_z = [(A_m e^{ik_x d} + C_m e^{-ik_x d})e^{-ik_y y} + (B_m e^{ik_x d} + D_m e^{-ik_x d})e^{ik_y y}]e^{\alpha_2(x+d)}$$

$$E_x = \frac{-1}{k_2^2 - \beta^2}[\{i\beta\alpha_2(A_e e^{ik_x d} + C_e e^{-ik_x d}) + \omega\mu k_y(A_m e^{ik_x d} + C_m e^{-ik_x d})\}e^{-ik_y y} + \{i\beta\alpha_2(B_e e^{ik_x d} + D_e e^{-ik_x d}) - \omega\mu k_y(B_m e^{ik_x d} + D_m e^{-ik_x d})\}e^{ik_y y}]e^{\alpha_2(x+d)}$$

$$H_y = \frac{-1}{k_2^2 - \beta^2}[\{i\omega\varepsilon_2\alpha_2(A_e e^{ik_x d} + C_e e^{-ik_x d}) + \beta k_y(A_m e^{ik_x d} + C_m e^{-ik_x d})\}e^{-ik_y y} + \{i\omega\varepsilon_2\alpha_2(B_e e^{ik_x d} + D_e e^{-ik_x d}) - \beta k_y(B_m e^{ik_x d} + D_m e^{-ik_x d})\}e^{ik_y y}]e^{\alpha_2(x+d)}$$

$$E_y = \frac{-1}{k_2^2 - \beta^2}[\{\beta k_y(A_e e^{ik_x d} + D_e e^{-ik_y d}) - i\omega\mu\alpha_2(A_m e^{ik_x d} + C_m e^{-ik_y d})\}e^{-ik_y y} - \{\beta k_y(B_e e^{ik_x d} + D_e e^{-ik_x d}) + i\omega\mu\alpha_2(B_m e^{ik_x d} + D_m e^{-ik_x d})\}e^{ik_y y}]e^{\alpha_2(x+d)}$$

$$H_x = \frac{-1}{k_2^2 - \beta^2}[\{-\omega\varepsilon_2 k_y(A_e e^{ik_x d} + C_e e^{-ik_x d}) + i\beta\alpha_2(A_m e^{ik_x d} + C_m e^{-ik_x d})\}e^{-ik_y y} + \{\omega\varepsilon_2 k_y(B_e e^{ik_x d} + D_e e^{-ik_x d}) + i\beta\alpha_2(B_m e^{ik_x d} + D_m e^{-ik_x d})\}e^{ik_y y}]e^{\alpha_2(x+d)}$$

The field in region 3 is similarly with $k_3^2 = \omega^2 \mu\varepsilon_3 = -\alpha_3^2 + k_y^2 + \beta^2$

$$E_z = [(A_e + C_e)e^{-ik_y y} + (B_e + D_e)e^{ik_y y}]e^{-\alpha_3 x}$$

$$H_z = [(A_m + C_m)e^{-ik_y y} + (B_m + D_m)e^{ik_y y}]e^{-\alpha_3 x}$$

$$E_x = \frac{-1}{k_3^2 - \beta^2}[\{-i\beta\alpha_3(A_e + C_e) + \omega\mu k_y(A_m + C_m)\}e^{-ik_y y} + \{-i\beta\alpha_3(B_e + D_e) - \omega\mu k_y(B_m + D_m)\}e^{ik_y y}]e^{-\alpha_3 x}$$

$$H_y = \frac{-1}{k_3^2 - \beta^2}[\{-i\omega\varepsilon_3\alpha_3(A_e + C_e) + \beta k_y(A_m + C_m)\}e^{-ik_y y} + \{-i\omega\varepsilon_3\alpha_3(B_e + D_e) - \beta k_y(B_m + D_m)\}e^{ik_y y}]e^{-\alpha_3 x}$$

$$E_y = \frac{-1}{k_3^2 - \beta^2}[\{\beta k_y(A_e + C_e) + i\omega\mu\alpha_3(A_m + C_m)\}e^{-ik_y y} + \{-\beta k_y(B_e + D_e) - i\omega\mu\alpha_3(B_m + D_m)\}e^{ik_y y}]e^{-\alpha_3 x}$$

$$H_x = \frac{-1}{k_3^2 - \beta^2}[\{-\omega\varepsilon_3 k_y(A_e + C_e) - i\beta\alpha_3(A_m + C_m)\}e^{ik_y y} + \{\omega\varepsilon_3 k_y(B_e + D_e) - i\beta\alpha_3(B_m + D_m)\}e^{ik_y y}]e^{-\alpha_3 x}$$

In region 4 with $k_4^2 = \omega^2 \mu\varepsilon_4 = k_x^2 - \alpha_4^2 + \beta^2$ we have

$$E_z = [(A_e e^{ik_y b} + B_e e^{ik_y b})e^{-ik_x x} + (C_e e^{-ik_y b} + D_e e^{ik_y b})e^{ik_x x}]e^{-\alpha_4(y-b)}$$

$$H_z = [(A_m e^{-ik_y b} + B_m e^{ik_y b})e^{ik_x x} + (C_m e^{ik_y b} + D_m e^{ik_y b})e^{ik_x x}]e^{-\alpha_4(y-b)}$$

$$E_x = \frac{-1}{k_4^2 - \beta^2}[\{\beta k_x(A_e e^{-ik_y b} + B_e e^{ik_y b}) - i\omega\mu\alpha_4(A_m e^{-ik_y b} + B_m e^{ik_y b})\}e^{-ik_x x} - \{\beta k_x(C_e e^{-ik_y b} + D_e e^{ik_y b}) + i\omega\mu\alpha_4(C_m e^{-ik_y b} + D_m e^{ik_y b})\}e^{ik_x x}]e^{-\alpha_4(y-b)}$$

$$H_y = \frac{-1}{k_4^2 - \beta^2}[\{\omega\varepsilon_4 k_x(A_e e^{-ik_y b} + B_e e^{ik_y b}) - i\beta\alpha_4(A_m e^{-ik_y b} + B_m e^{ik_y b})\}e^{-ik_x x} - \{\omega\varepsilon_4 k_x(C_e e^{-ik_y b} + D_e e^{ik_y b}) + i\beta\alpha_4(C_m e^{-ik_y b} + D_m e^{ik_y b})\}e^{ik_x x}]e^{-\alpha_4(y-b)}$$

$$E_y = \frac{-1}{k_4^2 - \beta^2}[\{-i\beta\alpha_4(A_e e^{-ik_y b} + B_e e^{ik_y b}) - \omega\mu k_x(A_m e^{-ik_y b} + B_m e^{ik_y b})\}e^{-ik_x x} + \{-i\beta\alpha_4(C_e e^{-ik_y b} + D_e e^{ik_y b}) + \omega\mu k_x(C_m e^{-ik_y b} + D_m e^{ik_y b})\}e^{ik_x x}]e^{-\alpha_4(y-b)}$$

$$H_x = \frac{-1}{k_4^2 - \beta^2}[\{i\omega\varepsilon_4\alpha_4(A_e e^{-ik_y b} + B_e e^{ik_y b}) + \beta k_x(A_m e^{-ik_y b} + B_m e^{ik_y b})\}e^{-ik_x x} + \{i\omega\varepsilon_4\alpha_4(C_e e^{-ik_y b} + D_e e^{ik_y b}) - \beta k_x(C_m e^{-ik_y b} + D_m e^{ik_y b})\}e^{ik_x x}]e^{\alpha_4(y-b)}$$

Finally in region 5 we have with

$k_5^2 = \omega^2 \mu\varepsilon_5 = k_x^2 - \alpha_5^2 + \beta^2$

$$E_z = [(A_e + B_e)e^{-ik_x x} + (C_e + D_e)e^{ik_x x}]e^{\alpha_5 y}$$

$$H_z = [(A_m + B_m)e^{-ik_x x} + (C_m + D_m)e^{ik_x x}]e^{\alpha_5 y}$$

$$E_x = \frac{-1}{k_5^2-\beta^2}[\{\beta k_x(A_e+B_e)+i\omega\mu\alpha_5(A_m+B_m)\}e^{-ik_x x} + \{-\beta k_x(C_e+D_e)+i\omega\mu\alpha_5(C_m+D_m)\}e^{ik_x x}]e^{\alpha_5 y}$$

$$H_y = \frac{-1}{k_5^2-\beta^2}[\{\omega\varepsilon_5 k_x(A_e+B_e)+i\beta\alpha_5(A_m+B_m)\}e^{-ik_x x} + \{-\omega\varepsilon_5 k_x(C_e+D_e)+i\beta\alpha_5(C_m+D_m)\}e^{ik_x x}]e^{\alpha_5 y}$$

$$E_y = \frac{-1}{k_5^2-\beta^2}[\{i\beta\alpha_5(A_e+B_e)-\omega\mu k_x(A_m+B_m)\}e^{-ik_x x} + \{i\beta\alpha_5(C_e+D_e)+\omega\mu k_x(C_m+D_m)\}e^{ik_x x}]e^{\alpha_5 y}$$

$$H_x = \frac{-1}{k_5^2-\beta^2}[\{-i\omega\varepsilon_5\alpha_5(A_e+B_e)+\beta k_x(A_m+B_m)\}e^{-ik_x x} + \{-i\omega\varepsilon_5\alpha_5(C_e+D_e)-\beta k_x(C_m+D_m)\}e^{ik_x x}]e^{\alpha_5 y}$$

3.2 Quasi TE Mode and Quasi TM Mode

The boundary conditions should be matched at $x=0, -d$. The components E_z and H_z have already been matched by the proper choice of the field amplitudes at the interfaces. It is required that the components H_x and H_y pass continuously through the core boundary at $x=0, -d$. This requirement also causes E_x and E_y to be continuous as required. With the notations

$$\begin{aligned} z_1 &= \omega[(k_3^2-\beta^2)\varepsilon_1 k_x + i(k_1^2-\beta^2)\varepsilon_3\alpha_3] \\ z_2 &= \omega[(k_2^2-\beta^2)\varepsilon_1 k_x + i(k_1^2-\beta^2)\varepsilon_2\alpha_2] \\ z_3 &= (k_3^2-\beta^2)k_x + i(k_1^2-\beta^2)\alpha_3 \\ z_4 &= (k_2^2-\beta^2)k_x + i(k_1^2-\beta^2)\alpha_2 \end{aligned} \tag{13}$$

$$\begin{aligned} a_1 &= \omega\beta k_y(\varepsilon_1-\varepsilon_3) \\ a_2 &= \omega\beta k_y(\varepsilon_1-\varepsilon_2) \\ a_3 &= -\beta k_y(k_1^2-k_3^2) \\ a_4 &= -\beta k_y(k_1^2-k_2^2) \end{aligned} \tag{14}$$

we have the following matrix equation :

$$\begin{bmatrix} z_1 & a_3 & 0 & 0 & -z_1^* & a_3 & 0 & 0 \\ 0 & 0 & z_1 & -a_3 & 0 & 0 & -z_1^* & -a_3 \\ z_2^* e^{ik_x d} & a_4 e^{ik_x d} & 0 & 0 & -z_2 e^{-ik_x d} & a_4 e^{-ik_x d} & 0 & 0 \\ 0 & 0 & -z_2^* e^{ik_x d} & a_4 e^{ik_x d} & 0 & 0 & z_2 e^{-ik_x d} & a_4 e^{-ik_x d} \\ a_1 & z_3 & 0 & 0 & a_1 & -z_3^* & 0 & 0 \\ 0 & 0 & -a_1 & z_3 & 0 & 0 & -a_1 & -z_3^* \\ a_2 e^{ik_x d} & z_4^* e^{ik_x d} & 0 & 0 & a_2 e^{-ik_x d} & -z_4 e^{-ik_x d} & 0 & 0 \\ 0 & 0 & a_2 e^{ik_x d} & z_4^* e^{ik_x d} & 0 & 0 & a_2 e^{-ik_x d} & z_4 e^{-ik_x d} \end{bmatrix} \begin{bmatrix} A_e \\ B_e \\ C_e \\ D_e \\ A_m \\ B_m \\ C_m \\ D_m \end{bmatrix} = 0 \tag{15}$$

This equation represents a system of homogeneous simultaneous equation. A solution is possible only if the determinant of the equation vanishes. We thus obtain the eigenvalue equation by using equation (13) and (14)

$$A\cos 2k_x d + B\sin 2k_x d = C \tag{16}$$

where

$$A = (k_1^2\alpha_2^2 - k_2^2k_x^2)(k_1^2\alpha_3^2 - k_3^2k_x^2) - k_x^2\alpha_2\alpha_3(k_1^2+k_2^2)(k_1^2+k_3^2)$$

$$B = -k_x\alpha_2(k_1^2+k_2^2)(k_1^2\alpha_3^2 - k_3^2k_x^2) - k_x\alpha_3(k_1^2+k_3^2)(k_1^2\alpha_2^2 - k_2^2k_x^2)$$

$$C = (k_1^2-k_2^2)(k_1^2-k_3^2)\{(k_1^2-k_x^2)^2 - k_x^2\alpha_2\alpha_3\}$$

Eq. (16) has two possible solutions. One is a TE mode solution and the other is a TM mode solution :

$$\begin{aligned} \tan k_x d &= \frac{B \pm \sqrt{B^2+A^2-C^2}}{A+C} \\ &= \frac{k_x(\alpha_2+\alpha_3)}{k_x^2-\alpha_2\alpha_3} \quad \cdots\cdot\ TE \text{ mode} \\ &= \frac{k_1^2k_x(k_2^2\alpha_3+k_3^2\alpha_2)}{k_x^2k_2^2k_3^2-k_1^4\alpha_2\alpha_3} \quad \cdots\cdot\ TM \text{ mode} \end{aligned} \tag{17}$$

We recognize these equations as the eigenvalue equations of TM modes (11) and TE modes (5) of the infinite slab respectively. It means that we can have quasi TE modes or quasi TM modes depending on the status of the boundary conditions at $y=0,\ b$.

For quasi TE modes, we can obtain the following two reduced matrix equations from the above equations (15), (16) and (17)

$$\begin{bmatrix} a_3 & -z_1^* & a_3 \\ z_3 & a_1 & -z_3^* \\ a_4 e^{ik_x d} & -z_2 e^{-ik_x d} & a_4 e^{-ik_x d} \end{bmatrix} \begin{bmatrix} A_m \\ C_e \\ C_m \end{bmatrix} = \begin{bmatrix} -z_1 \\ -a_1 \\ -z_2^* e^{ik_x d} \end{bmatrix} A_e \tag{18}$$

and

$$\begin{bmatrix} -a_3 & -z_1^* & -a_3 \\ z_3 & -a_1 & -z_3^* \\ a_4 e^{ik_x d} & z_2 e^{-ik_x d} & a_4 e^{-ik_x d} \end{bmatrix} \begin{bmatrix} B_m \\ D_e \\ D_m \end{bmatrix} = \begin{bmatrix} -z_1 \\ a_1 \\ z_2^* e^{ik_x d} \end{bmatrix} B_e \tag{19}$$

By solving these two simultaneous equations, we can have

$$C_e = e^{i2\theta_1} A_e$$

$$D_e = e^{i2\theta_1} B_e$$

$$A_m = \frac{-\beta}{\omega\mu} \frac{k_x}{k_y} A_e$$

$$B_m = \frac{\beta}{\omega\mu} \frac{k_x}{k_y} B_e$$

$$C_m = \frac{\beta}{\omega\mu} \frac{k_x}{k_y} e^{i2\theta_1} A_e$$

$$D_m = \frac{-\beta}{\omega\mu} \frac{k_x}{k_y} e^{i2\theta_1} B_e$$

where $\tan k_x d = \frac{k_x(\alpha_2 + \alpha_3)}{k_x^2 - \alpha_2 \alpha_3}$ and $\tan \theta_1 = \frac{\alpha_3}{k_x}$ are used.

For quasi TM modes, we can have the following coefficients exactly the same way

$$C_e = -e^{i2\theta_2} A_e$$

$$D_e = -e^{i2\theta_2} B_e$$

$$A_m = \frac{\omega\varepsilon_1}{\beta} \frac{k_y}{k_x} A_e$$

$$B_m = -\frac{\omega\varepsilon_1}{\beta} \frac{k_y}{k_x} B_e$$

$$C_m = \frac{\omega\varepsilon_1}{\beta} \frac{k_y}{k_x} e^{i2\theta_2} A_e$$

$$D_m = -\frac{\omega\varepsilon_1}{\beta} \frac{k_y}{k_x} e^{i2\theta_2} B_e$$

where $\tan k_x d = \frac{k_1^2 k_x(k_2^2 \alpha_3 + k_3^2 \alpha_2)}{k_x^2 k_2^2 k_3^2 - k_1^4 \alpha_2 \alpha_3}$ and $\tan \theta_2 = \frac{\varepsilon_1 \alpha_3}{\varepsilon_3 k_x}$ are used. We know that the eigenvalue equation can be used to determine k_x because we can express α_2 and α_3 in terms of k_x :

$$\alpha_2^2 = k_1^2 - k_2^2 - k_x^2 = (n_1^2 - n_2^2) k_0^2 - k_x^2$$

$$\alpha_3^2 = k_1^2 - k_3^2 - k_x^2 = (n_1^2 - n_3^2) k_0^2 - k_x^2$$

Then k_x is the only unknown quantity in $\tan k_x d = \frac{k_x(\alpha_2 + \alpha_3)}{k_x^2 - \alpha_2 \alpha_3}$ and

$$\tan k_x d = \frac{k_1^2 k_x(k_2^2 \alpha_3 + k_3^2 \alpha_2)}{k_x^2 k_2^2 k_3^2 - k_1^4 \alpha_2 \alpha_3}$$

At the core boundary in region 4 and region 5 we require that the electric field $\vec{E}$ and magnetic field $\vec{H}$ be continuous at $y=0,\ b$. The components E_z and H_z have already been matched by the proper choice of the field amplitudes. It is required that the components H_x and H_y should be continuous through the core boundary at $y=0,\ b$. This requirement also causes E_x and E_y to be continuous automatically. Let's use the notations

$$z_1 = \omega[(k_5^2 - \beta^2)\varepsilon_1 k_y + i(k_1^2 - \beta^2)\varepsilon_5 \alpha_5]$$

$$z_2 = \omega[(k_4^2 - \beta^2)\varepsilon_1 k_y + i(k_1^2 - \beta^2)\varepsilon_4 \alpha_4]$$

$$z_3 = (k_5^2 - \beta^2) k_y + i(k_1^2 - \beta^2)\alpha_5$$

$$z_4 = (k_4^2 - \beta^2) k_y + i(k_1^2 - \beta^2)\alpha_4 \qquad (20)$$

$$a_1 = -\omega\beta k_x(\varepsilon_1 - \varepsilon_5)$$

$$a_2 = -\omega\beta k_x(\varepsilon_1 - \varepsilon_4)$$

$$a_3 = -\beta k_x(k_1^2 - k_5^2)$$

$$a_4 = -\beta k_x(k_1^2 - k_4^2). \qquad (21)$$

Then we can obtain the following matrix equation

$$\begin{bmatrix} a_1 & z_3^* & a_1 & -z_3 & 0 & 0 & 0 & 0 \\ a_2 e^{-ik_y b} & z_4 e^{-ik_y b} & a_2 e^{ik_y b} & -z_4^* e^{ik_y b} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_1 & z_3^* & -a_1 & -z_3 \\ 0 & 0 & 0 & 0 & a_2 e^{-ik_y b} & -z_4 e^{-ik_y b} & a_2 e^{ik_y b} & z_4^* e^{ik_y b} \\ -z_1^* & a_3 & z_1 & a_3 & 0 & 0 & 0 & 0 \\ -z_2 e^{-ik_y b} & a_4 e^{-ik_y b} & z_2^* e^{ik_y b} & a_4 e^{ik_y b} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -z_1^* & -a_3 & z_1 & -a_3 \\ 0 & 0 & 0 & 0 & z_2 e^{-ik_y b} & a_4 e^{-ik_y b} & -z_2^* e^{ik_y b} & a_4 e^{ik_y b} \end{bmatrix} \begin{bmatrix} A_e \\ B_e \\ C_e \\ D_e \\ A_m \\ B_m \\ C_m \\ D_m \end{bmatrix} = 0 \qquad (22)$$

This equation represents a system of homogeneous simultaneous equation. A solution is possible only if the determinant D of the equation system vanishes. We thus obtain another eigenvalue equation

$$A' \cos 2k_y b + B' \sin 2k_y b = C' \qquad (23)$$

where

$$A' = (k_1^2 \alpha_4^2 - k_4^2 k_y^2)(k_1^2 \alpha_5^2 - k_5^2 k_y^2) - k_y^2 \alpha_4 \alpha_5 (k_1^2 + k_4^2)(k_1^2 + k_5^2)$$

$$B' = -k_y \alpha_4 (k_1^2 + k_4^2)(k_1^2 \alpha_5^2 - k_5^2 k_y^2) - k_y \alpha_5 (k_1^2 + k_5^2)(k_1^2 \alpha_4^2 - k_4^2 k_y^2)$$

$C' = (k_1^2 - k_4^2)(k_1^2 - k_5^2)\{(k_1^2 - k_y^2)^2 - k_y^2\alpha_4\alpha_5\}$

There are two possible solutions. One is a TE modes solution and the other is a TM modes solution :

$$\tan k_y b = \frac{B' \pm \sqrt{B'^2 + A'^2 - C'^2}}{A' + C'} = \frac{k_y(\alpha_4+\alpha_5)}{k_y^2 - \alpha_4\alpha_5} \cdots\cdot TE \text{ mode} = \frac{k_1^2 k_y(k_4^2\alpha_5 + k_5^2\alpha_4)}{k_y^2k_4^2k_5^2 - k_1^4\alpha_4\alpha_5} \cdots\cdot TM \text{ mode} \quad (24)$$

We recognize these equations as the eigenvalue equations of TM modes (11) and TE modes (5) of the infinite slab respectively in $y-$ direction. It means that we can have quasi TE modes or quasi TM modes depending on the status of the boundary conditions at $x=0, -d$. For quasi TE modes, we can obtain the following two reduced matrix equations from the above matrix equations (22) and (23)

$$\begin{bmatrix} z_3^* & a_1 & -z_3 \\ a_3 & z_1 & a_3 \\ z_4 e^{-ik_yb} & a_2 e^{ik_yb} & -z_4^* e^{ik_yb} \end{bmatrix} \begin{bmatrix} A_m \\ B_e \\ B_m \end{bmatrix} = \begin{bmatrix} -a_1 \\ z_1^* \\ -a_2 e^{-ik_yb} \end{bmatrix} A_e$$

and

$$\begin{bmatrix} z_3^* & -a_1 & -z_3 \\ -a_3 & z_1 & -a_3 \\ z_4 e^{-ik_yb} & -a_2 e^{ik_yb} & -z_4^* e^{ik_yb} \end{bmatrix} \begin{bmatrix} C_m \\ D_e \\ D_m \end{bmatrix} = \begin{bmatrix} a_1 \\ z_1^* \\ a_2 e^{-ik_yb} \end{bmatrix} C_e$$

By solving this simultaneous equations, we can have

$$B_e = e^{-i2\theta_3} A_e$$
$$D_e = e^{-i2\theta_3} C_e$$
$$A_m = \frac{\beta}{\omega\mu}\frac{k_y}{k_x} A_e$$
$$B_m = \frac{-\beta}{\omega\mu}\frac{k_y}{k_x} e^{-i2\theta_3} A_e$$
$$C_m = \frac{-\beta}{\omega\mu}\frac{k_y}{k_x} C_e$$
$$D_m = \frac{\beta}{\omega\mu}\frac{k_y}{k_x} e^{-i2\theta_3} C_e \quad (25)$$

where $\tan k_y b = \frac{k_y(\alpha_4+\alpha_5)}{k_y^2 - \alpha_4\alpha_5}$ and $\tan\theta_3 = \frac{\alpha_5}{k_y}$ are used.

For quasi TM modes, we can have the following coefficients exactly the same way

$$B_e = -e^{-i2\theta_4} A_e$$
$$D_e = -e^{-i2\theta_4} C_e$$
$$A_m = -\frac{\omega\varepsilon_1}{\beta}\frac{k_x}{k_y} A_e$$
$$B_m = \frac{-\omega\varepsilon_1}{\beta}\frac{k_x}{k_y} e^{-i2\theta_4} A_e$$
$$C_m = \frac{\omega\varepsilon_1}{\beta}\frac{k_x}{k_y} C_e$$
$$D_m = \frac{\omega\varepsilon_1}{\beta}\frac{k_x}{k_y} e^{-i2\theta_4} C_e \quad (26)$$

where $\tan k_y b = \frac{k_1^2 k_y(k_4^2\alpha_5 + k_5^2\alpha_4)}{k_y^2k_4^2k_5^2 - k_1^4\alpha_4\alpha_5}$ and $\tan\theta_4 = \frac{\varepsilon_1\alpha_5}{\varepsilon_5 k_y}$ are used.

We know that the eigenvalue equation can be used to determine k_y because we can express α_4 and α_5 in terms of k_y :

$$\alpha_4^2 = k_1^2 - k_4^2 - k_y^2 = (n_1^2 - n_4^2)k_0^2 - k_y^2$$
$$\alpha_5^2 = k_1^2 - k_5^2 - k_y^2 = (n_1^2 - n_5^2)k_0^2 - k_y^2$$

Then k_y is the only unknown quantity in $\tan k_y b = \frac{k_y(\alpha_4+\alpha_5)}{k_y^2 - \alpha_4\alpha_5}$ and

$$\tan k_y b = \frac{k_1^2 k_y(k_4^2\alpha_5 + k_5^2\alpha_4)}{k_y^2k_4^2k_5^2 - k_1^4\alpha_4\alpha_5}$$

3.3 The Wave Equations of Quasi TE and Quasi TM Modes

For quasi TE modes we can determine β_{QTE} from $\beta_{QTE}^2 = k_1^2 - (k_x^2 + k_y^2)$ where we get k_x and k_y from $\tan k_x d = \frac{k_x(\alpha_2+\alpha_3)}{k_x^2 - \alpha_2\alpha_3}$ and $\tan k_y b = \frac{k_y(\alpha_4+\alpha_5)}{k_y^2 - \alpha_4\alpha_5}$ respectively. The following first set of field components which is continuous at $x=0, -d$ satisfies the Maxwell's equation and describes the QTE_y modes in region 1.

$$E_z = A_y \frac{k_y \omega\mu}{k_x \beta_{QTE}} \cos(k_x x + \theta_1)\cos(k_y y - \theta_3)$$

$$H_z = A_y \sin(k_x x + \theta_1)\sin(k_y y - \theta_3)$$

$$E_y = iA_y \frac{\omega\mu}{k_x} \cos(k_x x + \theta_1)\sin(k_y y - \theta_3)$$

$$H_x = -iA_y \frac{k_1^2 - k_x^2}{k_x \beta_{QTE}} \cos(k_x x + \theta_1)\sin(k_y y - \theta_3)$$

$$H_y = iA_y \frac{k_y}{\beta_{QTE}} \sin(k_x x + \theta_1)\cos(k_y y - \theta_3)$$

$$E_x = 0 \qquad (27)$$

where $\tan\theta_1 = \frac{\alpha_3}{k_x}$ and $\tan\theta_3 = \frac{\alpha_5}{k_y}$. The wave impedance Z_{QTE} is defined by $Z_{QTE} = \frac{E_y}{-H_x} = \frac{\beta_{QTE}\omega\mu}{k_1^2 - k_x^2}$. Note that if $\beta_{QTE}^2 \approx k_1^2 - k_x^2$ with $k_y^2 \approx 0$, then Z_{QTE} becomes Z_{TE}. If θ_3 becomes $\frac{\pi}{2}$ as $k_y \approx 0$, components E_z and H_y become 0. Then the above wave equations are exactly those of slab dielectric waveguide's TE mode (6).

The QTE_x mode which is continuous at $y = 0,\ b$ and satisfies the Maxwell's equation is the following set of wave equations

$$E_z = -A_x \frac{k_x \omega\mu}{k_y \beta_{QTE}} \cos(k_x x + \theta_1)\cos(k_y y - \theta_3)$$

$$H_z = A_x \sin(k_x x + \theta_1)\sin(k_y y - \theta_3)$$

$$E_y = 0$$

$$H_x = iA_x \frac{k_x}{\beta_{QTE}} \cos(k_x x + \theta_1)\sin(k_y y - \theta_3)$$

$$E_x = -iA_x \frac{\omega\mu}{k_y} \sin(k_x x + \theta_1)\cos(k_y y - \theta_3)$$

$$H_y = -iA_x \frac{k_1^2 - k_y^2}{k_y \beta_{QTE}} \sin(k_x x + \theta_1)\cos(k_y y - \theta_3) \qquad (28)$$

where $\tan\theta_1 = \frac{\alpha_3}{k_x}$ and $\tan\theta_3 = \frac{\alpha_5}{k_y}$.

For quasi TM modes we can determine β_{QTM} from $\beta_{QTM}^2 = k_1^2 - (k_x^2 + k_y^2)$ where we get k_x and k_y from $\tan k_x d = \frac{k_1^2 k_x (k_2^2 \alpha_3 + k_3^2 \alpha_2)}{k_x^2 k_2^2 k_3^2 - k_1^4 \alpha_2 \alpha_3}$ and $\tan k_y b = \frac{k_1^2 k_y (k_4^2 \alpha_5 + k_5^2 \alpha_4)}{k_y^2 k_4^2 k_5^2 - k_1^4 \alpha_4 \alpha_5}$. The following first set of field components which is continuous at $x = 0,\ -d$ satisfies the Maxwell's equation and describes the QTM_y modes in region 1.

$$E_z = B_y \sin(k_x x + \theta_2)\sin(k_y y - \theta_4)$$

$$H_z = -B_y \frac{k_y}{k_x} \frac{\omega\varepsilon_1}{\beta_{QTM}} \cos(k_x x + \theta_2)\cos(k_y y - \theta_4)$$

$$E_y = iB_y \frac{k_y}{\beta_{QTM}} \sin(k_x x + \theta_2)\cos(k_y y - \theta_4)$$

$$H_x = 0$$

$$E_x = -iB_y \frac{k_1^2 - k_x^2}{k_x \beta_{QTM}} \cos(k_x x + \theta_2)\sin(k_y y - \theta_4)$$

$$H_y = -iB_y \frac{\omega\varepsilon_1}{k_x} \cos(k_x x + \theta_2)\sin(k_y y - \theta_4)$$

where $\tan\theta_2 = \frac{k_1^2 \alpha_3}{k_3^2 k_x}$ and $\tan\theta_4 = \frac{k_1^2 \alpha_5}{k_5^2 k_y}$. The wave impedance Z_{QTM} is defined by $Z_{QTM} = \frac{E_x}{H_y} = \frac{k_1^2 - k_x^2}{\beta_{QTM}\omega\varepsilon_1}$. Note that if $\beta_{QTM}^2 \approx k_1^2 - k_x^2$ with $k_y^2 \approx 0$, then Z_{QTM} becomes Z_{TM}. Since θ_4 becomes $\frac{\pi}{2}$ with $k_y \approx 0$, H_z and E_y become 0. Then the above wave equations are exactly those of slab dielectric waveguide's TM mode (10).

The QTM_x mode which is continuous at $y = 0,\ b$ and satisfies the Maxwell's equation is the following set of wave equations

$$E_z = B_x \sin(k_x x + \theta_2)\sin(k_y y - \theta_4)$$

$$H_z = B_x \frac{k_x}{k_y} \frac{\omega\varepsilon_1}{\beta_{QTM}} \cos(k_x x + \theta_2)\cos(k_y y - \theta_4)$$

$$E_y = -iB_x \frac{k_1^2 - k_y^2}{k_y \beta_{QTM}} \sin(k_x x + \theta_2)\cos(k_y y - \theta_4)$$

$$H_x = iB_x \frac{\omega\varepsilon_1}{k_y} \sin(k_x x + \theta_2)\cos(k_y y - \theta_4)$$

$$E_x = iB_x \frac{k_x}{\beta_{QTM}} \cos(k_x x + \theta_2)\sin(k_y y - \theta_4)$$

$$H_y = 0$$

where $\tan\theta_2 = \frac{k_1^2 \alpha_3}{k_3^2 k_x}$ and $\tan\theta_4 = \frac{k_1^2 \alpha_5}{k_5^2 k_y}$

Now let's compare above equations with the following equations which are the approximated

Marcatili's E^{y}_{pq} modes [1],[4]. With

$$\tan k_x d = \frac{k_x(\alpha_2 + \alpha_3)}{k_x^2 - \alpha_2\alpha_3} \text{ and}$$

$$\tan k_y b = \frac{k_1^2 k_y (k_4^2 \alpha_5 + k_5^2 \alpha_4)}{k_y^2 k_4^2 k_5^2 - k_1^4 \alpha_4 \alpha_5}$$

$$E_z = A\frac{k_y \beta^{y}_{pq}}{k_x \omega \varepsilon_1}\cos(k_x x + \theta_1)\cos(k_y y + \theta_4)$$

$$H_z = A\sin(k_x x + \theta_1)\sin(k_y y + \theta_4)$$

$$E_y = iA\frac{k_1^2 - k_y^2}{k_x \omega \varepsilon_1}\cos(k_x x + \theta_1)\sin(k_y y + \theta_4)$$

$$H_x = -iA\frac{\beta^{y}_{pq}}{k_x}\cos(k_x x + \theta_1)\sin(k_y y + \theta_4)$$

$$E_x \approx 0 = -iA\frac{k_y}{\omega\varepsilon_1}\sin(k_x x + \theta_1)\cos(k_y y + \theta_4)$$

$$H_y = 0$$

where $\tan\theta_1 = \frac{\alpha_3}{k_x}$ and $\tan\theta_4 = \frac{\varepsilon_5 k_y}{\varepsilon_1 \alpha_5}$ are used. Since θ_4 becomes 0 with $k_y \approx 0$, all E_z, H_z, E_y, H_x, E_x and H_y are 0 as k_y becomes 0.

For the case of Marcatili's approximated E^{x}_{pq} mode equations [1],[4], by using

$$\tan k_x d = \frac{k_1^2 k_x (k_2^2 \alpha_3 + k_3^2 \alpha_2)}{k_x^2 k_2^2 k_3^2 - k_1^4 \alpha_2 \alpha_3} \text{ and}$$

$$\tan k_y b = \frac{k_y(\alpha_4 + \alpha_5)}{k_y^2 - \alpha_4 \alpha_5}$$

$$E_z = A\cos(k_x x - \theta_2)\cos(k_y y - \theta_3)$$

$$H_z = -A\frac{k_y}{k_x}\frac{\omega\varepsilon_1}{\beta^{x}_{pq}}\sin(k_x x - \theta_2)\sin(k_y y - \theta_3)$$

$$E_y \approx 0 = -iA\frac{k_y}{\beta^{x}_{pq}}\cos(k_x x - \theta_2)\sin(k_y y - \theta_3)$$

$$H_x = 0$$

$$E_x = iA\frac{k_1^2 - k_x^2}{k_x \beta^{x}_{pq}}\sin(k_x x - \theta_2)\cos(k_y y - \theta_3)$$

$$H_y = iA\frac{\omega\varepsilon_1}{k_x}\sin(k_x x - \theta_2)\cos(k_y y - \theta_3)$$

where $\tan\theta_2 = \frac{\varepsilon_3 k_x}{\varepsilon_1 \alpha_3}$ and $\tan\theta_3 = \frac{\alpha_5}{k_y}$ are used. Since θ_3 becomes $\frac{\pi}{2}$ with $k_y \approx 0$, all E_z, H_z, E_y, H_x, E_x and H_y are 0 as $k_y \approx 0$. This result came from those Marcatili's approximation $E_y \approx 0$ and $H_x = 0$ for the case of E^{x}_{pq} modes and $E_x \approx 0$ and $H_y = 0$ for the case of E^{y}_{pq} modes. [1],[4].

IV. Propagation Constants and Comparisons with E^{y}_{pq} and E^{x}_{pq} Modes

We have evaluated the results of the exact field analysis and compared them with the Marcatili's results [5],[8] and the results of Effective Index Method(EIM) [10]. It is likely that we can have four sorts of solutions ; β_{QTE}, β_{QTM}, β^{y}_{pq} and β^{x}_{pq} [1],[5]. But only two kinds of solutions can be accepted : the quasi TE Mode β_{QTE} and quasi TM Mode β_{QTM}. The other solutions β^{y}_{pq} and β^{x}_{pq} should be discarded because of above discussions where

$$\beta_{QTE} = \sqrt{k_1^2 - k_{xTE}^2 - k_{yTE}^2}$$

$$\beta_{QTM} = \sqrt{k_1^2 - k_{xTM}^2 - k_{yTM}^2}$$

$$\beta^{y}_{pq} = \sqrt{k_1^2 - k_{xTE}^2 - k_{yTM}^2}$$

$$\beta^{x}_{pq} = \sqrt{k_1^2 - k_{xTM}^2 - k_{yTE}^2}.$$

To facilitate the comparison, the normalizations β and V for the propagation constant and the frequency, respectively, are used.

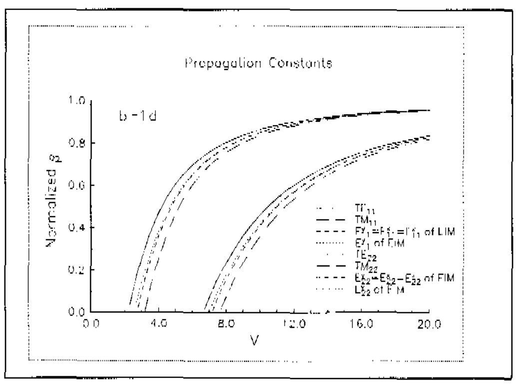

Fig 4.1 Propagation constant β as function of the normalized frequency V for dielectric

rectangular waveguide. [1],[3],[4]. $n_1^2=2.1$, $n_2=n_3=n_4=n_5=1.0$ and b=d. The normalized $\beta=\dfrac{\beta^2-k_2^2}{k_1^2-k_2^2}$ and $V=d\sqrt{k_1^2-k_2^2}$.

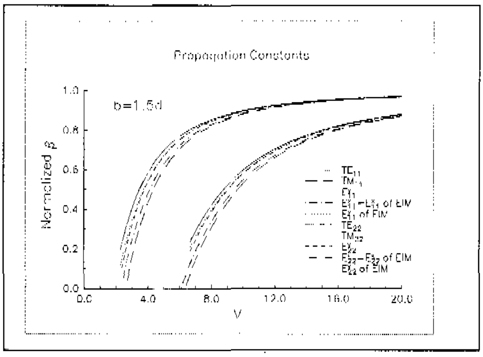

Fig 4.2 Propagation constant β as function of the normalized frequency V for dielectric rectangular waveguide. [1],[3],[4]. $n_1^2=2.1$, $n_2=n_3=n_4=n_5=1.0$ and b=1.5d. The normalized $\beta=\dfrac{\beta^2-k_2^2}{k_1^2-k_2^2}$ and $V=d\sqrt{k_1^2-k_2^2}$.

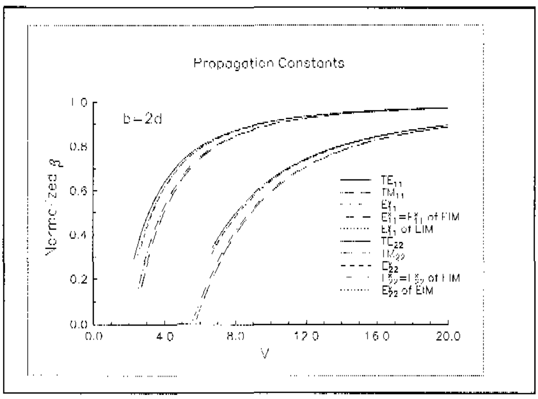

Fig. 4.3 Propagation constant β as function of the normalized frequency V for dielectric rectangular waveguide. [1],[3],[4]. $n_1^2=2.1$, $n_2=n_3=n_4=n_5=1.0$ and b=2d. The normalized $\beta=\dfrac{\beta^2-k_2^2}{k_1^2-k_2^2}$ and $V=d\sqrt{k_1^2-k_2^2}$.

Comparing with the propagation constants of E_{pq}^y and E_{pq}^x Modes which are $\beta_{pq}^y=\sqrt{k_1^2-k_{xTE}^2-k_{yTM}^2}$ and $\beta_{pq}^x=\sqrt{k_1^2-k_{xTM}^2-k_{yTE}^2}$ [1],[5] and EIM whose propagation constants are β_{pq}^y *of EIM* $=\sqrt{k_1^2-k_{xTE}^2-k_{yTMe}^2}$ where k_{yTMe} is obtained from $\tan k_y b=\dfrac{2k_y\varepsilon_{e1}k_0^2k_2^2\alpha_2}{k_y^2k_2^4-\varepsilon_{e1}^2k_0^4\alpha_2^2}$, $\varepsilon_{e1}=\varepsilon_1-\dfrac{k_{xTE}^2}{k_0^2}$, and $\beta_{pq}^x=\sqrt{k_1^2-k_{xTM}^2-k_{yTE}^2}$ the β_{pq}^x mode of EIM for $n_1^2=2.1$, $n_2=n_3=n_4=n_5=1.0$, slightly different results are observed in Fig 4.1, Fig 4.2 and Fig 4.3 for the dielectric rectangular waveguide. Therefore the EIM can be considered to be more accurate than the Marcatili's method in E_{pq}^y modes but no improvements are obtained in E_{pq}^x Modes due to the analytical method of EIM itself. That is the reason why the E_{pq}^x Modes did not appear in EIM. Increasing the aspect ratio while holding the other constants produces closer agreement between the exact solution and EIM. However, as E_{pq}^x of EIM coincides with E_{pq}^x of Marcatili's, it deviates slightly more from the exact solution. Note the normalized β_{pq}^y and β_{pq}^x approach that of TE and TM modes as the aspect ratio $\dfrac{b}{d}$ becomes larger and larger.

V. Conclusions

An exact analysis of dielectric rectangular waveguide structures is presented which is formulated in terms of the simple wave equation for quasi TE modes and quasi TM modes in each region. This leads to eigenvalue problems which are free from the troublesome problem of spurious modes [7],[8]. Our analysis does not hold near cutoff since the fields disconnect themselves from the core and reach strongly into the shaded regions of Fig. 3.1. The analyzing procedures of this manuscript are very complicated but the results are as simple as those of E_{pq}^y and E_{pq}^x Modes and EIM. Therefore the propagation constants of quasi TE and quasi TM modes can be used for comparison with other methods such as FDM or FEM in various structures and for dielectric waveguide fabrications.

We can apply these results to the well known directional couplers formed with two dielectric rectangular waveguides and the nonlinear distributed feedback laser array. The coupling coefficients between two adjacent dielectric waveguides can be derived very easily by introducing the quasi TE and quasi TM modes.

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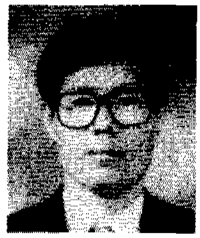

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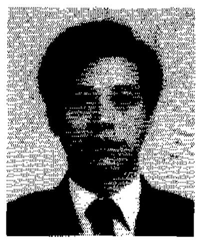

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