상관도가 있는 나카가미 채널에서 2D-RAKE 수신기의 성능 분석

정회원 문 철*, 강 창 훈*, 박 한 규*

Performance Analysis of 2D-RAKE Receiver over Correlated Nakagami Fading Channel

Cheol Mun*, Chang-Hoon Kang*, Han-Kyu Park* *Regular Members*

요 약

상관도가 있는 주파수 선택적 나카가미 페이딩 채널에서의 2D-RAKE 수신기의 평균 비트 에러율을 구하여 성능을 분석하였다. 동일한 RAKE 핑거의 배열 안테나에 수신되는 신호들은 동일한 페이딩 파라메터를 가지지만 서로 다른 평균 신호 대 잡음비를 갖는 것으로 가정하였다. 또한 서로 다른 RAKE 핑거에 수신되는 신호들은 서로 독립적이지만 서로 다른 평균 신호 대 잡음비를 가지고 서로 다른 페이딩 파라메터를 갖는다고 가정하였다. 위의 분석을 통하여 결합되는 다이버시티 브랜치 간의 상관 특성, 지연 확산 특성, 평균 신호 대 잡음비 분포 그리고 페이딩 파라메터들이 2D-RAKE 수신기의 성능에 밀접한 영향을 줌을 확인하였다.

ABSTRACT

The average bit error rate (BER) performance of 2D-RAKE receiver, operating in a correlated Nakagami fading channel, is analyzed. The analysis assumes correlated fading between the array elements with identical fading parameters but with unbalanced average signal-to-noise ratio (SNR). And independent but non-identically distributed frequency-selective fading channel with different fading parameters is assumed. The analyses show that fading correlation, delay profile, average SNR distribution, and fading parameters of combined branches affect the overall performance of 2D-RAKE receiver.

I. Introduction

Among the applications of adaptive array antenna to CDMA systems, 2D-RAKE receiver that exploits space and time domain structure of the received signal is known as a very suitable structure for CDMA[1]. A number of works have analyzed the BER performance of 2D-RAKE receiver, which are always limited to the case of correlated Rayleigh fading[1]. In the case of Nakagami fading channel, usually the analyses are limited to maximal ratio combining (MRC) with correlated branches[2]. Recently, we have analyzed a BER performance of 2D-RAKE receiver in an arbitrarily correlated Nakagami channel by deriving an approximated SNR probability density function (PDF) of 2D-RAKE receiver[3].

In this paper, we present an exact performance analysis of 2D-RAKE receiver, which corrects some approximate results of [3]. Furthermore, the analysis takes into account general cases such as

* 연세대학교 전기·컴퓨터공학과 전파통신연구실(mc97@yonsei.ac.kr)
논문번호 : 99489-1211, 접수일자 : 1999년 12월 11일

unbalanced average SNR distribution of combined diversity branches and non-identically distributed frequency-selective channel with different fading parameters.

II. Performance analysis

Consider a 2D-RAKE receiver composed of L-RAKE branches with K-elements antenna array for each RAKE branch. When 2D-RAKE receiver employs MRC in space and time domain, the instantaneous SNR γ_s at the output of 2D-RAKE receiver is given by

$$\gamma_s = \sum_{l=1}^{L} \gamma_l = \sum_{l=1}^{L}\sum_{k=1}^{K} \gamma_{l,k} \tag{1}$$

where $\gamma_{l,k}$ represents the instantaneous received SNR on the kth array element of the lth RAKE branch. In Nakagami channel, $\gamma_{l,k}$ is Gamma distributed random variables with the PDF of

$$p_{\gamma_{l,k}} = \left(\frac{m_{l,k}}{\overline{\gamma}_{l,k}}\right)\frac{\gamma^{m_{l,k}-1}}{\Gamma(m_{l,k})}\exp\left(-\frac{m_{l,k}}{\overline{\gamma}_{l,k}}\gamma\right) \tag{2}$$

where $m_{l,k}$ and $\overline{\gamma}_{l,k}$ are the fading parameter and average received SNR, respectively. And $\Gamma()$ is the standard Gamma function. Let us assume correlated fading between array elements with identical fading parameters but with unbalanced average SNR, namely, $m_{l,1} = \ldots = m_{l,K} = m_l$ and $\overline{\gamma}_{l,1} \neq \overline{\gamma}_{l,2} \neq \ldots \neq \overline{\gamma}_{l,K}$ for any fixed index l.

From the characteristic function of K-variate Gamma distribution whose marginal PDFs are given by (2), the characteristic function of γ_l, which is the sum of correlated Gamma distributed random variable $\gamma_{l,k}$, can be obtained as follows

$$\begin{aligned}\Phi_l(t) &= \left| \mathbf{I} - j\Theta_l \mathbf{R}_l t\right|^{-m_l} \\ &= \prod_{k=1}^{K}(1 - j\lambda_{l,k}t)^{-m_l}\end{aligned} \tag{3}$$

where $\mathbf{I}$, $\mathbf{R}_l$, and Θ_l are $K \times K$ identity matrix, correlation matrix of lth path, and diagonal matrix defined as $\Theta_l = \mathrm{diag}\{\overline{\gamma}_{l,1}/m_l, \ldots, \overline{\gamma}_{l,K}/m_l\}$, respectively. And $\lambda_{l,k}$ are eigenvalues of $\Phi_l \mathbf{R}_l$.

Let us assume independent but non-identically distributed frequency-selective channel with different fading parameters, that is to say, $m_1 \neq m_2 \neq \ldots \neq m_l$ and $\overline{\gamma}_1 \neq \overline{\gamma}_2 \neq \ldots \neq \overline{\gamma}_L$. And non-identical correlation matrices along the RAKE branches are assumed. Then the characteristic function of γ_s is given by

$$\begin{aligned}\Phi_s(t) &= \prod_{l=1}^{L}\prod_{k=1}^{K}(1 - j\lambda_{l,k}t)^{-m_l} \\ &= \prod_{d=1}^{LK}(1 - j\widehat{\lambda}_d t)^{-\widehat{m}_d}\end{aligned} \tag{4}$$

where $\{\lambda_{l,k}\}_{l=1\ldots L,\, k=1\ldots K} = \{\widehat{\lambda}_d\}_{d=(l-1)K+k}$ and $\{\widehat{m}_d\}_{d=(l-1)K+1\ldots lK} = m_l$.

Based on the approach of the work[4], the expression for the PDF of γ_s can be derived by inverse Laplace transform of (4) as follows

$$p_{\gamma_s}(\gamma) = \int_0^{\infty} \frac{\cos\left[\sum_{d=1}^{LK}\widehat{m}_d \tan^{-1}(\widehat{m}_d t) - t\gamma\right]}{\pi\prod_{d=1}^{LK}\left(1+(\widehat{\lambda}_d t)^2\right)^{0.5\widehat{m}_d}}\,dt \tag{5}$$

The average BER is obtained by averaging the conditional bit error probability over the PDF of γ_s given in (5). For coherent and non-coherent demodulation, we have

$$\begin{aligned}\overline{P}_e = &\frac{1}{2\pi}\int_0^{\infty}\prod_{d=1}^{LK}\left(1+(\widehat{\lambda}_d t)^2\right)^{-\widehat{m}_d/2} \cdot \\ &\left\{\cos\left[\sum_{d=1}^{LK}\widehat{m}_d \tan^{-1}(\widehat{\lambda}_d t)\right]\frac{a^b \sin(b\tan^{-1}(t/a))}{(t^2+a^2)^{b/2}}\right. \\ &+ \sin\left[\sum_{d=1}^{LK}\widehat{m}_d \tan^{-1}(\widehat{\lambda}_d t)\right] \\ &\left.\left[1 - \frac{a^b \sin(b\tan^{-1}(t/a))}{(t^2+a^2)^{b/2}}\right]\right\}\frac{dt}{t}\end{aligned} \tag{6}$$

where a=0.5 for frequency shift-keying (FSK), a=1 for phase shift-keying (PSK), b=0.5 for coherent detection, and b=1 for non-coherent detection. The analyses are focused on the case of coherent PSK case (a=1, b=0.5), but the analyses for other modulation type and detection mechanism can be obtained by proper choice of coefficients a and b.

III. Performance analysis under various conditions

Consider exponential and constant correlation models, defined as $R_l(i,j)=\rho_l^{|i-j|}$ and $R_l(i,j)=1\,(i=j),\ \rho_l$ (otherwise), respectively. $R_l(i,j)$ denotes the element of ith row and jth column of $\mathbf{R}_l$. The distribution of average SNR along the RAKE branches is assumed to be represented by the exponential power delay spread (PDS) model defined as $\bar{\gamma}_{l,0}=\bar{\gamma}_0 e^{-(l-1)\delta}$ where δ and $\bar{\gamma}_{l,0}$ are decaying constant and the average of $\{\bar{\gamma}_{l,i}\}_{i=1\cdots K}$ for fixed index l, respectively. And $\bar{\gamma}_0$ is an average SNR of one-dimensional RAKE receiver, namely, $\sum_{l=1}^{L}\bar{\gamma}_{l,0}$.

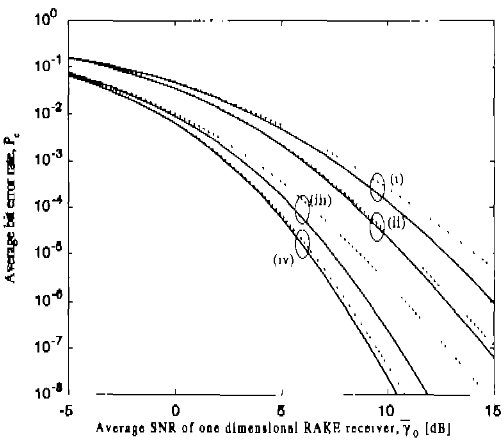

Fig. 1 Comparison of exact and approximate average BER for exponential correlation model
——— : Exact, ········ : Approximate (Ref. [3])
(i) K=2, $\rho_l=0.3(l=1,2,3)$, (ii) K=2, $\rho_l=0.7(l=1,2,3)$
(iii) K=4, $\rho_l=0.3(l=1,2,3)$, (iv) K=4, $\rho_l=0.7(l=1,2,3)$

Fig.1 compares average BER of our exact and approximate as a function of $\bar{\gamma}_0$ for $\{\rho_l\}_{l=1,2,3}=$ 0.3, 0.7 and K=2, 4 when L=3, $\delta=0$, and $\{m_l\}_{l=1,2,3}=1$. Exponential correlation model and balanced average SNR $\{\bar{\gamma}_{l,i}\}_{i=1\cdots K}=\bar{\gamma}_{l,0}$ are assumed. While the approximation is very good in case of relatively weak correlation, overestimates of the SNR for a given BER increase with the SNR and with the level of correlation but still quite small. For $\bar{P}_e=10^{-4}$, the approximation gives an SNR overestimate of about 1 dB.

Fig.2 shows the effect of unbalanced SNR among the combined antenna elements and non-identical fading parameters along the RAKE branches for K=2, 4 with exponential correlation model when L=3, $\{\rho_l\}_{l=1,2,3}=0.5$, and $\delta=1$. Comparisons show that unbalanced SNR degrades the BER due to the loss in diversity gain and the effect of non-identical fading severity on the BER is very large. This result indicates that the effect of unbalanced SNR and non-identical fading severity should be taken into account for accurate prediction of the performance of 2D-RAKE receiver.

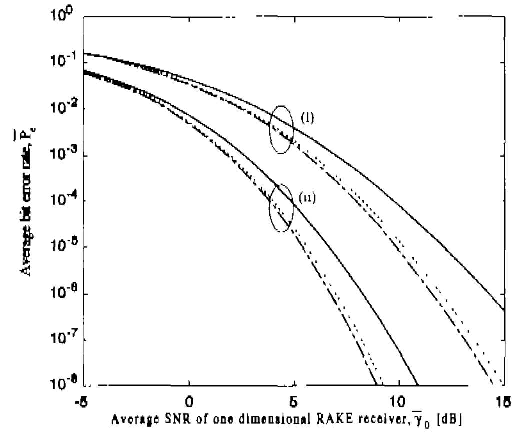

Fig. 2 Average BER for non-identical fading parameters along the RAKE branches and unbalanced SNR
(i) K=2, (ii) K=4
——— : $\{m_1,m_2,m_3\}=\{1,1,1\}$, $\{\bar{\gamma}_{l,i}\}_{i=1,\cdots K}=\bar{\gamma}_{l,0}$
············ : $\{m_1,m_2,m_3\}=\{2,1.5,1\}$, $\{\bar{\gamma}_{l,i}\}_{i=1,\cdots K}=\bar{\gamma}_{l,0}$
—·—·—· : $\{m_1,m_2,m_3\}=\{2,1.5,1\}$, $\{\bar{\gamma}_{l,i}\}_{i=odd}=0.5\,\bar{\gamma}_{l,0}$, $\{\bar{\gamma}_{l,i}\}_{i=even}=1.5\,\bar{\gamma}_{l,0}$

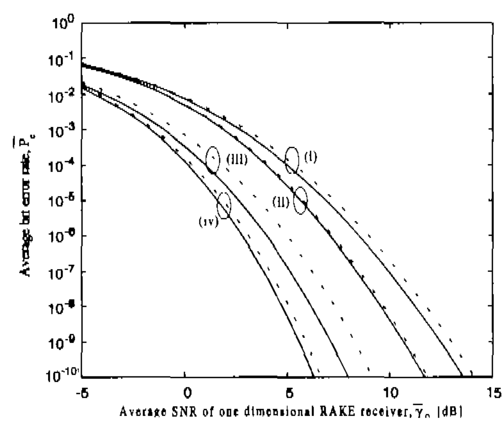

Fig. 3 Average BER for exponential and constant correlation model and various values of correlation coefficients
——— : Exponential correlation
············ : Constant correlation
(i) K=4, $\{\rho_1,\rho_2,\rho_3\}=\{0.5,0.3,0.1\}$
(ii) K=4, $\{\rho_1,\rho_2,\rho_3\}=\{0.9,0.7,0.5\}$
(iii) K=8, $\{\rho_1,\rho_2,\rho_3\}=\{0.5,0.3,0.1\}$
(iv) K=8, $\{\rho_1,\rho_2,\rho_3\}=\{0.9,0.7,0.5\}$

Fig.3 shows the effect of correlation model for the cases of $\{\rho_1, \rho_2, \rho_3\}=\{0.5,\ 0.3,\ 0.1\}$, $\{0.9,\ 0.7,\ 0.5\}$ and $K=4,8$ when $L=3$, $\{m_l\}_{l=1,2,3}=1$ and $\delta=0$. Comparisons show that the difference between the BERs for different correlation model becomes larger as the correlation coefficient and diversity order increase. And for a given correlation model, larger correlation coefficient makes larger variation among eigenvalues of correlation matrix, which leads to lower diversity gain.

Fig.4 shows the effect of decaying constant δ on BER performance for the cases of $\delta=0$, 0.5, 1 and $K=1$, 2, 4 when $L=3$, $\{m_l\}_{l=1,2,3}$, and $\{\rho_1, \rho_2, \rho_3\}=\{0.9,\ 0.7,\ 0.5\}$ for exponential correlation model. $\delta=0$ is equivalent to constant PDS model, which has superior performance of path diversity to other $\delta=0.5$ and $\delta=1$ cases. δ reflects frequency selectivity of the channel and therefore the smaller δ is, the larger gain of path diversity is obtained. The effects of fading parameter, average SNR distribution, and fading correlation of combined diversity branches on the BER performance indicate that the performance of 2D-RAKE receiver highly depends on its deploying environment.

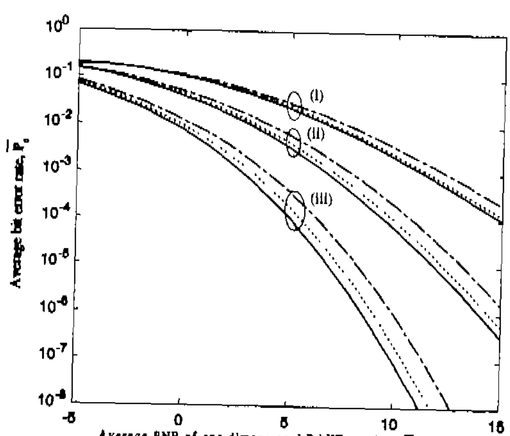

Fig.4 Average BER for various values of decaying constant δ

———— : $\delta=0$

············ : $\delta=0.5$

–·–·–·– : $\delta=1$

(i) K=1, (ii) K=2, (iii) K=4

V. Conclusions

In this paper, the average BER performance of 2D-RAKE receiver, operating in a correlated Nakagami fading channel, is analyzed. The analysis assumes correlated fading between the array elements with identical fading parameters but with unbalanced average signal-to-noise ratio. And independent but non-identically distributed frequency-selective fading channel with different fading parameters is assumed. The analyses show that fading correlation profile, average SNR distribution, and fading parameters of combined branches affect the overall performance of 2D-RAKE receiver and therefore have to be taken into account for the accurate evaluation of the performance of 2D-RAKE receiver.

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문 철(Cheol Mun) 정회원

현재 : 연세대학교 전기 · 컴퓨터공학과 박사과정

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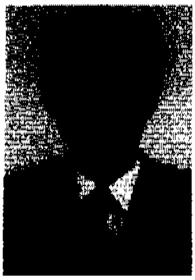
강 창 훈(Chang-hoon Kang) 정회원



1984년 2월 : 연세대학교 전자공학과 졸업

1986년 2월 : 연세대학교 전자공학과 석사

1995년 8월~현재 : 연세대학교 전기·컴퓨터공학과 박사과정

<주관심 분야> 이동통신, 안테나, 다이버시티

박 한 규(Han-kyu Park) 정회원

현재 : 연세대학교 전기·컴퓨터공학과 교수

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