

### 무선 통신에 사용되는 AD 변환기의 샘플-앤드-홀드 회로의 보상

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# Compensation of the Sample-and-Hold Circuit in an AD Converter Used in Radio Telecommunications

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요 익

이 논문에서는 AD 변환기의 앞에 설치되는 샘플·앤드·홀드 회로의 비선형성을 보상하기 위해 신경 회로망 기법과 볼테라 급수 모델을 직접적으로 적용하는 기법을 제안한다. 제안하는 기법들의 성능을 비교하기 위해, 볼테라급수 모델에 기반을 둔 전통적인 p 차 역산 방식의 결과와 비교 검토한다. 비교 검토를 위해서는 모노·톤과 투·톤 신호를 사용하여 출력의 고조과 및 혼변조 레벨을 살펴 보았다. p 차 역산 방식이 역(逆) 시스템을 구하는 것이라면 제안하는 기법들은 최적화 기법에 바탕을 두고 있다고 할 수 있다. 결과를 보면 어떤 한 방식이 다른 방식보다성능이 월등하다고 할 수 없는데, 그 이유는 각 방식마다 나름대로의 장단점을 갖고 있기 때문이다. 보상 방식의 선택은 신호의 통계적 성질, 신호 레벨, 비선형성의 정도 등을 고려해야 한다.

#### **ABSTRACT**

In this paper, we propose a neural network approach and a direct Volterra series model approach to the compensation for the nonlinearity of a sample-and-hold circuit placed forward of A/D converters. We compare the performance of the proposed approaches with that of the conventional pth-order inverse method based on the Volterra series model. The proposed approaches use optimization techniques whereas the pth-order inverse method is basically a system inversion technique. The results show that any one approach can not be said to perform better than the others since each approach has its own pros and cons. One should choose a method based on the signal statistics, levels, and the degree of the nonlinearity.

#### I. Introduction

The trend in telecommunications is clearly from analog to digital. Thus, it is inevitable that analog-to-digital converters (ADC) be used in radio receivers. However, the nonlinear distortion generated by the ADC greatly reduces the dynamic range of the receiver. It is well known that placing a sample-and-hold circuit forward of the ADC significantly

improves the ADC performance since the distortion is strongly dependent on the analog input signal variation during the conversion. Despite the great improvement in overall ADC performance, there is still room for research in the compensation of nonlinearities introduced by the sample-and-hold circuits.

To eliminate or compensate the nonlinear effects in ADC, many researchers have developed methods based on a look-up table. In this method, the distorted

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output code of an ADC is mapped to the correct code through the look-up table. Rebold and Irons used a two-dimensional phase-plane approach for the nonlinear compensation of ADC at the output [1]. Later, Irons et al improved this method by using three frequencies for calibration [2]. Dent and Cowan developed a technique called threshold tracking to set up a look-up table [3]. Hummels et al utilized a method using sub-samples to design an error correction table at the ADC output [4].

Usually, the output of the sample-and-hold circuit is dependent upon the previous sample values, meaning the sample-and-hold circuit possesses system memory. Nonlinear systems with memory can be modeled with a Volterra series model [5]. Tsimbinos and Lever modeled the sample-and-hold circuit with a Volterra series model and applied the *p*th-order inverse method to compensate the nonlinearities [6].

In this paper, we use three approaches to the compensation of a nonlinear sample-and-hold circuit used in radio communications. The three approaches are the neural network approach, the direct Volterra series model approach, and the *pth*-order inverse method based on the Volterra series model. The first two approaches are newly attempted in this work. We will compare the three approaches using the monotone and two-tone tests.

This paper consists of the following steps. In section II, we will describe the sample-and-hold circuit model used in this study. In section III, the Volterra series model of the sample-and-hold circuit is presented along with a brief discussion of the conventional pth-order inverse method. In section IV, we present the neural network approach to the sample-and-hold circuit compensation. In section V, the direct Volterra series model approach is presented. The results of the three methods are compared and discussed in section VI. In section VII, we will conclude our work.

#### II. Sample-and-Hold Circuit Model

The nonlinearity in a sample-and-hold circuit is caused by input-indepedent and input-dependent timing jitters [6]. Tsimbinos and Lever obtained a

mathematical model for the sample-and-hold circuit output  $f(\cdot)$  as follows [6]:

$$f(x(t)) \approx x(t+\tau(t)+q(x(t)), \tag{1}$$

where x(t) is the input to the sample-and-hold circuit,  $\tau(t)$  is the input-independent timing jitter and q(x(t)) is the input-dependent timing jitter. The input-independent timing jitter is due to the sampling interval variations, whereas the input-dependent timing jitter occurs due to the imperfect operation of switching diodes [7].

If  $\tau(t)$  and q(x(t)) are very small compared with t (in the discrete time case, the sampling interval T), we may use a Taylor expansion of Eq. (1). Taking the first two terms of the Taylor expansion, we have

$$f(x(t)) \approx x(t) + x'(t) \tau(t) + x'(t)q(x(t)),$$
 (2)

Since  $q(x(t)) \approx c(1 - |x(t)|)$  [7], and as we are only interested in the distortion term, Eq. (2) can be simplified to

$$f(x(t)) \approx x(t) + cx'(t)(1 - |x(t)|)$$
 (3)

where c is a constant determined by circuit parameters. We can approximate x'(t) using sampled values as in

$$x'(t) \approx \frac{x[n] - x[n-1]}{T} \tag{4}$$

where T is the sampling interval. Therefore, in discrete time form, Eq. (3) can be approximated by

$$f(x[n]) \approx x[n] + \frac{c}{T}(x[n] - x[n-1])(1 - |x[n]|)$$
 (5)

From Eq. (5), we can see that the output of the ADC comprises a linear term and nonlinear terms.

### III. The Volterra Series Model and the pth-order Inverse Method

To apply the pth-order inverse method, we must have a Volterra model of the system. Accordingly, Eq. (5) should be converted to a Volterra series form which, in the discrete form, is expressed by

$$y[n] = \sum_{k=0}^{N_1} f_k^{(1)} x[n-k] +$$

$$+ \sum_{k_1=0}^{N_2} \sum_{k_2=0}^{N_2} f_{k_1, k_2}^{(2)} x[n-k_1] x[n-k_2]$$

$$+ \sum_{k_1=0}^{N_1} \sum_{k_2=0}^{N_2} \sum_{k_3=0}^{N_2} f_{k_1, k_2, k_3}^{(3)} x[n-k_1]$$

$$\times x[n-k_2] x[n-k_3] + \cdots$$
(6)

where  $N_1$ ,  $N_2$ ,  $N_3$ , ... are the memory duration of the first order, the second order, the third order terms, and so on.  $f_k^{(1)}$ ,  $f_{k_1, k_2, k_3}^{(2)}$  and  $f_{k_1, k_2, k_3}^{(3)}$ , are the first, second, and third order Volterra kernels, respectively. Tsimbinos and Lever used two methods to calculate the Volterra kernels of the sample-and-hold circuit of Eq. (5); the Lee-Schetzen method [5] and an adaptive method [8]. With the adaptive method, they obtained the Volterra kernels up to the fifth order as shown in Table 1 [6]. The even order kernels are zero as we can see from Eq. (3) that the nonlinearity is basically of type x|x|. Even if they were not zero, they would not generate in-band interference signals.

Once the Volterra model is obtained, we can apply the pth-order inverse method to get the inverse Volterra model. The pth-order inverse method connects another Volterra system in tandem to the nonlinear system to be compensated such that the overall system kernels are zero up to the pth-order except the first order kernels which should be a unit impulse [5].

Table 1. Volterra kernels obtained from the mathematical model with an adaptive method, from [6].

Linear Part	3rd-order part	5th-order part		
$f_0^{\text{(I)}}=1.007628$ $f_1^{\text{(I)}}=-0.013607$	$f_{000}^{(3)} = -0.008109$ $f_{100}^{(3)} = 0.003858$ $f_{000}^{(3)} = 0.003858$ $f_{000}^{(3)} = 0.003858$	$f_{00000}^{(5)} = 0.000313$ $f_{10000}^{(5)} = -0.000133$ $f_{00000}^{(5)} = -0.000133$ $f_{00100}^{(5)} = -0.000133$ $f_{00010}^{(5)} = -0.000133$		
		$f_{00001}^{(5)}$ = -0.000133		

In Fig. 1, we show the concept of the *pth*-order inverse method. **F** is the Volterra model to be compensated, the kernels of which are known, and  $G_{(p)}$ 

is the Volterra system used for nonlinear compensation the kernels of which are to be determined. From the condition that the overall system must be linear up to the pth order, we can calculate the  $G_{(p)}$  system kernels in terms of the F system kernels [5]. In Fig. 2, the block diagrams to obtain those unknown kernels are shown up to the third order. We can see that to obtain the kernels of the inverse system involves complex, successive multi-dimensional convolutions. Using the pth-order inverse, Tsimbinos and Lever obtained more than a 20 dB suppression of the third harmonic using a mono-tone test input signal [6].

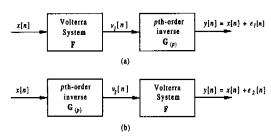


Fig. 1 A block diagram showing the idea of pth-order inverse method. The pth-order inverse system G<sub>(p)</sub> is connected in tandem to the Volterra series model F.

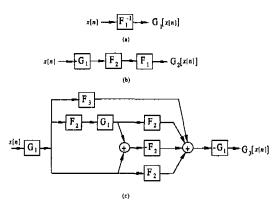


Fig. 2 Block diagrams of the inverse system. (a) first-order system (b) second-order system (c) third-order system.

#### IV. Neural Network Approach

In this section, we propose a new alternative method for nonlinear ADC compensation using a neural network. To use the neural network method, there is no need to convert the existing model to a Volterra model. The key idea is just to determine the mapping from the distorted output data set to the undistorted input data set. Since the sample-and-hold circuit has system memory, we used a modified time-delayed neural network (TDNN) shown in Fig. 3. This network can be shown to approximate the Volterra series model if we have enough hidden units [9].

In Fig 4, we illustrate the idea of ADC compensation using a neural network in a block diagram form. The input to the neural network y[n] is the output of the system to be compensated and the desired output is the input of the system x[n]. As the error x[n] - o[n] goes to zero, the output of the network approaches the input, which is the desired output of the overall system.

Simple as it is, the neural network method also has limitations. The neural network does not provide any physical insight into the system. For example, the neural network compensator can not be decomposed into a linear, a cubic systems, and so forth. Moreover, since the training algorithm is based on the stochastic gradient method, the convergence speed is slow and there is no guarantee of converging to the optimal solution. On the other hand, the neural network method, in principle, can approximate any function to an arbitrary accuracy [10, 11].

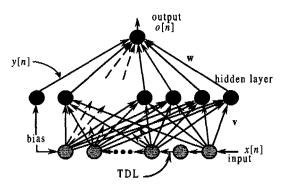


Fig. 3 A neural network diagram with delay elements used for nonlinear system compensation.

## V. Direct Volterra Series Model Approach

If the nonlinearity of the system is not of a very high order, we may use a Volterra series model as a compensator. In Fig. 5, we show a block diagram to compensate a nonlinearity using a Volterra series model. For training the Volterra series model equalizer, we use the RLS algorithm[12] since the Volterra series model equation is linear with respect to its kernel coefficients. In Fig. 5,  $\alpha$  [n] is called the innovation since it contains the information which can not be predicted by the previous kernel coefficients.

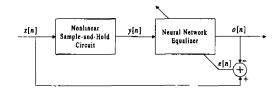


Fig. 4 A block diagram showing the training of a neural network compensator.

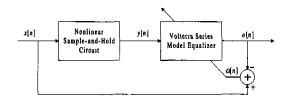


Fig. 5 A block diagram showing the training of a Volterra series model compensator.

The direct Volterra series model approach has some advantages over neural network appraoch and the not be predicted by the previous kernel coefficients. pth-order inverse method. Since the direct Volterra series model approach is based on an optimization technique, it does not generate higher-order nonlinearities as is the case for the pth-order inverse method. Since the direct Volterra series model approach requires fewer coefficients than the neural network approach, the required data set is smaller. Moreover, training is faster with the RLS algorithm than with the error backward propagation algorithm.

However, if the order of the major nonlinearity is high (i.e.,  $\geq 5$ ) and the memory length is long (i.e.,  $\geq 5$ ), the number of kernel coefficients becomes very large. In such cases, the neural network approach will be more suitable to apply. Since the number of weights of neural networks can be increased linearly instead of exponentially as opposed to the Volterra series model approach, the redundancy of weights

becomes less significant when compensating higherorder nonlinearities. In contrast, the number of Volterra kernels grows tremendously when applied to higher-order (say,  $\geq 5$ ) nonlinear compensation.

#### VI. Results

We used Eq. (5) to model a sample-and-hold circuit with c = 0.02 as in [6]. The input to this model was a random signal uniformly distributed between -3.0 and 3.0. The output of this model was used as the input to the compensators and the input of the model was used as the desired output for training. The dimension of the neural network was  $3 \times 5 \times 5 \times 1$  (three input units, five first hidden-layer units, five second hidden-layer units, one output unit). The memory of the third-order direct Volterra series model compensator was set to one. Therefore, the total kernel coefficients were 6. This compares with the 45 weights of the neural network compensator.

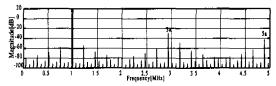


Fig. 6 Spectral distribution of the uncompensated output of the sample- and-hold circuit corresponding to the mono-tone input. The mono-tone input frequency is about 0.98 MHz. The "a" represent the fundamental frequency.

We tested the trained compensators with a mono-tone signal. We assumed a sampling frequency of 10 MHz. The test mono-tone signal was about 0.98 MHz. A mono-tone was used to facilitate comparison with the mono-tone test results of Tsimbinos and Lever using the *pth*-order inverse.

In Fig. 6, we show the spectral distribution of the output signal of the sample-and-hold circuit model fed an input containing a mono-tone of frequency approximately 0.98 MHz (marked "a"). We observe a third-order harmonic (3a) and a fifth-order harmonic (5a) with levels of 40.87 dB and 53.45 dB below the reference tone. The other frequency components are the higher-order harmonics aliased back into the lower frequency band.

In Fig. 7, we show the spectral distributions of the compensated mono-tone signal output. In Fig. 7(a), we show the neural network result. Placing the neural network compensator right after the sample-and-hold circuit, the amplitudes of the third- and fifth-order harmonics (3a and 5a, respectively) are reduced to -58.65 dB and -62.84 dB. With the direct Volterra series model compensator, as shown in Fig. 7(b), the third-order harmonic (3a) is reduced almost to -90 dB. However, the fifth-order harmonic is not suppressed since we used a third-order Volterra series model. Fig. 7(c) shows the result of the pth-order inverse method. Harmonic suppression to -71.87 dB and -63.31 dB for the third-order and the fifth-order harmonics, respectively, were obtained. Even with the pth-order inverse up to fifth order, the third- and fifth-order harmonics were not completely removed. This may be due to the fact that, to apply the pth-order inverse method, we must convert the existing model to a Volterra series model. Modeling error may cause imperfect compensation.

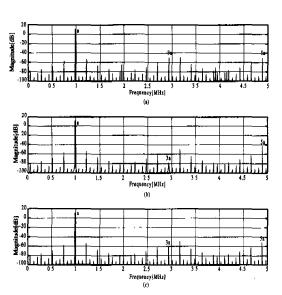


Fig. 7 Spectral distributions of the compensated monotone sample-and-hold circuit output signal. (a) the neural network approach (b) the direct Volterra series model approach (c) the pth-order inverse method. the "a" represents the fundamental frequency.

Third-order harmonic suppression using the neural network approach is worse than that of the *p*th-order inverse method. A possible explanation may be

destructive interference between the third harmonic and a higher order harmonic aliased back at the third harmonic frequency in the *p*th-order inverse case. To investigate further, we used a two-tone signal test and a random input test.

In Fig. 8, we show the spectral distribution of the distorted two-tone signal. We used 0.98 MHz and 1.47 MHz tones. We marked the harmonics and their inter-modulation frequencies. The frequency components of 0.98 MHz and 1.47 MHz are marked as "a" and "b", respectively.

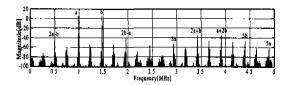


Fig. 8 Spectral distribution of the uncompensated output of the sample -and-hold circuit corresponding to the two-tone input. The two-tone input frequencies are about 0.98 MHz and 1.47 MHz. 0.98 MHz is marked as "a" and 1.47 MHz as "b".

We tested the neural network, the Volterra series model, and the pth-order inverse compensators using the two-tone signal. In Fig. 9, we show the spectral distributions of the compensated outputs. The neural network appears to have a better performance than the pth-order inverse method in this test. In Table 2, we compared the suppression of inter-modulation products using the three approaches. The intermodulation components 2b-a, 2a+b, and a+2b are lower for the neural network approach than for the other approaches. The direct Volterra series model approach has larger inter-modulation components. However, considering the number of kernel coefficients and the design simplicity of the direct Volterra series model approach this should not be a problem.

For the random input test, we used a uniformly distributed input signal with amplitudes between -3.0 and 3.0. We input the random signal to the sample-and-hold circuit model represented by Eq. (5). The output is then fed to the equalizers obtained using the three approaches. We measured the normalized mean-squared-error (NMSE) of the equalizer outputs

which is defined by

$$NMSE = \frac{\sum_{k=1}^{K} |x[k] - o[k]|^2}{\sum_{k=1}^{K} |x[k]|^2}$$
 (7)

where x[k] is the input to the sample-and-hold circuit

Table 2. Comparison of suppression (in dB) of the inter-modulation products using various approaches.

	ъ	2b-a	2a+b	2b+a
Neural network	0	-69.4	-70.3	-65.1
Volterra model	0	-64.1	-57.4	-57.9
pth-order inverse	0	-66.4	-63.6	-60.6

which is the desired output of the equalizer and o[k] is the equalizer output. K is the total number of test data.

The normalized mean-squared-errors of the test results were  $5.0 \times 10^{-6}$ ,  $2.0 \times 10^{-5}$ , and  $9.0 \times 10^{-6}$  for the neural net approach, for the direct Volterra series model approach, and for the *p*th-order inverse method,

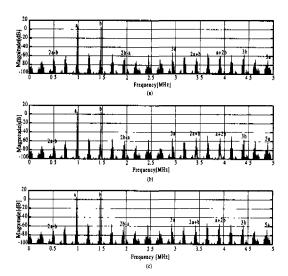


Fig. 9 Spectral distribution of the compensated two-tone sample-and-hold circuit output signals. (a) the neural network approach (b) the direct Volterra model series approach (c) the pth-order inverse method. 0.98 MHz is marked as "a" and 1.47 MHz as "b".

respectively. These results show that we can not say that any one approach is better than the others. Depending on the signal statistics and levels, the performance of each approach may vary.

For low-order nonlinearities, the direct Volterra series model is very simple and easy to implement. Although the neural network performs better, it requires many weights since they are redundant. These two approaches are based on optimization techniques whereas the *p*th-order inverse method is based on a system inversion technique. The design procedure of the *p*th-order inverse method is very complex including a linear system inversion and successive multi-dimensional convolutions.

#### VII. Conclusion

We described different methods of nonlinear compensation using a neural network or a direct Volterra series model to implement the mapping from output data to input data. Compared to the *p*th-order inverse method, the neural network and the direct Volterra series model compensators are extremely simple to implement. Furthermore, the harmonic suppression is quite substantial, comparable to that of the *p*th-order inverse approach. We achieve nearly the same performance as in the *p*th-order inverse model despite the much simpler design procedure.

The direct Volterra series model approach requires very small number of filter coefficients when the order of the nonlinearity is low and the memory span of the system is short. However, to compensate for the higher-order nonlinearity, the direct Volterra series model approach as well as the *pth*-order inverse method requires tremendous number of kernel coefficients. When the order of nonlinearity is high, the neural network method is highly preferable.

#### References

[1] T. A. Rebold, F. H. Irons, "A Phase Plane Approach to the Compensation of High-Speed Analog-to-Digital Converters," *Proceeding - IEEE International Symposium on Circuits and Systems*, vol. 2, pp. 455-458, 1987.

- [2] F. H. Irons, D. M. Hummels, S. P. Kennedy, "Improved Compensation for Analog-to-Digital Converters," *IEEE Transactions on Circuits and Systems*, vol. 38, no. 8, pp. 958-961, August 1991.
- [3] A. C. Dent and Colin F. N. Cowan, "Linearization of Analog-to-Digital Converters," *IEEE Trans*actions on Circuits and Systems, vol. 37, no. 6, pp. 729-737, June 1990.
- [4] D. M. Hummels, F. H. Irons, S. P. Kennedy, "Using Adjacent Sampling for Error Correcting Analog-to-Digital Converters," Proceeding -IEEE International Symposium on Circuits and Systems, vol. 2, pp. 589-592, San Diego, CA, 1992.
- [5] Martin Schetzen, The Volterra and Wiener Theories of Nonlinear Systems, John Wiley & Sons, Inc., 1980.
- [6] John Tsimbinos, Kenneth V. Lever, "Application of Higher-Order Statistics to Modelling, Identification and Cancellation of Nonlinear Distortion in High Speed Samplers and Analogue-to-Digital Converters Using the Volterra and Wiener Models," Proceeding - IEEE Signal Processing Workshop on High Order Statistics, pp. 379-383, South Lake Tahoe, CA, June 1993.
- [7] J. R. Gray and S. C. Kitsopoulos, "A Precision Sampling and Hold Circuit with Subnanosecond Switching," *IEEE Transactions on Circuit Theory*, pp. 389-396, September 1964.
- [8] V. John Mathews, "Adaptive Polynomial Filters," IEEE Signal Processing Magazine, pp. 10-26, July 1991.
- [9] Hassan M. Ahmed, Fawad Rauf, "NADINE A Feed-forward Neural Network for Arbitary Nonlinear Time Series," Proceeding of international Joint Conference on Neural Networks, pp. II 721-726, Seattle, WA, USA, July 1991.
- [10] G. Cybenko, "Approximation by Superpositions of a Sigmoidal Function," Mathematics of Control, Signals, and Systems, 2:303-314, 1989.
- [11] Kurt Hornik, "Multilayer Feedforward Networks are Universal Approximators," *Neural Networks*, vol. 2, pp.359-366, 1989.

[12] Simon Haykin, Adaptive Filter Theory, 2nd Ed., Prentice Hall, Inc., 1991.

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