Runlength Limited Codes based on Convolutional Codes

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ABSTRACT

We present a modification method for runlength limited codes based on convolutional codes. This method is based on cosets of convolutional codes and can be applied to any convolutional code without degradation of error control performance of the codes. The upper bound of maximum zero and/or one runlength are provided. Some convolutional codes which have the shortest maximum runlength for given coding parameters are tabulated.

I. INTRODUCTION

In many communication systems and digital data storing systems, it is desirable to use codes whose coded sequences have limited zero and/or one runlength to facilitate self synchronization. Moreover, error correcting codes are also essential to improve reliability of communication against channel noise. In this paper, a method of constructing combined zero and/or one runlength limited error control codes using cosets of convolutional codes is presented.

The idea using cosets for convolutional codes embedded runlength limited property has a long history[1]-[6]. Since the known methods have some restrictions in designing an encoder, the proposed codes may have rather lower error control capability than that of known good-convolutional code for given number of memory. In [6], some runlength limited convolutional codes which can be designed without degradation of error control performance are introduced. The design technique introduced in [6], however, can not be applied in any convolutional code, because it requires rather strict condition of code rate and memory order of convolutional codes.

The method presented herein can be applied to any convolutional codes for limited zero and/or one runlength property. The conditions of convolutional codes for the property of limited runlength are first shown, and then a method for runlength limited convolutional codes is presented. Through some examples, it is demonstrated that the codes designed with this method combine good runlength property with their own error control capability.

II. CONDITION FOR RUNLENGTH LIMITED CONVOLUTIONAL CODES

Let us consider an (n, k, m) binary convolutional code where n is the number of output bits, k is the number of input bits and m is memory order of the encoder. The n-tuple output at time t of the convolutional encoder, v_t, can be defined as

$$v_t = \mathfrak{J}_m(u_t, u_{t-1}, \cdots, u_{t-m}) = (v_{t,1}, v_{t,2}, \cdots, v_{t,n}),\ t>m, \qquad (1)$$

here $\mathfrak{J}(\cdot)$ is an encoder of (n, k, m) binary convolutional code and $u_t = (u_{t,1}, u_{t,2}, \cdots, u_{t,k})$ is k-tuple input message at time t. From (1), we can see that n-tuple output v_t, $t>m$, can be generated by $(m+1)k$-tuple input message $(u_t, u_{t-1}, \cdots, u_{t-m})$. It implies that there are at most $2(m+1)k$ distinct n-tuple outputs according to all combinations of possible input message and encoder states.

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Now we define a n-tuple codeword set C which consist of $2(m+1)k$ distinct n-tuple outputs of the convolutional code. Since convolutional codes are a linear code, if a convolutional code satisfies $2n>2(m+1)k$ then, there exist non-trivial cosets of the code C,

$$A = C \oplus \mu = \{ v \oplus \mu | v \in C \},$$

which do not have all zero codeword. Here, $\mu = (\mu_1, \mu_2, \cdots, \mu_n)$ is n-tuple coset representative. In practical, the convolutional codes satisfying the condition $n>(m+1)k$, however, may be an inappropriate code.

Now considering a codeword set, C^2, whose $2n$-tuple codewords consist of two consecutive outputs of (n, k, m) convolutional code, $v_t^2=(v_{t-1}, v_t)$. Then, we can see that 2n-tuple codeword, v_t^2, $t>m+1$, is generated by $(m+2)k$-tuple input message, $(u_t, u_{t-1}, \cdots, u_{t-m}, u_{t-m-1})$. Similarly, if an (n, k, m) convolutional code satisfies the condition $2n>(m+2)k$, then, with $2n$-tuple coset representative, we can also avoid all zero codeword in the coset of the code, C^2.

In a similar way, we can expand this idea to a set of B consecutive outputs of (n, k, m) convolutional code, C^B. Thus, a condition for runlength limited convolutional code is

$$Bn > (m+B)k \quad \text{i.e.,} \quad B > \frac{mk}{n-k}. \tag{2}$$

This condition implies that any convolutional code can get limited runlength property with a Bn-tuple coset representative of the code C^B as a modification vector, here the number B should satisfy the condition (2). That is, a code A^B is runlength limited code, here

$$A^B = C^B \oplus M = \{ v^B \oplus M | v^B \in C^B, M \notin C^B \},$$

$$M = (\mu_1, \mu_2, \cdots, \mu_B).$$

Let us consider both zero run and one run in a code sequence, simultaneously. If the code C^B is a transparent code, that is, $(11\cdots1) \in C^B$, it is clear that there doesn't exist all zero code and all one codewords in the coset, A^B, of code C^B. In the state diagram of a convolutional code C which is the origin code of C^B, if there exist B sequentially connected branches generating all one codeword, then the code C^B is transparent. Else the code C^B is not transparent code and, in this case, all one codeword can be exist in the coset, A^B.

Let A^B and A'^B both be non-trivial cosets of the non-transparent code C^B, and assume that all one codeword is in the coset, A^B. Then there are no all one codeword as well as all zero codeword in the coset, A'^B, because $A^B \cap A'^B = \varnothing$. Therefore, we can say that if a linear non-transparent code, C^B, has more than 1 non-trivial cosets, there exists a coset which does not include all zero and all one codewords. The condition that the code, C^B, has more than 1 coset, is

$$Bn > (m+B)k+1$$
$$\text{i.e.,} B > \frac{mk+1}{n-k}. \tag{3}$$

Ⅲ. COSETS FOR LIMITED ZERO AN ONE RUNLENGTH

A code sequence of nB-tuple codewords set A^B is depicted in Fig. 1, where the number B is the smallest number satisfying (2) and A^B is a coset of C^B with a proper coset representative, M. In Fig. 1, assume that $(B-1)n$ bits of the code sequence are all zero as the worst case, then it is clear that the leftmost and the rightmost n-bit should be not all zero because both the left Bn-tuple codeword and right Bn-tuple codeword are not all zero codeword. Thus, for a given (n, k, m) binary convolutional code, maximum zero runlength, R_{max0}, of its coset is

$$R_{max0} \le \left\lfloor \frac{mk}{n-k} + 2 \right\rfloor n - 2. \tag{4}$$

Similarly, from (3), both the maximum zero runlength, R_{max0}, and maximum one runlength, R_{max1}, in the code sequence of (n, k, m) convolutional codes with proper modification

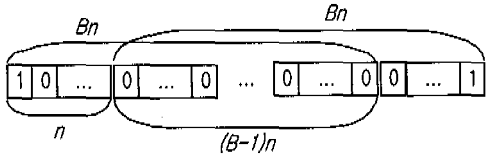

Fig. 1. A code sequence of the longest zero run.

vector are

$$(R_{\max 0}, R_{\max 1}) \leq \begin{cases} \left\lfloor \frac{mk}{n-k} + 2 \right\rfloor n-2, & C^B \textit{ is transparent}, \\ \left\lfloor \frac{mk+1}{n-k} + 2 \right\rfloor n-2, & \textit{otherwise}. \end{cases} \quad (5)$$

For example, let us consider (2, 1, 2) binary convolutional code whose polynomial generator matrix is

$$G(D) = [\,1+D+D^2 \;\; 1+D^2\,]. \quad (6)$$

By the condition (3), we can construct a codewords set C^4 which consist of all possible four consecutive outputs v_{t-3}, v_{t-2}, v_{t-1}, and v_t. There are $(2^{2 \cdot 4}-2^{(2+4) \cdot 1})$ 8-tuple binary vector, $M \not\in C^4$, which can be used as a modification vector. Among them, we can search a vector, M, which minimizes the longest sequence of consecutive zeros and ones in any code sequence. In the case of our example, 32 vectors lead the maximum zero and one runlength, $(R_{max0}, R_{max1})=(8, 8)$. For simple implementation, we can select the vector (00 01 00 01) as a modification vector of the code generated by (6).

Our method to limit zero and/or one runlength in transmitting code sequence is shown in Fig. 2. When the encoder of convolutional code is already implemented, the codeword reordering can be used because the number of consecutive zero and/or one tailing bits of leftmost n-bit codeword and number of consecutive zero and/or one leading bits of rightmost n-bit codewords in Fig. 1 can be varied according to codeword order. For example, $G(D)=[D+D^2+D^3 \;\; 1+D+D^3]$ leads (R_{ma} R_{max1})=(7, 7) but $G(D)=[1+D+D^3 \;\; D+D^2+D^3]$ leads $(R_{max0}, R_{max1})=(8, 8)$.

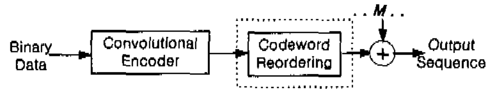

Fig. 2. Block diagram of proposed scheme.

Ⅳ. SOME EXAMPLES AND DISCUSSION

The modification vectors and R_{max0} and R_{max1} for the best codes introduced in [7] are listed in Table Ⅰ. The generator sequences are reordered for short runlength and expressed in octal form. For example, the generator polynomial $[1+D^3+D^4 \;\; 1+D+D^2+D^4]$ is expressed (46 72) in Table I. The modification vector, 010, for the code of generator sequence (52 66 76) is a shortened expression for 010 010 010.

When we consider zero runlength only, R_{max0} of the codes denoted by * can be smaller than the value listed in Table Ⅰ, because the codes, C^B, generated by the code denoted * are non- transparent. In the case of the code denoted **, however, even if C^B is non-transparent, R_{max0} is equal to the value presented in Table Ⅰ, because the smallest numbers of B satisfying (2) and (3) are equal.

Table Ⅱ shows the modification vector leading the shortest R_{max0} for the codes noted by * in Table Ⅰ. The R_{max0} listed in Table Ⅱ are smaller than the value listed in Table Ⅰ. Table Ⅲ reports the codes which have the shortest (R_{max0}, R_{max1}) for given n, k, m, K and d_{free}. Listed codes show good runlength properties as well as good error control capability.

Ⅴ. CONCLUSIONS

The author investigated a method to obtain limited runlength property for convolutional codes. Since the proposed method is based on cosets of the convolutional codes, with only simple modification, we can achieve good runlength properties without degradation of error control performance of the code. As a result, the codes presented in tables show good error control

Table 1. The shortest maximum zero and one runlength for given convolutional codes.

k/n	m	K	generator sequence	d_{free}	M	(R_{max0}, R_{max1})
1/4	2	2	5 7 7 7	10	0010	(3, 3)
1/4	3	3	54 64 64 74	13	0010	(4, 4)
1/4	4	4	52 56 66 76	16	0001	(6, 6)
1/4	5	5	71 67 53 75	18	0010	(7, 7)
1/3	2	2	5 7 7	8	001	(3, 3)
1/3	3	3	64 74 54	10	001	(5, 5)
1/3	4	4	52 76 66	12	010	(8, 9)**
1/3	5	5	53 47 75	13	010	(10, 11)*
1/2	2	2	5 7	5	00 11	(8, 8)*
1/2	3	3	64 74	6	00 11	(10, 10)*
1/2	4	4	46 72	7	00 11	(12, 12)*
1/2	5	5	65 57	8	00 11	(14, 14)*
2/3	1	2	6 2 6 2 4 4	3	001	(9, 9)
2/3	2	3	4 2 6 1 4 7	4	010	(13, 13)
2/3	2	4	7 1 4 2 5 7	5	001	(16, 16)

Table 2. The shortest maximum zero runlength for given convolutional codes.

k/n	m	K	Generator sequence	d_{free}	M	R_{max0}
1/3	5	5	47 53 75	9	100	9
1/2	2	2	5 7	5	10	6
1/2	3	3	64 74	6	01	8
1/2	4	4	46 72	7	01	10
1/2	5	5	65 57	8	10	12

Table 3. Some convolutional code with shortest maximum zero and one runlength for given coding parameters and free distance.

k/n	m	K	Generator sequence	d_{free}	M	(R_{max0}, R_{max1})
1/3	2	2	7 7 5	8	100	(3, 3)
1/3	3	3	74 54 54	10	001	(3, 3)
1/3	4	4	76 62 52	11	001	(5, 5)
1/3	4	4	76 66 52	12	011	(8, 9)
1/3	5	5	65 73 47	13	001	(5, 5)
1/2	2	2	7 4	4	01	(5, 5)
1/2	2	2	7 5	5	00 11	(8, 8)
1/2	3	3	34 64	6	01	(7, 7)
1/2	4	4	72 46	7	00 11	(12, 12)
1/2	5	5	67 52	8	01	(11, 11)
2/4	1	2	6 6 2 0 2 2 4 6	5	0100	(4, 4)
2/4	2	3	7 2 2 1 2 6 6 4	6	0010	(4, 4)
2/4	2	4	7 7 4 2 6 1 7 5	8	0001	(7, 7)
2/3	1	2	2 6 4 6 4 0	3	001	(8, 8)
2/3	2	3	7 7 4 6 4 2	4	001	(12,12)
2/3	2	4	4 5 5 3 1 7	5	001	(14, 14)

capability as well as good runlength property. Proposed codes are suitable in systems with high demands on synchronization information conveyed in the transmitted signal.

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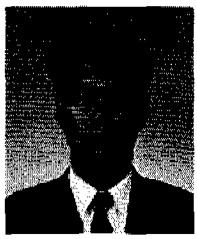

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