부하가 있는 전송 선로의 입력 반사 계수의 스미스 차트 상에서의 궤적

정회원 은 창 수*

On the Loci of the Input Reflection Coefficient of a Loaded Transmission Line on the Smith Chart

Chang-Soo Eun* *Regular Members*

요 약

이 논문에서는 부하가 걸려 있는 전송 선로의 길이가 변화할 경우 입력반사 계수가 스미스 차트 상에서 그리는 궤적의 방정식을 유도한다. 결과에 따르면, 전송 선로의 특성 임피던스가 정규화 임피던스와 다를 경우 궤적은 원을 그리게 되며, 그 원의 중심과 반지름은 부하의 임피던스와 전송 선로의 특성 임피던스에 의해 결정된다. 이 결과는 마이크로스트립 선로를 사용하여 정합 회로를 설계할 때, 선로의 길이와 너비를 조정할 필요가 있을 경우 유용하게 사용될 수 있다.

키워드: 입력 반사 계수, 궤적, 전송선로, 마이크로스트립라인, 스미스차트

ABSTRACT

We present the derivation of the loci of the reflection coefficient of a loaded transmission line on the Smith chart as the length of the transmission line is varied. The results show that the loci constitute a circle when the characteristic impedance of the transmission line differs from the normalizing one, whose center and radius are determined by the impedances of the load and the transmission line. The results can be used in the matching circuit design using microstriplines when line width and length should be compromised.

I. Introduction

When designing a matching circuit using microstriplines, one often encounters a case in which the width and length of the microstripline are unfit to the board size or are unrealistic to implement. One way around this situation is to use a transmission line of characteristic impedance different from the normalizing one which is usually 50 W. Even though one can do this using a simple CAD program such as in [1], it is necessary that we can design matching circuits even when such CAD programs are not available. Moreover, it is substantial to understand the behavior of the transmission lines of different impedances. Microstriplines of arbitrary impedances have been used as a quarter wave impedance transformer[2, 3, 4, 5] or as a segment of transmission line whose impedance transformation is calculated on the Smith chart by changing the normalizing impedance[2].

The use of transmission lines of arbitrary characteristic impedances provides us with flexibility in the microstripline matching circuit designs. For example, we can design a match

* 충남대학교 전기정보통신공학부 이동통신연구실 (eun@ieee.org)
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ing circuit on the Smith chart not only along the constant- resistance or conductance circles and/or the concentric circles (i.e., constant-reflection coefficient circles) but also along the loci derived for the microstripline of arbitrary impedance.

In this paper, we present the derivation of the loci of the input reflection coefficient of a loaded transmission line on the Smith chart as the length of the transmission line is varied. The characteristic impedance of the transmission line and the load impedance are assumed arbitrary with respect to the normalizing impedance.

In Sec. 2, we derive the relations between the input reflection coefficient and the normalized load and transmission line impedances, which is a circle on the Smith chart. We will also derive the relation between the transmission line length and the corresponding angle on the circle.

In Sec. 3, we will summarize the result and conclude.

II. Derivation of the Equations

1. The Loci of the Reflection Coefficient

Assume a circuit consisting of a source, a section of transmission line, and a load. In Fig. 1, we show such a circuit in which the source impedance is Z_s, the transmission line impedance Z_t, and the load impedance Z_l. We will take Z_s as the normalizing impedance and the transmission line is assumed to be lossless.

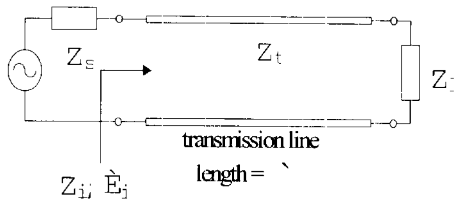


Fig. 1. A circuit consisting of a source, a section of transmission line, and a load.

The input impedance Z_i looking into the transmission line is given by [6]

$$Z_i(\ell)=Z_t\frac{Z_l+jZ_t\tan\beta\ell}{Z_t+jZ_l\tan\beta\ell}, \tag{1}$$

where ℓ is the length of the transmission line segment and β is the propagation constant of the transmission line. Normalizing the impedances with Z_s, we can re-write Eq. (1) as

$$z_i=z_t\frac{z_l+jz_t\tan\beta\ell}{z_t+jz_l\tan\beta\ell}, \tag{2}$$

where z_i, z_t, and z_l are the normalized impedances of $Z_i(\ell)$, Z_t, and Z_l, respectively. We drop the argument ℓ for brevity. The input reflection coefficient Γ_i looking into the transmission line is given by

$$\Gamma_i=\frac{Z_i-Z_s}{Z_i+Z_s}=\frac{z_i-1}{z_i+1}. \tag{3}$$

If $Z_s = Z_t$, that is, the source impedance equals the characteristic impedance of the transmission line, the *magnitude* of the reflection coefficient should be constant regardless of the length ℓ, i.e.,

$$\frac{z_i-z_t}{z_i+z_t}=re^{j\theta}, \tag{4}$$

where r is a constant and given by

$$r=\left|\frac{Z_i-Z_t}{Z_i+Z_t}\right|=\left|\frac{Z_l-Z_t}{Z_l+Z_t}\right| =\left|\frac{z_l-z_t}{z_l+z_t}\right|\le 1 \tag{5}$$

and θ is a function of ℓ. Solving Eq. (3) for z_i, we obtain

$$z_i = \frac{1+\Gamma_i}{1-\Gamma_i}. \qquad (6)$$

Substituting Eq. (6) into Eq. (4), we get

$$\frac{\Gamma_i - \frac{z_t - 1}{z_t + 1}}{1 - \Gamma_i \frac{z_t - 1}{z_t + 1}} = re^{j\theta}. \qquad (7)$$

Taking the magnitude squares of both sides, we have

$$\left| \frac{\Gamma_i - \gamma}{1 - \Gamma_i \gamma} \right|^2 = r^2, \qquad (8)$$

where

$$\gamma = \frac{z_t - 1}{z_t + 1} = \frac{Z_t - Z_s}{Z_t + Z_s}. \qquad (9)$$

Solving Eq. (8) for Γ_i leads to

$$\left| \Gamma_i - \frac{\gamma(1-r^2)}{1 - r^2\gamma^2} \right|^2 = \frac{r^2(1-\gamma^2)^2}{(1-r^2\gamma^2)^2}, \qquad (10)$$

$$|\Gamma_i - C|^2 = R^2, \qquad (11)$$

where

$$C = \frac{\gamma(1-r^2)}{1-r^2\gamma^2}, \qquad (12)$$

$$R = \frac{r(1-\gamma^2)}{1-r^2\gamma^2}. \qquad (13)$$

The result shows that the loci of the reflection coefficient looking into the loaded transmission line of length ℓ whose characteristic impedance is different from the normalizing impedance constitutes a circle on the Smith chart whose center and radius are given by Eq. (12), (13), respectively. From Eq. (12), we find that the center of the circle lies on the real axis since γ and r are real such that C is real. In addition, the location of thecenter is determined by γ, that is, the center lies on the negative side of the real axis if γ is negative, and vice versa. This means the center is on the negative side if the characteristic impedance of the transmission line is smaller than the normalizing impedance, and on the positive side if the characteristic impedance is larger than the normalizing impedance. In Fig. 2, we show the variation of the location of the center and radii with respect to g and r. We can verify the discussion mentioned above from the figure: if γ is positive, the center is on the positive real axis, and vice versa. We also observe that, when the center is at ± 1, the radius of the loci is zero. This means that the reflection coefficient remains always less than one if the load is passive. The loci of the reflection coefficients never go out of the unit circle when the network is passive.

2. Relation between Angle and Length, $\phi(\ell)$

We now investigate the relation between the length of the transmission line and the phase of the reflection coefficient measured at the derived circle. We define ϕ as the angle made by the real axis of the Smith chart and the line connecting the center of the circle and a point on the circle. In Fig. 3, we show the definition of the angle ϕ. The circle shown represents the loci of the reflection coefficient on the Smith chart whose center and radius are given by Eq. (12) and (13).

From Eq. (11), we have

$$\Gamma_i - C = Re^{j\phi}. \qquad (14)$$

On the other hand, from Eq. (7), (9), we have

$$\frac{\Gamma_i - \gamma}{1 - \Gamma_i\gamma} = re^{j\theta}. \qquad (15)$$

Solving Eq. (15) for G_i, we obtain

$$\Gamma_i = \frac{re^{j\theta}+\gamma}{1+\gamma re^{j\theta}}. \quad (16)$$

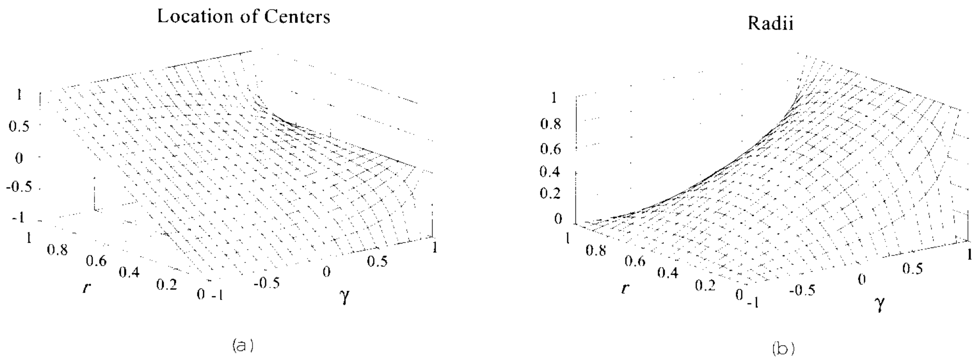
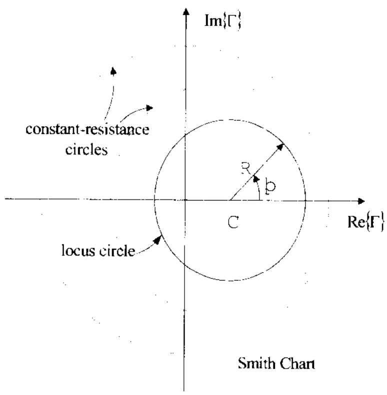


Fig. 3. Definition of the phase ϕ.

Replacing Eq. (16) for G_i and Eq. (12) for C in Eq. (14), and using Eq. (13) , Eq. (14) reduces to



$$R\frac{e^{j\theta}+\gamma r}{1+\gamma re^{j\theta}} = Re^{j\phi}. \quad (17)$$

The numerator of the left side of Eq. (17) is manipulated to

$$e^{j\theta}+\gamma r = e^{j\theta}(1+\gamma re^{-j\theta})$$

$$= e^{j\theta}(1+\gamma re^{j\theta})^*, \quad (18)$$

where * denotes the complex conjugate. From this result and Eq. (17), we obtain tlhe angle relation as

$$\phi = \theta - 2\angle(1+\gamma re^{j\theta})$$
$$= \theta - 2\arctan\left(\frac{\gamma r\sin\theta}{1+\gamma r\cos\theta}\right). \quad (19)$$

The relation between the two angles is shown in Fig. 4 for several values of r and γ. We see here that the relation between θ and ϕ is not linear. We can also see that as θ approaches $\pm\pi$, the var iation of ϕ becomes steeper. As the value of r an d γ increases, the nonlinearity becomes more evid ent. If we let the length of the transmission line be ℓ, and let the reflection coefficient when ℓ = 0 be $re^{j\theta_0}$, then $\phi_0(\ell = 0)$ is given by

$$\phi_0 = \theta_0 - 2\arctan\left(\frac{\gamma r\sin\theta_0}{1+\gamma r\cos\theta_0}\right). \quad (20)$$

When the length of the transmission line is ℓ and $Z_s = Z_t$, the reflection coefficient is

$$re^{j\theta_0}e^{-j2\beta\ell} = re^{j(\theta_0-2\beta\ell)}. \quad (21)$$

Fig. 2. The variation of the center and radius of the loci. (a) The variation of the location of the center with respect to γ and r. (b) The variation of the radius with respect to γ and r.

Therefore, when the length is ℓ, $\phi(\ell)$ is given by

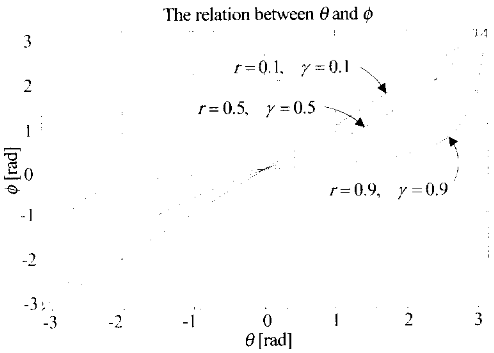


Fig. 4. The nonlinear relation between θ and ϕ.

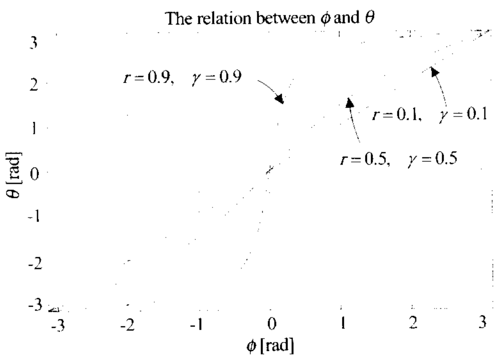


Fig. 5. The nonlinear relation between ϕ and θ.

$$\phi(\ell)=\theta_0-2\beta\ell-2\arctan\left[\frac{\gamma r\sin(\theta_0-2\beta\ell)}{1+\gamma r\cos(\theta_0-2\beta\ell)}\right]. \quad (22)$$

Fig. 4 shows, depending on the initial angle θ_0, ther curvature of the θ-ϕ curve varies.

3. Relation between Length and Angle, $\ell(\phi)$

Next, we find the the variation of the length ℓ as the phase ϕ varies on the Smith chart. From Eq. (14) we get

$$\Gamma_i=Re^{j\phi}+C. \quad (23)$$

Substituting Eq. (23) into Eq. (15), and replacing Eq. (12) for R and Eq. (13) for C, we are led to

$$re^{j\phi}\frac{(1-\gamma re^{j\phi})^*}{1-\gamma re^{j\phi}}=re^{j\theta}. \quad (24)$$

Similarly as with Eq. (17), from Eq. (24), we obtain the angle relation as

$$\begin{aligned}\theta&=\phi-2\angle(1-\gamma re^{j\phi})\\&=\phi+2\arctan\left(\frac{\gamma r\sin\phi}{1-\gamma r\cos\phi}\right).\end{aligned} \quad (25)$$

We see here that Eq. (25) is the inverse of Eq. (19). In Fig. 5, we show the relation between θ a nd ϕ for several values of r and γ. It is found th at, as ϕ approaches $\pm\pi$, the variation of θ becom es slow, which is the inverse action of Eq. (19). Assuming $\theta=\theta_0$, $\phi=\phi_0$ when $\ell=0$, we have, from Eq. (25),

$$\theta_0-2\beta\ell=\phi_0+\triangle\phi+2\arctan\left[\frac{\gamma r\sin(\phi_0+\triangle\phi)}{1-\gamma r\cos(\phi_0+\triangle\phi)}\right], \quad (26)$$

where $\Delta\phi$ is the angle of the movement on the locus circle. Solving Eq. (26) for ℓ, we get

$$\ell=\frac{\theta_0-\phi_0-\triangle\phi-2\arctan\left[\frac{\gamma r\sin(\phi_0+\triangle\phi)}{1-\gamma r\cos(\phi_0+\triangle\phi)}\right]}{2\beta}. \quad (27)$$

Eq. (27) gives the corresponding length of the tra nsmission line (with a characteristic impedance dif ferent from the normalizing impedance) when we move on the locus circle on the Smith chart by $\Delta\phi$.

III. Conclusions

This paper presented the derivation of the locus on the Smith chart of the input reflection coeffici ent of the loaded transmission line the characteristic impedance of which is arbitrary with respect to the normalizing and the load impedances. The result shows that the loci of the input reflection coefficient constitute a circle whose center and radius are determined by the normalized load impedance and the characteristic impedance of the transmission line. The angular variation on the loci on the Smith chart corresponding to the length variation is not linear: as the deviation of the impedances of the transmission line and/or the load from the normalizing impedance increases, the nonlinearity becomes more evident. The relative magnitude of the characteristic impedance of the transmission line with respect to the normalizing impedance determines the location of the center: if the characteristic impedance of the transmission line is larger than the normalizing impedance, the center resides on the positive side of the real axis, and vice versa.

The results will be useful in the matching circuit design using microstriplines when the length or width of the lines should be compromised for realization or for the board size, even more when a CAD program is not available.

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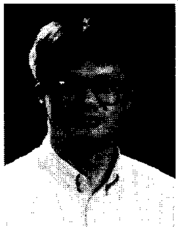
은 창 수(Chang-Soo Eun) 정회원



1985년 2월: 서울대학교 전자공학과 졸업
1987년 2월 : 서울대학교 전자공학과 석사
1995년 12월: 텍사스 오스틴대학교 전기공학과 박사
1997년 9월~현재: 충남대학교 전기정보통신공학부 부교수

<주관심분야> 통신 신호 처리, RF 및 초고주파 회로 설계