

# An Efficient Method to Obtain MCF in Millimeter Wave Systems

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ABSTRACT

Millimeter waves are potentially useful for high resolution ranging and imaging in low optical visibility conditions such as fog and smoke. Also, They can be used for wide band communications since the currently used spectrum bands are already crowded. However, it is necessary to develop a theoretical and experimental understanding of millimeter wave propagation to assess the performance of millimeter wave systems. The intensity fluctuations and mutual coherence function (MCF) describe atmospheric effects on the millimeter wave propagation. Using the quasi-optical method (QOM), a practical and efficient method is suggested to obtain MCF from the flux measurement in the antenna focal plane.

## I. Introduction

Millimeter waves are being used for wide band communication and other special purposes. However, it is necessary to develop a theoretical and experimental understanding of electromagnetic wave propagation at millimeter wavelengths through the atmosphere in order to assess the performance of millimeter wave systems. A general theory of beam wave propagation through an atmosphere was presented in [1]. This theory accounts for the finite aperture size of the source and describes the propagation of five different wave field configurations through an atmosphere taken to have mean and fluctuating dispersive and absorbing components. In [2] a general model of beam wave propagation was extended and related to common experimental quantities capable of being measured via a parabolic reflector antenna. These experimentally measurable quantities i.e., intensity covariance angle of arrival and the mutual coherence function are very important because atmospheric effects on the propagation of electromagnetic waves are

usually described in terms of these quantities.

Of particular interest here is the mutual coherence function defined as the cross-correlation function of the complex fields in a direction transverse to the direction of propagation. Assuming that the fluctuations of electric permittivity are small and following the Rytov method<sup>[3]</sup>, the MCF,  $\Gamma$  is given by

$$\Gamma(\vec{r}_1, \vec{r}_2) = E_0(\vec{r}_1)E_0^*(\vec{r}_2)\exp\left[-\frac{1}{2}D(\vec{r}_1, \vec{r}_2)\right] \quad (1)$$

where  $E_0$  and  $D$  are an unperturbed field component and a wave structure function respectively. Here  $\vec{r}$  is the position vector and  $E$  represents the electric field. From Eq.(1), we see that the MCF describes the loss of coherence of an initially coherent wave propagating in a turbulent medium. As a result, the MCF is important for a number of practical applications. It determines the S/N ratio of an optical heterodyne detector, the limiting resolution obtainable along an atmospheric path and the mean irradiance distribution from an initially coherent wave emanating from a finite aperture.

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## II. Quasi-Optical method (QOM)

The MCF is related to fluctuating inhomogeneities in the index of refraction of the propagation medium, so meteorological information is an important factor in assessing all-weather performance of millimeter wavelength systems. The transverse coherence length (defined as the transverse separation at which the MCF is reduced by  $e^{-1}$ ) can be significantly reduced by fog, smoke, etc. If we define  $\rho_0$  as the transverse separation at which the atmospheric MCF is reduced by  $e^{-1}$ , the minimum resolvable length at distance  $Z$  from an observer is well known to be  $Z/(k\rho_0)$ . Thus, a decrease of coherence length means a decrease in the resolution and an increase of beam wave spreading. The QOM can be used here to get the MCF from the instantaneous focal plane irradiance distribution, that is, the spatial Fourier transform of the product of the electromagnetic fields at the aperture plane.

Under certain assumptions, the electric field in the focal plane of a parabolic reflector antenna can be represented as the Fourier transform of fields in the aperture plane. The relationship is given by

$$E(q) = (ik/2\pi f) \exp(ikL) \times \iint_{S_a} E(\vec{r})W(\vec{r}) \exp[-ik(\vec{q} \cdot \vec{r})/f] d^2\vec{r} \quad (2)$$

where

- $\vec{q}$  : a position vector in the focal plane
- $\vec{r}$  : a position vector in the aperture plane
- $k$  : the wave number
- $\lambda$  : the wave length
- $f$  : the focal plane length of the parabolic reflector antenna
- $L$  : a constant phase factor
- $S_a$  : the area of the aperture plane

$W(\vec{r})$ : the aperture function whose value is one for  $|\vec{r}| \leq d/2$  and zero otherwise where  $d$  is the diameter of the reflector at its aperture.

The assumptions under which Equation (2) holds are as follows.

- a. the diameter and radius of curvature of the antenna and incident wave front must all be much larger than the wavelength of the incident
- b.  $q < \sqrt{\lambda f/2}$
- c.  $F \gg 1/4$  where  $F$  is the F number defined by  $f$ /aperture diameter

Therefore, the intensity  $I(q)$  at a point  $q$  on the focal plane is given by

$$I(q) = [k/(2\pi f)]^2 \iint E(\vec{r})E^*(\vec{r}')W(\vec{r})W^*(\vec{r}') \times \exp[-ik\vec{q} \cdot (\vec{r} - \vec{r}')/f] d^2\vec{r} d^2\vec{r}' \quad (3)$$

The electric field  $E$  is, however, a random function due to the statistical nature of the atmosphere through which it propagated. Thus only the ensemble averaged intensity can be considered. Ensemble averaged equation of (3) yields

$$\langle I(q) \rangle = [k/(2\pi f)]^2 \iint \Gamma(\vec{r}, \vec{r}')W(\vec{r})W^*(\vec{r}') \times \exp[-ik\vec{q} \cdot (\vec{r} - \vec{r}')/f] d^2\vec{r} d^2\vec{r}' \quad (4)$$

where  $\Gamma(\vec{r}, \vec{r}') = \langle E(\vec{r})E^*(\vec{r}') \rangle$  is the mutual coherence function of the electric field at the aperture plane. It is the mutual coherence function that describes the effects of the fluctuating electrical parameters of the atmosphere on the propagating electromagnetic wave. Thus, if one can solve the integral equation (4) for  $\Gamma$ , the MCF can be obtained from intensity data on the focal plane. This is the essence of the Quasi-optical method<sup>[2]</sup>.

To solve Eq.(4) for  $\Gamma$ , the inverse Fourier transform will be taken with respect to the  $q$

coordinate. After some manipulations, The optical transfer function of the total system (of the atmosphere and the antenna),  $H_T(\vec{\rho})$  is given by

$$H_T(\vec{\rho}) = \int \Gamma(\vec{r}, \vec{r} - \vec{\rho}) W(\vec{r}) W^*(\vec{r} - \vec{\rho}) d^2\vec{r} \quad (5)$$

where  $\vec{\rho} = \vec{r} - \vec{r}'$ . If the statistical inhomogeneity is negligible, Eq.(5) decouples and becomes

$$\begin{aligned} H_T(\vec{\rho}) &= \Gamma(\vec{\rho}) \int W(\vec{r}) W^*(\vec{r} - \vec{\rho}) d^2\vec{r} \\ &= \Gamma(\vec{\rho}) H_A(\vec{\rho}) \end{aligned} \quad (6)$$

where  $H_A(\vec{\rho})$  is the antenna transfer function which simply represents the area of overlap of two circles of diameter  $d$ , one having a center at the origin of the  $\vec{r}$  coordinate system and the other having its origin at  $\vec{\rho}$  in the same system. Thus the  $r$  integration yields the area of overlap between the two circles given by

$$\begin{aligned} H_A(\rho) &= (d^2/2) \\ &\times [\cos^{-1}(\rho/d) - (\rho/d)(1 - (\rho/d)^2)^{1/2}] \end{aligned} \quad (7)$$

while the value of  $H_A(\rho)$  is 0 for  $\rho > d$ .

### III. Signal analysis on the focal plane

It remains to determine  $H_T(\rho)$  given by

$$H_T(\rho) = \int \langle I(q) \rangle \exp(ik\vec{\rho} \cdot \vec{q} / f) d^2\vec{q} \quad (8)$$

from measurements of  $\langle I(q) \rangle$  in the focal plane.  $\langle I(q) \rangle$  can be obtained by placing a circular iris (to conform to the azimuth symmetry of the intensity distribution) on the focal plane and measuring the rate of increase of flux as the iris is opened or closed. Quantitatively, the flux  $F(R)$  transmitted through an iris of radius  $R$  is described by

$$\begin{aligned} F(R) &= \int_0^{2\pi} \int_0^R \langle I(q) \rangle q dq d\theta \\ &= 2\pi \int_0^R \langle I(q) \rangle q dq \end{aligned} \quad (9)$$

To be able to write the average intensity  $\langle I(q) \rangle$  in terms of the experimentally measured flux, Eq.(9) can be differentiated in the radial coordinate, i.e.,

$$\frac{1}{R} \frac{\partial}{\partial R} F(R) = 2\pi \langle I(R) \rangle \quad (10)$$

Thus, the radial derivative of the flux at  $R$  yields the intensity there. However, the need to take the derivative of the flux (a rather difficult task to do experimentally) can be circumvented by the following considerations.

Taking into account the circular symmetry of the intensity distribution  $\langle I(q) \rangle$ , Eq.(8) becomes

$$\begin{aligned} H_T(\rho) &= \iint \langle I(q) \rangle \exp(ik\rho q \cos\theta / f) q dq d\theta \\ &= 2\pi \int_0^\infty \langle I(q) \rangle J_0(k\rho q / f) q dq \end{aligned} \quad (11)$$

which is the Fourier-Bessel transform of  $\langle I(q) \rangle$ <sup>[4]</sup>. Here,  $J_n$  represents the Bessel function of the first kind of order  $n$ . Equation (11) can be rewritten in terms of the flux  $F(R)$  transmitted through an iris of radius  $R$  which is given by

$$H_T(\rho) = \int_0^\infty dF(q) / dq J_0(k\rho q / f) dq \quad (12)$$

Integrating by parts and using the recurrence relation for Bessel functions,  $H_T(\rho)$  can be represented by

$$\begin{aligned} H_T(\rho) &= J_0(k\rho q / f) F(q) \Big|_0^\infty \\ &\quad + k\rho / f \int_0^\infty F(q) J_1(k\rho q / f) dq \end{aligned} \quad (13)$$

The first term in Eq.(13) vanishes since

$J_0(k\rho q/f)$  goes to zero as  $q$  goes to infinity and  $F(0)=0$  by definition. However, the actual interval of integration is not from zero to infinity but from zero to the maximum radius of the focal plane scanner, i.e.,  $q_{\max}$ . Thus, the first term of Equation (13) cannot be ignored and Eq.(13) becomes

$$H_T(\rho) \approx J_0(k\rho q_{\max}/f)F(q_{\max}) + k\rho/f \int_0^{q_{\max}} F(q)J_1(kq\rho/f)dq \quad (14)$$

Even though the definition of Fourier transform formally requires the intensity  $\langle I(q) \rangle$  to be considered in the interval  $[0, \infty]$  on the focal plane, the above procedures are also valid for the actual case because most of the received power is within the area bounded by the maximum radius,  $q_{\max}$ . Thus using the experimentally determined flux, one can find  $H_T(\rho)$  through the use of Equation (14).

#### IV. MCF from simulated flux data

Since the flux or intensity cannot be known without actual measurements, the MCF will be computed from simulated flux data. For simulation purposes, the intensity function may be represented by

$$I(q) = A \exp(-a^2 q^2) \quad (15)$$

where the constants,  $A$  and  $a$ , are determined based on the conditions that the intensity must decrease by  $e^{-3}$  at the maximum focal plane point,  $q_{\max}$ , and the total detected power in the focal plane is one milliwatt. Therefore, the flux function for the simulation is given by

$$F(r) = 2\pi \int_0^r \langle I(q) \rangle q dq = \frac{\pi A}{a^2} (1 - \exp(-a^2 r^2)) \quad (16)$$

where  $a = 78.7 \text{ m}^{-1}$  and  $A = 1.97 \text{ watt/m}^2$  for  $q_{\max} = 2.2 \times 10^{-2} \text{ m}$  satisfying the previously mentioned conditions for simulation. Instead of using a Riemann sum, the approximated total transfer function represented by Eq.(14) was computed numerically using the Gauss-Legendre Quadrature formula<sup>[4]</sup> for the improved integration accuracy. The results are shown in Fig. 1. Figure 2 shows the MCF,  $\Gamma$  which can be simply obtained from Eq.(6). From the figures, it can be seen that the coherence length is very small because of a widely spreaded intensity. This coherence length depends on the shape of intensity distribution. Here, another concern is the accuracy of the approximated results,  $H_T(\rho)$ . The results were tested for a case where an analytic solution is available. In the case of Gaussian beam waves (the assumed function for the simulation), an analytic solution for  $H_T(\rho)$  can be obtained by Fourier-Bessel transformation method<sup>[5]</sup>. Referring to the original transfer function in Eq.(11), the exact solution of  $H_T(\rho)$  becomes

$$H_T(\rho) = \frac{\pi A}{a^2} \exp\left[-\left(\frac{k}{2fa}\right)^2 \rho^2\right] \quad (17)$$

This exact solution and the approximated one were plotted in Fig.3. The approximated results shown in Fig.3 is the same plot given in Fig.1. As it can be expected, errors are not a voidable because of the upper range approximation,  $q_{\max}$  instead of infinity and some numerical integration errors. However, as seen from Fig. 3, the results show that the approximation method is good enough to be applied in practice.

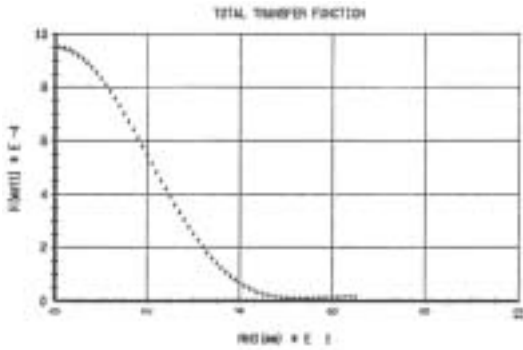


Fig.1 the computed total transfer function,  $H_T(\rho)$

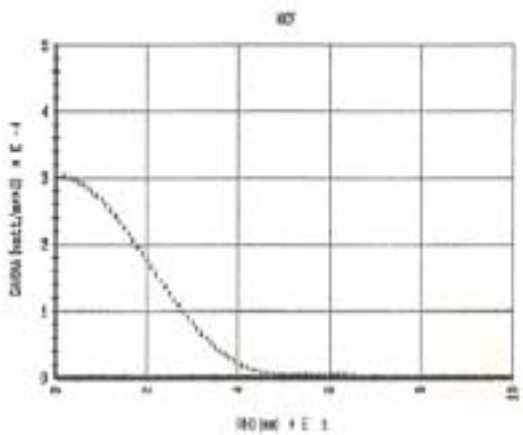


Fig. 2 The obtained mutual coherence function,  $\Gamma(\rho)$

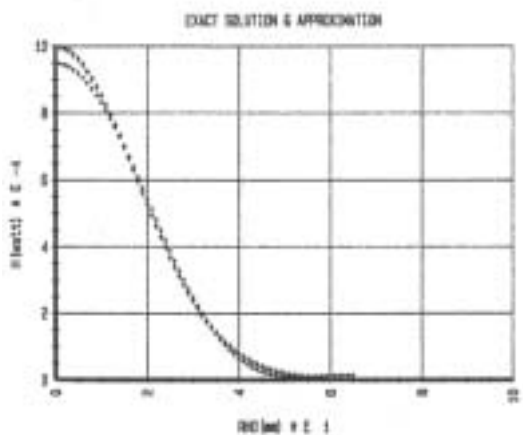


Fig. 3 the exact solution and approximation of  $H_T(\rho)$

## V. Conclusion

The MCF is a very important factor in assessing the all-weather performance of millimeter wave systems. In this paper, a practical method is derived and suggested to obtain MCF more efficiently since the measurement of an average intensity is a rather difficult task to do experimentally. The suggested method is very convenient since the flux measurement can be done easily using a circular iris. Also, it is shown that errors of this approximation method can be considered to be negligible practically.

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