# Blind Multiuser Receiving Scheme Based on Oriented Minimum Output Energy Criterion

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ABSTRACT

Based on oriented minimum output energy (OMOE) criterion, a new blind multiuser receiving scheme of linear minimum mean square error (MMSE) detector is proposed, then an adaptive algorithm for implementing the new scheme is given by the steepest descent method, and the convergence property of the algorithm is analyzed in this paper. The numerical results demonstrate the performance of the proposed scheme.

## I. Introduction

Blind multiuser detector [1][2], which doesn't require the prior knowledge of the signature waveform and timing of all users, has received considerable attention. Honig M etc. [2] proposed blind multiuser detection earlier, and basing on constrained minimum output energy (MOE) criterion, they obtained a blind adaptive Scheme of linear minimum-mean-square-error (MMSE) detector [3] which is realized by solving a constrained optimization problem, that is, it requires detector to satisfy the form of standard criterion. However, in the iteration algorithm, the projection of the quadrature component often needs to be calculated, which brings up difficulty in solving problem

In fact, the form of standard criterion is nonessential to a linear detector. The substance is that the detector is required to be the same quadrant with the user address code, that is, the projection on the user address code is positive. Based on the idea, a non-constrained optimization criterion is given in this paper A new blind multiuser receiving scheme of linear MMSE is obtained by solving the optimization problem.

Notations Column vectors and matrices are denoted by boldface lower and upper case letters, respectively The superscript $^T$ stands for transpose, and $I_n$ denotes the $n\times n$ identity matrix.

## II. Signal Model

Consider a synchronous baseband DS-CDMA system with $K$ users. The received signal is given by

$$r(t)=\sum_{k=1}^{K}A_k b_k s_k(t)+n(t),\quad t\in[0,T] \tag{1}$$

where $A_k$, $b_k$ and $s_k(t)$ denote, respectively, the received amplitude, transmitted symbol and normalized signaling waveform of the $k$th user, and $n(t)$ is the additive white Gaussian noise (AWGN) with zero mean and variance $\sigma^2$. For simplicity, it is assumed that $b_k$ is a binary phase-shift keying (BPSK) signal, that is, $b_k\in\{\pm1\}$ is assumed to be independent equiprobable random variables, and that $s_k(t)$ is real and supported only on the interval $[0,T]$, which is of the form

$$s_k(t)=\sum_{j=0}^{N-1}c_j^k\phi(t-jT_c),\quad t\in[0,T] \tag{2}$$

where $N$ is the processing gain, $(c_0^k, c_1^k,\ldots,c_{N-1}^k)$ is a signature sequence of $\pm1$'s assigned to the kth user, and $\phi$ is a

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normalized chip waveform of duration $T_c$ where $NT_c = T$. The extension to the MPSK signal and complex signaling is straight-forward.

At the receiver, chip-matched filtering followed by chip rate sampling yields a $N$-vector of chip-matched filter output samples within a symbol interval $T$.

$$r = \sum_{k=1}^{K} A_k b_k s_k + n, \qquad (3)$$

where $s_k = (1/\sqrt{N})[c_0^k, \ldots, c_{N-1}^k]^T$ is the normalized signature waveform vector of the kth user, $n$ is an AWGN vector with mean $0$ and covariance matrix $\sigma^2 I_N$ Moreover, assume that the vectors $\{s_k\}_{k=1}^{K}$, also called signal vectors, are independent.

## Ⅲ. The New Blind Multiuser Detection Scheme

Linear multiuser detector is composed of a correlator and a decision threshold, that is

$$b = \mathrm{sgn}(w^T r), \qquad (4)$$

where $w \in R^N$. For any $\mu > 0$, linear detector $w$ and $\mu w$ are equivalent It's easy to deduce that their error rates, asymptotic efficiencies and signal-to-interference rates are the same.

The data receiving of user 1 is considered. A linear MMSE detector $w_{MSE}$ is obtained by minimum mean square error (MSE) [3].

$$MSE(w) = E\{(w^T r - A_1 b_1)^2\} \qquad (5)$$

and

$$w_{MSE} = A_1^2 R^{-1} s_1 \qquad (6)$$

where

$$\mathbf{R} = E\{\mathbf{r}\mathbf{r}^T\} = \sum_{k=1}^{K} A_k^2 \mathbf{s}_k \mathbf{s}_k^T + \sigma^2 \mathbf{I}_N \qquad (7)$$

is autocorrelation matrix of receiving signal When directly solving detector $w_{MSE}$, we require to know the training sequence, so there is no method to realize the blind multiuser receiving

According to constrained MOE criterion [2],

$$MOE(w) = E\{(w^T r)^2\},\ s.t.\ w^T s_1 = 1 \qquad (8)$$

A blind multiuser receiving scheme of linear MMSE detector is given. The optimization detector satisfying above criterion is

$$\mathbf{w}_{MOE} = \frac{1}{\mathbf{s}_1^T \mathbf{R}^{-1} \mathbf{s}_1} \mathbf{R}^{-1} \mathbf{s}_1 \qquad (9)$$

MOE criterion is a constrained optimization problem which requires detector to satisfy standard criterion form, that is, $w^T s_1 = 1$. But in fact, the criterion form is nonessential, and its substance is to require that $w$ and $s_1$ locate in the same quadrant in space, that is, $w^T s_1 > 0$. According to the idea, we consider the following optimization criterion

$$J(w) = \mathrm{E}\{(w^T r)^2\} + (w^T s_1 - m)^2, \qquad (10)$$

where $m > 0$ is any constant. And it may be proved that $J(\mathbf{w})$ has unique extreme point

$$\mathbf{w}_{min} = \frac{m}{1 + \mathbf{s}_1^T \mathbf{R}^{-1} \mathbf{s}_1} \mathbf{R}^{-1} \mathbf{s}_1 \qquad (11)$$

Comparing the expressions (6) and (11), we know that linear detector $w_{min}$ is equivalent to MMSE detector $w_{MSE}$. Therefore, according to optimization (10), we can obtain a blind multiuser receiving scheme of linear MMSE detector

The derivation process of expression (11) is given in the following. According to grads of $J(w)$, we get

$$\begin{aligned} \nabla J(\mathbf{w}) &= 2E\{\mathbf{r}\mathbf{r}^T\}\mathbf{w} + 2(\mathbf{w}^T \mathbf{s}_1 - m)\mathbf{s}_1 \\ &= 2(\mathbf{R} + \mathbf{s}_1 \mathbf{s}_1^T)\mathbf{w} - 2m\mathbf{s}_1 \end{aligned} \qquad (12)$$

Therefore we know

$$\begin{aligned} \mathbf{w}_{min} &= m(\mathbf{R} + \mathbf{s}_1 \mathbf{s}_1^T)^{-1} \mathbf{s}_1 \\ &= m(\mathbf{I}_N + \mathbf{R}^{-1} \mathbf{s}_1 \mathbf{s}_1^T)^{-1} \mathbf{R}^{-1} \mathbf{s}_1 \end{aligned}$$

$$= m[\mathbf{I}_N - \mathbf{R}^{-1}\mathbf{s}_1(1+\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s}_1)^{-1}\mathbf{s}_1^T]\mathbf{R}^{-1}\mathbf{s}_1$$
$$= m[\mathbf{R}^{-1}\mathbf{s}_1 - \mathbf{R}^{-1}\mathbf{s}_1(1+\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s}_1)^{-1}\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s}_1]$$
$$= \frac{m}{1+\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s}_1}\mathbf{R}^{-1}\mathbf{s}_1 \qquad (13)$$

The above third equation is obtained by inverse operation formula of the following matrix.

$$(I+ab^T)^{-1}=I-a(1+b^Ta)^{-1}b^T \qquad (14)$$

***Remark 1***: Optimization criterion (10) without constrained condition doesn't require to detector to satisfy standard criterion form Its second term insures that directions of $w$ and $s_1$ are identical, which plays the pole of orientation, that is, its projection $w^Ts_1>0$ Therefore, optimization criterion (10) may be named as oriented minimum output energy (OMOE) criterion

***Remark 2***: Linear detector multiplies a positive scaling factor, and its performance doesn't change Therefore, according to optimization (10), we know the selection of the parameter $m$ doesn't affect the performance of the detector

***Remark 3***:

When $m=1+1/(\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s})$, $\mathbf{w}_{min}^T\mathbf{s}_1=1$ where $\mathbf{w}_{min}=w_{MOE}$.

***Remark 4***:

When $m=A_1^2(1+\mathbf{s}_1^T\mathbf{R}^{-1}\mathbf{s}_1)$, $w_{min}=w_{MSE}$.

## Ⅳ Blind Adaptive Algorithm

Using steepest-descent algorithm [3], we can get a blind adaptive algorithm of linear MMSE detector basing on optimization criterion (10).

$$\mathbf{w}(n+1)=\mathbf{w}(n)-\frac{1}{2}\mu\nabla J(\mathbf{w}(n)) \qquad (15)$$

where $\nabla J(\mathbf{w})$ is obtained from (12) It can be proved that above algorithm converges if and only if

$$0<\mu<\frac{2}{\lambda_{\max}}, \qquad (16)$$

where $\lambda_{\max}$ is the maximum eigenvalue of matrix $\mathbf{R}+\mathbf{s}_1\mathbf{s}_1^T$

The convergence of the algorithm is proved in the following. Define vector error

$$\mathbf{e}(n)=\mathbf{w}(n)-\mathbf{w}_{min} \qquad (17)$$

Because of $\nabla J(\mathbf{w}_{min})=0$, equation (14) is equivalent to

$$\mathbf{e}(n+1)=\mathbf{e}(n)-\mu[\nabla J(\mathbf{w}(n))-\nabla J(\mathbf{w}_{min})] \qquad (18)$$

According to (12), we further get

$$\mathbf{e}(n+1)=[\mathbf{I}_N-\mu(\mathbf{R}+\mathbf{s}_1\mathbf{s}_1^T)]\mathbf{e}(n) \qquad (19)$$

Because matrix $\mathbf{R}+\mathbf{s}_1\mathbf{s}_1^T$ is positive definite, there exists orthogonal matrix $\mathbf{Q}$ which makes $\mathbf{R}+\mathbf{s}_1\mathbf{s}_1^T=\mathbf{Q}^T\Lambda\mathbf{Q}$ where $\Lambda=diag(\lambda_1,\lambda_2,\Lambda,\lambda_N)$ is diagonal matrix, and $\lambda_i$ is the eigenvalue of matrix $\mathbf{R}+\mathbf{s}_1\mathbf{s}_1^T$ If we define $\mathbf{u}(n)=\mathbf{Q}\mathbf{e}(n)$, then equation (18) may be changed as

$$\mathbf{u}(n+1)=(\mathbf{I}_N-\mu\Lambda)\mathbf{u}(n) \qquad (20)$$

or

$$u_i(n+1)=(1-\mu\lambda_i)u_i(n), \quad i=1,\ldots,N. \qquad (21)$$

Therefore, iteration process (19) converges if and only if

$$0<\mu<\frac{2}{\lambda_i}, \quad i=1,\ldots,N, \qquad (21)$$

that is, condition (15) comes into existence Because the convergence of algorithm (14) and iteration process (18), (19) are equivalent, algorithm (14) converges if and only if condition (15) comes into existence

In the real application, the autocorrelation $\mathbf{R}$ of receiving signal can use its estimate value

$$\hat{R}(n)\triangleq\frac{1}{n}\sum_{i=1}^{n}r(i)\,r(i)^T \qquad (23)$$

to calculate. The blind adaptive algorithm of linear MMSE detector based on optimization criterion (10) is summarized as the following:

**Algorithm:**

$$\hat{R}(n) = \frac{n-1}{n}\hat{R}(n-1) + \frac{1}{n} r(n) r(n)^T \qquad (24)$$

$$\frac{1}{2}\nabla J(\mathbf{w}(n)) = [\hat{\mathbf{R}}(n) + \mathbf{s}_1\mathbf{s}_1^T]\mathbf{w}(n) - m\mathbf{s}_1 \qquad (25)$$

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2}\mu\nabla J(\mathbf{w}(n)) \qquad (26)$$

$$\hat{R}(0) = \mathbf{I}_N \qquad (27)$$

$$\mathbf{w}(0) = \mathbf{w}_0 \qquad (28)$$

## V Simulation Analysis

In the following, the performance of blind adaptive detector given in this paper is analysed through computer simulation The performance measure is the output signal-to-interference ratio (*SIR*)

$$SIR(n) = \frac{E^2\{\mathbf{w}(n)^T r\}}{Var\{\mathbf{w}(n)^T r\}} = \frac{A_1^2[\mathbf{w}(n)^T s_1]^2}{\sum_{k=2}^{K} A_k^2[\mathbf{w}(n)^T s_k]^2 + \sigma^2 \mathbf{w}(n)^T \mathbf{w}(n)}. \qquad (29)$$

We consider a synchronous DS/CDMA system with processing gain $N$=10 and number of user $K$=5 The spreading sequences $\{c_j^k, j=0, 1, \ldots, N-1\}$ are randomly generated The desired user is user 1, and other users are multiple-access interference (MAI) The MAI intensity is $MAI_k = 10\log_{10}(A_k^2/A_1^2)$=20 dB ($k$=2~6) The channel interference is white Gaussian noise, and the signal-to-noise ratio is $SNR_1 = 10\log_{10}(A_1^2/\sigma^2)$=20 dB Simulation results are shown in Fig.1 where all data is the averaged value of 30 simulation results Curves represent SIRs of OMOE and MOE, and straight dashed represents optimization value of MMSE detector.

In order to compare conveniently, we use the steepest descent methods for all iteration algorithms and choose the same steps From Fig.1, the performance of OMOE and MOE corresponds and is close to optimization value of MMSE detector

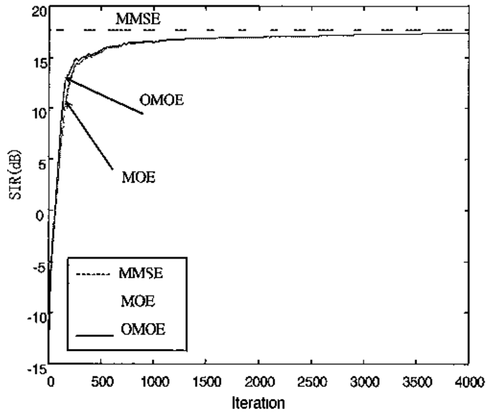


Fig 1 Output SIR of detector

## Ⅵ. Conclusion

In this paper, basing on an OMOE criterion, we have presented a new blind receiving scheme of linear MMSE detector. OMOE without constrained condition doesn't require detector to satisfy standard criterion form, which makes it convenient to find solution The steepest descent method is used, corresponding blind adaptive detection algorithm is given, and the convergence condition of the algorithm is analysed It is shown by simulation results that the performance of OMOE and MOE are equivalent, and OMOE possesses strong capability to constrain interference

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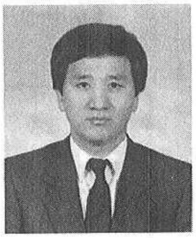

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