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# 멀티웨이브렛 시스템의 전후처리 보간 필터 설계 및 영상 압축 응용

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# Interpolation Prefilters and Postfilters for Multiwavelet Systems and Application for Image Compression

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## ABSTRACT

As a multiwavelet filter bank has multiple channels of inputs, we investigate the data initialization problem by considering prefilters and postfilters that may give more efficient representations of the decomposed data. The interpolation postfilter and prefilter are formulated, which are capable of providing a better approximate image at each coarser resolution level. A design process is given to obtain both filters having compact supports. Computer simulations are performed to assess the performance of the proposed multiwavelet systems using prefiltering and postfiltering, and we have found that the image compression performances of proposed multiwavelet systems are superior than those of the single wavelet systems in terms of PSNR.


## I. Introduction

Introduced in early 1980's, the wavelets have become an intensely studied subject for its applications to signal processing. One of the most common applications of wavelets has been audio or visual data compression as multiresolutional analysis decomposes a signal over multiple time-frequency channels [1-5].

In signal analysis, we desire a symmetric or antisymmetric wavelet that gives a high order approximation. However, the orthogonal wavelets with compact support are not symmetric. Recently, multiwavelet systems have been introduced, where multiple scaling functions and multiple wavelets are used. A multiwavelet system can combine symmetry and shorter supports with a high approximation order and high regularity, which was not possible with a single wavelet system [6-9].

As the filters of multiwavelet systems are of matrix form and each filter has multiple channels of inputs, we encounter a new problem of how to input the data. This initialization problem has been taken up by several researches for applications to signal compression [10-12], but very little experimental result was reported for image compression because of the requirement for an excessive computational effort.

In this paper, we present an application of multiwavelet analysis to image data and undertake a general study on preconditioning a multiwavelet system. Our objective is to apply a multiwavelet transform on a 2-D image for data compression. As matrices do not commute in multiplication, we derive multiwavelet decomposition and reconstruction algorithms with a great care to yield the perfect reconstruction condition of the analysis and synthesis matrix filters. Conditions on inter-

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polation prefiltering and postfiltering are given. These are applied to several specific multiwavelet systems to examine their image compression performances in comparison to single wavelet systems.

The paper is organized as follows. In Section 2, multiwavelet theories are reviewed, and multiwavelet decomposition and reconstruction algorithms are presented. In Section 3 we exploit the problem of preconditioning multiwavelet systems where we define a compact data structure for vector-valued sequences, and design prefilters and postfilters for severalmultiwavelets systems. The experimental results on image compression using the prefilters and postfilters on several multiwavelet systems are given in Section 4. Finally, the conclusions are made in Section 5.

## Ⅱ. Multiwavelet System

In this section basic theories on multiwavelet systems are presented. In order to obtain a complete characterization of multiwavelet analysis, multiscaling functions and multiwavelets are introduced, and the multiresolutional decomposition and reconstruction relations are discussed.

### 2.1 Multiscaling Functions and Multiwavelets

Let us define a multiresolution analysis of $L^2(R)$ generated by several scaling functions, with an increasing sequence of function subspaces $\{V_j\}_{j\in Z}$ in $L^2(R)$:

$$\{0\}\subset \ldots \subset V_{-1}\subset V_0 \subset V_1 \subset \ldots \subset L^2(R) \tag{1}$$

Subspaces $V_j$ are generated by a set of scaling functions $\phi^1,\phi^2,\ldots,\phi^r$ (namely, multiscaling functions) such that

$$V_j := clos_{L^2(R)} < \phi^m_{j,k} : 1\le m\le r, k\in Z >, \quad \forall j\in Z \tag{2}$$

i.e., $V_j$ is the closure of the linear span of $\{\phi^m_{j,k}\}_{1\le m\le r,k\in Z}$ in $L^2(R)$, where

$$\phi^m_{j,k}(x) := 2^{j/2}\phi^m(2^j x-k), \ \forall x\in R \tag{3}$$

Then we have a sequence of multiresolution subspaces $\{V_j\}$ generated by a set of multiscaling functions, where the resolution gets finer and finer as j increases.

Let us define inter-spaces $W_j\subset L^2(R)$ such that $V_{j+1} := V_j \oplus W_j, \ \forall j\in Z,$ where the operator $(\oplus)$ denotes a nonorthogonal direct sum. $W_j$ is the complement to $V_j$ in $V_{j+1}$, and thus $W_j$ and $W_l$ with $j\neq l$ are disjoint but may not be orthogonal to each other. If $W_j \perp W_l, \ \forall j\neq l$, we call them semi-orthogonal wavelet spaces[13]. By the nature of construction, subspaces $W_j$ can be generated by r base functions, $\psi^1,\psi^2,\ldots,\psi^r$ that are multiwavelets. The subspace $W_j$ is the closure of the linear span of $\{\psi^m_{j,k}\}_{1\le m\le r,k\in Z}$ :

$$W_j := clos_{L^2(R)} < \psi^m_{j,k} : 1\le m\le r, k\in Z >, \quad \forall j\in Z \tag{4}$$

where

$$\psi^m_{j,k}(x) := 2^{j/2}\psi^m(2^j x-k), \ \forall x\in R \tag{5}$$

We may express multiscaling functions and multiwavelets as vector functions:

$$\phi(x) := \begin{pmatrix}\phi^1(x)\\ \vdots \\ \phi^r(x)\end{pmatrix} \quad \psi(x) := \begin{pmatrix}\psi^1(x)\\ \vdots \\ \psi^r(x)\end{pmatrix}, \quad \forall x\in R \tag{6}$$

Also, in a vector form, let us define

$$\phi_{j,k}(x) := 2^{j/2}\phi(2^j x-k) \text{ and}$$
$$\psi_{j,k}(x) := 2^{j/2}\psi(2^j x-k), \quad \forall x\in R \tag{7}$$

Since the multiscaling functions $\phi^m\in V_O$ and the multiwavelets $\psi^m\in W_O$ are all in $V_1$, and since $V_1$ is generated by

$\{\phi_{1,k}^m(x) = 2^{1/2}\phi^m(2x-k)\}_{1\le m\le r, k\in Z}$,

there exist two $l^2$ matrix sequences $\{H_n\}_{n\in Z}$ and $\{G_n\}_{n\in Z}$ such that we have a two-scale relation for the multiscaling functions $\phi(x)$:

$$\phi(x) = 2\sum_{n\in Z} H_n\phi(2x-n), \quad x\in R \tag{8}$$

which is also called as a two-scale matrix refinement equation (MRE), and for multiwavelet $\psi(x)$:

$$\psi(x) = 2\sum_{n\in Z} G_n\phi(2x-n), \quad x\in R \tag{9}$$

where $G_n$ and $H_n$ are r×r square matrices. Moreover, since all elements of both $\phi(2x)$ and $\phi(2x-1)$ are in $V_1$ and $V_1 = V_0 \oplus W_0$, there exist two $l^2$ matrix sequences $\{H_n\}_{n\in Z}$ and $\{G_n\}_{n\in Z}$ such that

$$\phi(2x-k) = \sum_{n\in Z}\left[H_{k-2n}^T\phi(x-n) + G_{k-2n}^T\psi(x-n)\right], \quad \forall k\in Z, \tag{10}$$

which is called the decomposition relation of $\phi$ and $\psi$. We here intentionally transposed the matrices of $H$ and $G$ and reversed indexing instead of 2n-k, for some convenience in representing formulas of dual relationship.

In the middle of 1990's Geronimo, Hardin, and Massopust successfully constructed a very important multiwavelet system using the fractal interpolation [6,14]. The GHM multiwavelet is a unique pair of sequences $(\{H_n\},\{G_n\})$ that can generate multiscaling functions and multiwavelets and thus multiwavelet subspaces. Its scaling functions and wavelets are orthogonal, very shortly supported, symmetric or anti-symmetric, and it has second order approximation so that locally constant and locally linear functions are in $V_j$. Another example of orthogonal multiwavelet is cardinal m-balanced orthogonal multiscaling and multiwavelet systems, where m stands for the approximation order of the cardinal balanced orthogonal multiwavelet systems. For more details on cardinal balanced orthogonal multiwavelets, refer to the paper written by Selesnick [15].

### 2.2 Multiwavelet Decomposition and Reconstruction

From the formulas (8), (9) and (10), the following signal decomposition and reconstruction algorithms can be derived. Let $v_j \in V_j$ and $w_j \in W_j$ so that

$$v_j(x) := \sum_{k\in Z} c_{j,k}\cdot\phi(2^j x-k) = \sum_{k\in Z} c_{j,k}^T\phi(2^j x-k): \tag{11}$$

$$w_j(x) := \sum_{k\in Z} d_{j,k}\cdot\psi(2^j x-k) = \sum_{k\in Z} d_{j,k}^T\psi(2^j x-k) \tag{12}$$

where (.) denotes a dot product between two vectors and $(^T)$ denotes the transpose operator. The scale factor $2^{j/2}$ is not explicitly shown here for simplicity but incorporated into data sequences $c_{j,k}$ and $d_{j,k}$. By the relation $V_j := V_{j-1}\oplus W_{j-1}$

$$\begin{aligned} v_j(x) &:= v_{j-1}(x) + w_{j-1}(x) \\ &= \sum_{k\in Z} c_{j-1,k}\cdot\phi(2^{j-1}x-k) + \sum_{k\in Z} d_{j-1,k}\cdot\psi(2^{j-1}x-k), \quad \forall j\in Z. \end{aligned} \tag{13}$$

Thus we have the following recursive decomposition (analysis) formulas:

$$c_{j-1,k} = \sum_n H_{n-2k}c_{j,n} = \sum_n H_{-n}c_{j,2k-n}, \quad \forall j\in Z \tag{14}$$

$$d_{j-1,k} = \sum_n G_{n-2k}c_{j,n} = \sum_n G_{-n}c_{j,2k-n}, \quad \forall j\in Z \tag{15}$$

An original data sequence $c_0 (= \{c_{0,k}\}_k)$ is decomposed into $c_1$ and $d_1$ data sequences, and the sequence $c_1$ is further decomposed into $c_2$ and $d_2$ data sequences. Keeping this process recursively, the original sequences $c_0$ is decomposed into $d_1$, $d_2$, and $d_3$, and on. Note that this process continuously reduces the data size by half for each decomposed sequence but it conserves the total data size.

Let $D_K, K \geq 1$, be the subsampling (downsampling) operator defined by

$$(D_K x)[n] := x[Kn] \tag{16}$$

where $K$ is a subsampling rate and $x$ is a sequence of vector-valued samples.

The decomposition formulas can be rewritten in the Z-transform domain as

$$c_{j-1}(z) = D_2 H^-(z) c_j(z) \tag{17}$$

$$d_{j-1}(z) = D_2 G^-(z) c_j(z) \tag{18}$$

where the superscript (-) denotes reverse indexing, i.e., $H^- := H^{*T}$.

From the two-scale relations (8), (9) and form (11), (12), we have the following recursive reconstruction (synthesis) formula:

$$c_{j,k} = 2\sum (H^T_{k-2n} c_{j-1,n} + G^T_{k-2n} d_{j-1,n}) \tag{19}$$

Let $U_K, K \geq 1$, be the upsampling operator defined by

$$(U_K x)[n] := \begin{cases} x\left[\frac{n}{K}\right], & \text{if } \frac{n}{K} \text{ is an interger} \\ 0, & \text{otherwise} \end{cases} \tag{20}$$

where $K$ is an upsampling rate and $x$ is a sequence of vector-valued samples.

Then the reconstruction formula can be rewritten in the Z-transform domain as

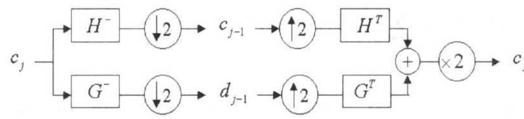
$$c_j(z) = 2\left[H^T(z) U_2 c_{j-1}(z) + G^T(z) U_2 d_{j-1}(z)\right] \tag{21}$$

The decomposition and reconstruction systems implemented by multiwavelet filter banks are shown in Figure 1, where the system (a) is the exact implementation of our equations derived. If we take reverse indexing for all filters, we have the system (b), and the multiwavelet decomposition formulas become

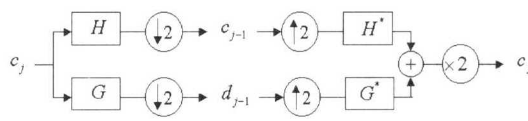
$$c_{j-1}(z) = D_2 H(z) c_j(z) \tag{22}$$

$$d_{j-1}(z) = D_2 G(z) c_j(z) \tag{23}$$

and the reconstruction formula becomes



(a) Filter banks derived from multiwavelet analysis



(b) Multiwavelet filter banks by reverse indexing

Fig. 1 The multiwavelet transform filter banks. Filters are $r \times r$ matrices and data paths are r lines, where $r = 2$ in our examples. The multiwavelet systems (a) and (b) are equivalent, except that filter indices are all reversed between the two systems.

$$c_j(z) = 2\left[H^*(z) U_2 c_{j-1}(z) + G^*(z) U_2 d_{j-1}(z)\right] \tag{24}$$

Note that the input data $c_j$ is a sequence of vector-valued data. Every data path has r lines, and filters are $r \times r$ matrices. We restrict $r = 2$ in this paper.

Constructing a vector valued sequence $c_j$ from a signal or an image is nontrivial. As a 1-D input signal is vectorized, the direction of filter indexing will affect the reconstructed signal in an undesirable way, if the vectorization scheme does not match with filter indexing. This effect does not happen in a scalar wavelet system, whose filters are not matrices. As we do not take reverse indexing for data sequences, we will take the system (a) of Figure 1 in our implementation.

## Ⅲ. Preconditioning Multiwavelet Systems

In this section we consider multiwavelet systems that analyze discrete data, and investigate how to precondition a multiwavelet system by prefiltering input data, which is not necessary for the case of single (or scalar) wavelet systems.

### 3.1 Prefilters and Postfilters

Consider the multiwavelet series expansion :

$$f_i(t) := \sum_k c_{j,k}^T \phi(2^j t - k) \tag{25}$$

From a given 1-D signal $x[n]$, construct a vector-valued sequence $\mathbf{x}[n]$ by

$$\mathbf{x}[n] := \begin{bmatrix} x[nr] \\ \vdots \\ x[nr+r-1] \end{bmatrix}, \quad r \geq 1 \tag{26}$$

Let us define a prefilter $Q(z)$, which maps a vector-valued sequence space onto itself, such that the coefficient vector sequence $c_{0,k}$ is obtained by filtering $\mathbf{x}[n]$:

$$c_0(z) = Q(z)x(z) \tag{27}$$

For any $j \leq 0$, $c_{j,k}$ is decomposed to $\{c_{j-1,k}, d_{j-1,k}\}$ by a layer of multiwavelet decomposition. Recursive multiwavelet decompositions down to a resolution level $J < 0$ give us a set of decomposed data sequences $c_{J,k}$ and $\{d_{j,k}\}_{J \leq j < 0}$. Recursive multiwavelet reconstruction from the decomposed data set gives the original coefficient vector $c_{o,k}$.

Then $x(z)$ is reconstructed by applying a postfilter $P(z)$:

$$x(z) = P(z)c_o(z) \tag{28}$$

The postfilter $P$ must be an inverse of the prefilter $Q$ up to some unit delay for the perfect reconstruction:

$$P(z)Q(z) = z^{-l}\mathrm{I}, \quad \text{for some integer } l. \tag{29}$$

We may assume $l = 0$ (no delay) for convenience.

Define

$$x_o(z) := x(z) \quad \text{and} \quad x_j(z) := P(z)c_j(z) \tag{30}$$

Then $\{x_j\}_{j<0}$ are the projections of $x$ onto (discrete-time) multiscaling spaces at lower resolutions. This implies that a postfilter should be

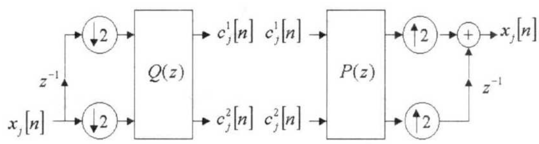



Fig. 2 Prefilter and postfilter blocks. A unit delay and downsampling in a prefilter block (a) vectorize the 1-D input data sequence $x_j[n]$ to a vector-valued sequence, where the prefilter output $[c_j^1[n]\, c_j^2[n]]$ is the input to multiwavelet decomposition filter banks. A unit delay and upsampling in a postfilter block (b) serialize the two-channel postfilter output vector sequence to the 1-D output signal $x_j[n]$, where $[c_j^1[n]\, c_j^2[n]]$ are from the outputs of multiwavelet reconstruction filter banks.

applied to a coefficient vector $c_j$ if we want to see a decomposed signal at the resolution level $j < 0$.

For an r-channel multiwavelet system, the construction of a vector-valued input sequence from an 1-D signal can be implemented in a prefilter block by serial-to- parallel conversion (vectorization) by using r-1 unit delays and then downsampling each channel at the rate r. The block diagrams of prefilter and postfilter for a 2-channel multiwavelet system are shown in Figure 2.

### 3.2 Interpolation Prefilter and Postfilter

In the multiwavelet case, however, in order to avoid the undesirable visual effect, we need a prefilter that computes multiscaling coefficient sequence $c_{o,k}$ from a discrete-time input signal before starting the multiwavelet decomposition [10-12]. Here, we develop a process of finding a pair of prefilter and postfilter such that

$$f_o(t) := \sum_k c^T{}_{o,k}\phi(t-k) \tag{31}$$

interpolates an original signal $x_o[n]$. Since we have r scaling functions, a continuos-time signal $f_o(t)$ is sampled at the interval of 1/r at the 0-th resolution level:

$$f_o(\frac{n}{r}) = \sum_{k\in Z} c^T{}_{o,k}\phi(\frac{n}{r}-k) = \sum_{k\in Z}\phi(\frac{n}{r}-k)^T c_{o,k} \tag{32}$$

and we impose an interpolation property by $f_o[\frac{n}{r}] = x_o[n]$. We construct vector-valued sequences $f_o[n]$ and $x_o[n]$ from the sampled sequence $f_o(n/r)$ and the 1-D signal $x_o[n]$, respectively:

$$f_o[n] := \begin{bmatrix} f_o(n) \\ f_o(n+\frac{1}{r}) \\ \mathrm{M} \\ f_o(n+\frac{r-1}{r}) \end{bmatrix}$$

$$x_o[n] := \begin{bmatrix} x_o[nr] \\ x_o[nr+1] \\ \mathrm{M} \\ x_o[nr+r-1] \end{bmatrix} \tag{33}$$

then the interpolation condition $f_o[\frac{n}{r}] = x_o[n]$ gives the following relation :

$$f_o[n] = x_o[n] = \sum_{k\in Z} P_{n-k}c_o[k] = \sum_{k\in Z} P_k c_o[n-k] \tag{34}$$

where $P_n$ is an r×r matrix sequence and defined by

$$P_n := \begin{bmatrix} \phi(n)^T \\ \phi(n+\frac{1}{r})^T \\ \mathrm{M} \\ \phi(n+\frac{r-1}{r})^T \end{bmatrix} \tag{35}$$

This is an interpolation postfilter that maps the space of scaling coefficients $c_j[k]$ to the space of sampled signals $f_j[n]$. At any resolution level $j$, a decomposed signal can be obtained by filtering scaling coefficients $c_j[k]$ by the postfilter $P_n$:

$$x_j[n] = \sum_{k\in Z} P_{n-k}c_j[k] = \sum_{k\in Z} P_k c_j[n-k] \tag{36}$$

This relation is expressed in the Z-transform domain as

$$x_j(z) = P(z)c_j(z) \tag{37}$$

where $P(z) := \sum_n P_n z^{-n}$. By (35), $P_n$ is a finite sequence (FIR filter) if the scaling vector function $\phi$ is compactly supported.

Furthermore, we define a prefilter $Q(z)$ such that

$$Q(z)P(z) = P(z)Q(z) = \mathrm{I}_r \tag{38}$$

Then the scaling coefficient $c_j(z)$ is obtained by filtering the signal $x_j(z)$ :

$$c_j(z) = Q(z)x_j(z) \tag{39}$$

To have an FIR solution to the above condition (38), det ($P(z)$) must have the form of det $(P(z)) = \alpha z^{-l}$, where $\alpha$ is a constant and $l$ is an integer.

For the GHM orthogonal multiwavelet system, an interpolation postfilter $P$ is obtained from the GHM scaling functions:

$$P(z) = \begin{bmatrix} 0 & 1.73210618z^{-1} \\ 1.95965444 & -0.51963185 - 0.519631854z^{-1} \end{bmatrix} \tag{40}$$

The corresponding prefilter $Q$ is computed from the condition $P(z)Q(z) = \mathrm{I}$,

$$Q(z) = P^{-1}(z) = \begin{bmatrix} 0.15309232z + 0.15309232 & 0.51030773 \\ 0.57735174z & 0 \end{bmatrix} \tag{41}$$

For the cardinal 2-balanced orthogonal multi-wavelet system, we obtain the postfilter and prefilter as

$$P(z)=\begin{bmatrix}0 & \sqrt{2}z^{-2}\\ \sqrt{2}z^{-1} & 0\end{bmatrix} \qquad Q(z)=\begin{bmatrix}0 & z/\sqrt{2}\\ z^{2}/\sqrt{2} & 0\end{bmatrix} \quad (42)$$

## Ⅳ. Computer Simulations

With the prefilters and postfilters that we have designed for multiwavelet systems, we have investigated the applications of these systems to image compression and examined their compression performances.

Computer simulations are performed on several levels of compression ratios using multiwavelet systems (two GHMs and cardinal 2-balanced) in comparison to some single wavelet systems (Daubechies'D4 and D6 orthogonal wavelets). We consider a simple compression scheme with a uniform quantizer, which removes a certain number of small values from high-passed subimages but keeps the larger values to achieve a specified compression ratio. We used six test images (515×512×8-bit) including Lena, Airplane, Baboon, Peppers, Sailboat, and Wavy in our experiments. Our experiments suggested that wavelet decomposition up to the 3rd or 4th level would give a reasonably high compression ratio and a good reconstruction.

To describe the image fidelity, PSNR (peak signal-to noise ratio) is defined by

$$\text{PSNR(dB)}:=20\ \log\left(255/\sqrt{\frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{M}(f[i,j]-s[i,j])^2}\right) \quad (43)$$

where $f$ is a M×N reconstructed image and $s$ is the M×N original image. The PSNR values shown in Table 1 are the average values taken from the experimental results for the six test images at each given compression ratio.

From the simulation results, we observe that the 4-coefficient GHM orthogonal multiwavelet systems perform better than Daubechies' D4 wavelet (4 coefficients). The cardinal 2-balanced orthogonal multiwavelet filter (6 coefficients) gives a better compression performance than Daubechies' D6 (6 coefficients). When a certain degree of image fidelity, i.e., PSNR = 28 dB, is desired, the cardinal 2-balanced multiwavelet system with interpolation prefiltering-postfiltering gives the highest compression ratio than any other wavelet systems. For some GHM orthogonal multiwavelet systems, the interpolation prefiltering-postfiltering provides a slightly better compression performance than an orthogonal 2nd-order approximation prefiltering. In general, orthogonal multiwavelet systems perform better than single wavelet systems with comparable support lengths, but with a small overhead of computation.

Table 1. Performance comparison of wavelets in image compression

| Compr Ratio | PSNR(dB) | | | | |
|---|---|---|---|---|---|
| | Multiwavelets | | | Single Wavelets | |
| | GHM (o) | GHM (i) | CardBal2 | D4 | D6 |
| 2 | 47.93 | 48.93 | 48.31 | 47.41 | 48.23 |
| 4 | 40.50 | 41.01 | 41.33 | 39.48 | 41.23 |
| 8 | 35.72 | 36.13 | 37.04 | 34.76 | 36.87 |
| 16 | 31.89 | 32.26 | 32.92 | 30.96 | 32.57 |
| 32 | 28.35 | 28.79 | 29.30 | 27.62 | 28.81 |
| 64 | 25.60 | 26.03 | 26.07 | 24.99 | 25.53 |
| 128 | 23.04 | 23.38 | 23.38 | 26.69 | 23.11 |
| 256 | 20.57 | 20.57 | 20.79 | 20.56 | 20.67 |
| Prefilter | Orthogonal | Interpolation | Interpolation | N/A | |

## Ⅴ. Conclusions

In this paper data intialization problem for disctete-time multiwavelet systems has been approached by preconditioning the multiwavelet systems using interpolation prefilters and postfilters. A design process for interpolation prefilter-postfilter for GHM and cardinal m-balanced multiwavelet systems has been developed. These filters must be of the finite impulse response type, or else, an orthogonal prefilter of some approximation order can be designed. Using these filters, image compression performances of orthogonal multiwavelet systems have been assessed in comparison to the single wavelet systems. In general, the orthogonal multiwavelet systems perform better than the single wavelet systems with comparable support lengths, but with a small overhead of computation.

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