

Blind Equalization using Higher-Order Statistics and Fuzzy-ARTMAP

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ABSTRACT

This paper discusses a blind equalization technique for FIR channel system, that might be minimum phase or not, in digital communication. The proposed techniques consist of two parts. One is to estimate the original channel coefficients based on fourth-order cumulants of the channel output, the other is to employ fuzzy-ARTMAP neural network to model an inverse system for the original channel. Here, the estimated channel is used as a reference system to train the fuzzy-ARTMAP. The proposed fuzzy-ARTMAP equalizer provides fast and easy learning, due to the structural efficiency of fuzzy-ARTMAP; a small number of parameters, automatic increase of hidden units, and the capability of adding new data without retraining previously trained data. In simulation studies, the performance of the proposed blind equalizer is compared with both linear and other neural basis equalizers.

I. Introduction

Intersymbol Interferences (ISI) arises in pulse modulation systems whenever the energy of one received pulse does not die away completely before the beginning of the next. ISI may be caused by band limiting (as, for example, in telephone channels) or frequency selectivity (fading or multipath propagation) as in digital microwave radio and in mobile communication systems. The most widely known equalizer is an adaptive transversal equalizer, in which output signal is compared to the expected signal and FIR filter coefficients are adjusted in accordance with the error between the desired and actual filter output.

For the last three decades, many of blind equalizers that do not use the known training sequence have been proposed in the literature beginning with Sato^[1], because there are some practical situations when the conventional adaptive algorithms are not suitable for wireless communications during an outage (caused by severe fading).

Most current blind equalization techniques use higher order statistics (HOS) of the received sequences, directly or indirectly, because they are efficient tools for identifying that may be the nonminimum phase^[2,3]. The HOS based techniques have the capability to identify a nonminimum phase system simply from its output because of the property of polyspectra to preserve not only the magnitude but also the phase information of the signal.

As a new approach for channel equalization, many researchers have been concerned with applying neural networks, such as multilayer perceptrons (MLP) and radial basis functions (RBF), to both the conventional and blind equalization systems^[4-7]. However, each of these networks internally has significant shortcomings. MLP basis equalizer typically require long training and are sensitive to the initial choice of network parameters (especially initial weights). On the other hand, RBF basis equalizer is simpler and fast to train, but usually require a large number of centers, which increases the complexity of computation. In addition, it is not easy to

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determine with the number and the location of centers required for training.

This paper develops a new method to solve the problems of blind equalization, by combining the advantages of HOS and a fuzzy-ARTMAP neural network. The main purpose of the proposed blind fuzzy-ARTMAP equalize is to solve the obstacles of long time training and complexity that are often encountered in the blind MLP equalizers^[7]. The proposed techniques firstly estimates the order and coefficients of the original channel based on the autocorrelation and the fourth-order cumulants of the received signals. Then, an equalizer system using a fuzzy-ARTMAP neural network is trained with input sequences from the estimated channel models. Section II presents the problem assumptions and the cumulant-based channel estimation algorithms are described. In the Section III, a brief summary of fuzzy-ARTMAP network is presented. Section IV gives the learning procedure for the blind fuzzy-ARTMAP equalizer. Simulation results are provided in Section V and Section VI gives the conclusions.

II. Channel estimation

The block diagram of a base-band communication system subject to Intersymbol interference (ISI) and additive white Gaussian noise (AWGN) is shown in Fig. 1.

Assume that the received signal $\{y_k\}$ is generated by an FIR system described by

$$y_k = \sum_{i=0}^p h_i s_{k-i} + n_k = \hat{y}_k + n_k \quad (1)$$

Where $\{h_i\}, 0 \leq i \leq p$ is the impulse response of the channel and $\{s_k\}$ is i.i.d., nonGaussian. $\{s_k\}$ could be a two-level symmetric PAM sequence. The additive noise $\{n_k\}$ is zero mean, Gaussian, and statistically independent of $\{s_k\}$.

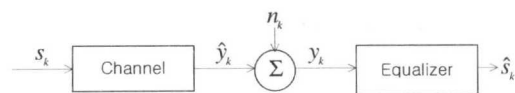


Fig. 1 Equalizer system

1. Estimation of channel order

The autocorrelation technique is used to estimate the channel order that is required to specify the number of centers in an RBF equalizer. Consider the autocorrelation ρ_l .

$$\rho_l = \begin{cases} 1, & l=0 \\ \frac{\sum_{i=0}^{p-l} h_i h_{i+l}}{\left(\sum_{i=0}^p h_i^2 + \sigma_n^2\right)}, & 1 \leq l \leq p \\ 0, & l > p \end{cases} \quad (2)$$

where $\sigma_n^2 = E[n_k^2]$ is noise variance, l is correlation lag. As shown in (2), the basic idea of the autocorrelation technique for channel order estimation is that when the lag l exceeds the correct order of the channel model, the autocorrelation becomes zero.

For convenience, the normalized sample autocorrelation is considered.

$$\hat{\rho}_l = \frac{\sum_{k=1}^{N-l} (y_k - \bar{y})(y_{k+l} - \bar{y})}{l \sum_{k=1}^N (y_k - \bar{y})^2}, l \geq 0 \quad (3)$$

where N is the number of samples, and \bar{y} is the sample mean of y_k

$$\bar{y} = \frac{\sum_{k=1}^N y_k}{N} \quad (4)$$

A sample autocorrelation, $\hat{\rho}_l$ is regarded as meaningful if it is outside the 95 percent confidence interval

$$-1.96 \frac{1}{\sqrt{N}} \leq \hat{\rho}_l \leq 1.96 \frac{1}{\sqrt{N}} \quad (5)$$

The technique selects the last meaningful sample autocorrelation as an estimate $\hat{\rho}_l$ for the channel order.

2. Channel coefficient estimation

Using the properties of higher order cumulants and the problem assumptions, the following expressions can be written for the channel described by (1)

$$C_y(l, m, n) = C_{y_3}(l, m, n) + C_n(l, m, n) \quad (6)$$

$C_y(l, m, n)$ is the fourth-order cumulant sequence of $\{y_k\}$, which is defined as

$$C_y(l, m, n) = M_y(l, m, n) - M_y(l)M_y(n-m) - M_y(m)M_y(n-l) - M_y(n)M_y(m-l) \quad (7)$$

where the second-order moments $M_y(j)$ and the fourth-order moments $M_y(l, m, n)$ of y_k are defined as

$$M_y(j) = E[y_k, y_{k+j}], \\ M_y(l, m, n) = E[y_k, y_{k+l}, y_{k+m}, y_{k+n}] \\ , j, l, m, n = 0, \pm 1, \pm 2, \dots \quad (8)$$

since $\{n_k\}$ is Gaussian, its fourth-order cumulants has zeros, which means

$$C_y(l, m, n) = C_{y_3}(l, m, n) \quad (9)$$

with knowing this fact, it is easy to show that,

$$C_{y_3}(l, m, n) = \gamma_s \sum_{i=0}^{\rho - \max(l, m, n)} h_i \cdot h_{l+i} \cdot h_{l+m} \cdot h_{l+n} \dots \quad (10)$$

(13) can be represented as

$$CH = c \quad (11)$$

where C and c are the matrix and vector consisting of the estimated fourth-order cumulants, and H is the unknown coefficient vector. The solution of (11) can be given in an explicit form as

$$H = (C^H C)^{-1} C^H c \quad (12)$$

III. Background of the Fuzzy-ARTMAP Neural Network

Since the advent of ART (adaptive resonance theory) as a cognitive and neural theory^[9], a number of ART neural network architectures have been progressively developed. These models include ART2, fuzzy-ART, ARTMAP^[10,11].

Recently, a growing number of models

computationally synthesize properties of neural networks, and fuzzy logic. Fuzzy-ARTMAP is one such model, combined with ARTMAP and fuzzy logic^[11].

Fuzzy-ARTMAP utilizes a minimax learning rule that conjointly minimizes prediction error and maximizes generalization. As learning proceeds, the input and stored prototype of a category are said to resonate when they are sufficiently similar. When an input pattern is not sufficiently similar to any existing prototype, a new node (or hidden unit) is created to represent a new category with the input patterns as the prototype. The meaning of similarity depends on a vigilance parameter ρ , with $0 < \rho \leq 1$. If ρ is small, the similarity condition is easier to meet, resulting in a coarse categorization. On the other hand, if ρ is set close to 1, many finely divided categories are formed. By selecting the desired level for the vigilance parameter, the user has control over the performance of the work. Details of the fuzzy-ARTMAP network are given in [8].

IV. Implementation of the blind fuzzy ARTMAP Equalizer

1. Training Patterns for Blind fuzzy-ARTMAP Equalizer

A blind fuzzy-ARTMAP equalizer is implemented in this section. It uses higher-order cumulants and a fuzzy-ARTMAP neural network. Fig. 2 shows the block diagram of the blind fuzzy-ARTMAP equalizer system.

In order to train a neural network to serve as a channel equalizer, it is necessary to generate appropriate training data. The following is the training input to fuzzy-ARTMAP network, which is generated by the receiver instead of being provided by the transmitter

$$r_k = \sum_{i=0}^{\rho} h_i x_{k-i} \quad (13)$$

where x_k and r_k are input signal to the estimated channel model $\{h_i\}$ and input signal to

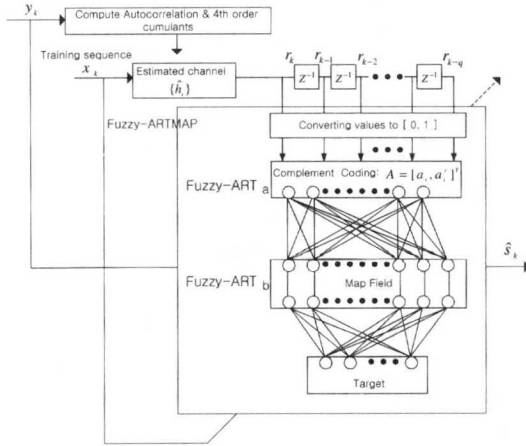


Fig. 2 The structure of blind fuzzy-ARTMAP equalizer system

the fuzzy-ARTMAP equalizer respectively. Therefore, input patterns for the network is the r_k , and the corresponding target is x_k . In this study, it is assumed that the network is trained to reconstruct the originally transmitted binary signal (1 or 1).

The estimated channel is characterized by its transfer function, which in general has the form

$$H(Z) = \sum_{i=0}^p h_i Z^{-i} \quad (14)$$

where p is the estimated channel order. If q denotes the equalizer order (number of tap delay elements in the equalizer), then there are $M = 2^{p+q+1}$ different sequences

$$x_k = [x_k, x_{k-1}, \dots, x_{k-p-q}]^T \quad (15)$$

and the corresponding received signal vectors, R_k , are represented as

$$r_k = [r_k, r_{k-1}, \dots, r_{k-q}]^T \quad (16)$$

where

$$r_k = \sum_{i=0}^q h_i x_{k-i} \quad (17)$$

The training input patterns, $\{R_i\}$, can be obtained by the following procedures

$$\text{if } (x_k = x_i) \{ \\ R_i = r_k ; \\ \}$$

where x_i and R_i , $i=1,2,\dots,M$, are the combination of x_k , and training input pattern, respectively

$$R_i = [R_{i,0}, R_{i,1}, \dots, R_{i,q}]^T \quad (18)$$

In order for the training patterns to be suitable for fuzzy-ARTMAP, firstly conversion process is needed. Conversion process is to convert the component of input patterns to values between 0 and 1. Thus, the following binary sigmoid function is used to convert the actual interval of received signal to [0,1]

$$f(x) = \frac{1}{1 + e^{-\alpha x}} \quad (19)$$

where σ is the steepness of sigmoid function. Secondly, the complement coding process is used to prevent category proliferation, as described in [8].

Thus, The final training input vectors after converting and complementing coding procedures are

$$A_i = [a_i, a_i^c]^T, \quad i = 1, 2, \dots, M \quad (20)$$

where

$$a_i = [a_{i,0}, a_{i,1}, \dots, a_{i,q}], \\ a_i^c = [1 - a_{i,0}, 1 - a_{i,1}, \dots, 1 - a_{i,q}], \\ a_{i,j} = \frac{1}{1 + \exp(-\sigma R_{i,j})}, \quad j = 0, 1, \dots, q \quad (21)$$

The target value for each generated training pattern is the correct value for x_{k-d} for the desired delay, d . A training output pattern with target value 1 is represented by the vector (1,0); a training output pattern with target value 0 is represented by the vector (0,1).

The following is the training algorithms.

- (1) Determine the training patterns
Set the input pattern for fuzzy-ART (a), and the

corresponding output pattern for fuzzy-ART (b).

(2) Create the categories:

When training starts, no category is created. For this reason, in the beginning, a category can be made without any competition by fuzzy rule. However, when more than one category have already been created and a new input comes to fuzzy-ARTMAP equalizer, the category will be created by the following rule, and category choice is indexed by J

$$C_j(A_i) = \frac{|A_i \wedge w_j|}{\alpha + |w_j|}$$

$$C_j = \max \{C_j; j = 1, 2, \dots, N_{cat}\} \quad (22)$$

where the C_j is the choice function, and N_{cat} denotes the total number of categories created.

(3) Check if resonance occurs:

When $|A_i \wedge w_j| |A_i|^{-1}$ is greater than or equal to ρ , a match happens. Otherwise, a mismatch occurs. Despite that a match happens, corresponding target for the introduced input pattern may not be matched with the selected category. In this case, vigilance parameter is increased until it is slightly larger than $|A_i \wedge w_j| |A_i|^{-1}$. Then, the search for another category starts, except the previously selected categories. The search process continues until the chosen category satisfies the above conditions. If all the trial fail, a new category is created.

(4) update weights

Once search ends, the weight vector is updated according to the equation

$$w_J^{new} = \beta(A_i \wedge w_J^{old}) + (1 - \beta)w_J^{old} \quad (23)$$

where J denotes the selected category index. When β is set to 1, that leads to the fast learning.

(5) Stop the learning

If any new category is created for all patterns

throughout the steps(1-4) above, retraining for all patterns begins until no category is created.

V. Simulation Results

In this section, some results of computer simulations are presented to demonstrate how higher order cumulants and layered feed forward networks can be utilized to form a blind adaptive equalizer. For the computational convenience, it is assumed that the binary signals (+1 or 1) are generated at random with an additive white Gaussian noise. Firstly, the channel order is estimated with two different channel models. Autocorrelations of channel observations were computed using (3) and the results of them are illustrated in Fig. 3. As shown in the Fig. 3, channel orders were correctly revealed from their normalized sample autocorrelations.

For the estimates of the channel coefficients, 5 different realizations with the training sequences equal to 512 are performed with SNR equal to 10 db. The mean value of the estimates is shown in the Table 1.

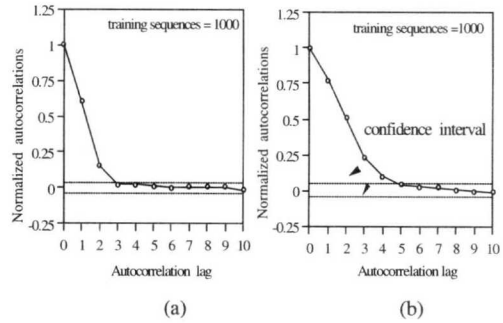


Fig. 3 Channel order estimation

$$(a) H(z) = 0.348 + 0.87z^{-1} + 0.348z^{-2}$$

$$(b) H(z) = 0.227 + 0.46z^{-1} + 0.688z^{-2} + 0.46z^{-3} + 0.227z^{-4}$$

The results show that the channel is almost correctly estimated from the channel output observations.

Finally, the fuzzy-ARTMAP equalizer is trained with the estimated channel model. The following

channel is assumed in the training procedure:

$$H(Z) = 0.5 + 1.0Z^{-1} \tag{24}$$

Among the favorable characteristics of this network is the fact that there are relatively few network parameters to be determined. The steepness of the sigmoid function, σ , used to convert the input patterns into the required interval [0,1] and the vigilance parameter for the network must be set by user. The network is not particularly sensitive to the values of either of these parameters. Sigmoid steepness parameter values in the range[0.7, 1.0] were used.

Figure 4 shows the error rate comparison of three kinds of neural network equalizers.

As shown in the graph, the performance of the blind fuzzy-ARTMAP equalizer is superior to that of the blind MLP equalizer, while producing results as favorable as those in the blind RBF equalizer. Even if the blind fuzzy-ARTMAP equalizer's performance is almost the same as that of the blind RBF equalizer, it has more advantages over the blind RBF equalizer considering the cost and effort required in neural network implementation.

Table 1. Estimation of channel coefficients

Original channel model	estimated channel coefficient
$H(Z) = 0.5 + 1.0Z^{-1}$	$h_0 = 0.50854$ $h_1 = 1.00076$
$H(z) = 0.348 + 0.87z^{-1} + 0.348z^{-2}$	$h_0 = 0.35706$ $h_1 = 0.87566$ $h_2 = 0.34682$

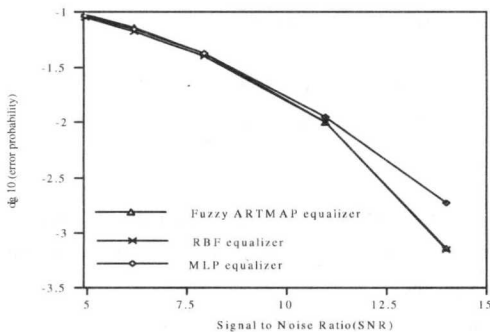


Fig. 4 Error rate comparison $H(z) = 0.5 + 1.0Z^{-1}$

VI. Conclusion

In this paper, a blind equalization technique is discussed based on higher-order statistics and a fuzzy-ARTMAP. The main procedures of the proposed blind equalizer consist of two parts. One is to estimate the order and coefficients of original channel using higher-order-cumulants; the estimated channel is used to generate the reference signal. The other part is to reconstruct the originally transmitted symbols (signals) after training the fuzzy-ARTMAP neural network. The main purpose of the blind fuzzy-ARTMAP equalizer is to solve the obstacles of long time training and complexity that are often encountered in the both MLP and RBF equalizers. The blind fuzzy-ARTMAP equalizer is fast and easy to train and includes capabilities not found in other neural network approaches; a small number of parameters, no requirements for the choice of initial weights, automatic increase of hidden units, and capability of adding new input data without retraining previously trained data. Throughout the simulation studies, it was found that the blind fuzzy-ARTMAP equalizer performed favorably better than the blind MLP equalizer, while requiring the relatively smaller computation steps in training. The superiority of fuzzy-ARTMAP to other neural networks makes the blind fuzzy-ARTMAP equalizer feasible to implement.

References

- [1] Y. Sato, "A Method of Self-Recovering Equalization for Multilevel Amplitude Modulation Systems," *IEEE Trans. Commun.*, vol. COM-23, pp. 679-682, Jun. 1975.
- [2] J. Mendel, "Tutorial on Higher-Order Statistics (Spectra) in Signal Processing and System Theory: Theoretical Results and Some Applications," *Proc. IEEE*, vol. 79, pp. 277-305, Mar. 1991.
- [3] F. B. Ueng and Y. T. Su, "Adaptive Blind Equalization Using Second and Higher Order

Statistics," *IEEE J. Select. Areas Commun.*, vol. 13, pp. 132-140, Jan. 1995.

[4] S. Chen, G. J. Gibson, C. F. N. Cowan, and P. M. Grant, "Adaptive Equalization of Finite Non-Linear Channels Using Multilayer Perceptrons," *Signal Processing*, vol. 20, pp. 107-119, 1990

[5] S. Chen, B. Mulgrew, and P. M. Grant, "A Clustering Technique for Digital Communications Channel Equalization Using Radial Basis Function Networks," *IEEE Trans. Neural Networks*, vol. 4, pp. 570-579, 1993.

[6] J. Lee, "A New Algorithm for Reducing the Number of Centers in Operating the RBF Neural Net Equalizer", *The Journal of the Korean Institute of Communication Sciences*, vol. 24, 4B, pp. 771-776, April, 1999.

[7] S. Mo and B. Shafai, "Blind Equalization Using Higher Order Cumulants and Neural Network," *IEEE Trans. Signal Processing*, vol. 42, pp. 3209-3217, Nov. 1994.

[8] G. A. Carpenter, S. Grossberg, N. Markuzon, J.H. Reynolds, and D.B. Rosen, "Fuzzy ARTMAP: A Neural Network Architecture for Incremental Supervised Learning of Analog Multidimensional Maps," *IEEE Trans. Neural Networks*, vol. 3, pp. 698-713, Sep. 1992.

[9] S. Grossberg, "Adaptive Pattern Classification and Universal Recording: I. Parallel Development and Coding of Neural Feature Detectors," *Biological Cybernetics*, pp. 121-134, 1976.

[10] G. A. Carpenter, S. Grossberga, and D. B. Rosen, "Fuzzy-ART: Fast Stable Learning and Categorization of Analog Patterns by an Adaptive Resonance System," *Neural Networks*, vol. 4, pp. 759-771, 1991.

[11] G. A. Carpenter, S. Grossberg, J. H. Reynolds, "ARTMAP: Supervised Real-Time Learning and Classification of Nonstationary Data by a Self-Organizing Neural Network," *Neural Networks*, vol. 4, pp. 565-588, 1991.

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