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DMT 시스템에서의 정수 비트 할당 알고리즘

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Integer Bit Loading Algorithm for Discrete Multitone Systems

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요 약

이 논문에서는 최적이면서 실용적인 정수 비트 할당 알고리즘을 제안한다. 정수 계획법을 이용하여, 주어진 에너지 조건들을 만족시키면서 비트율을 최대화하도록 모델링하고 성능을 분석한다. 제안된 정수 계획법 수식은 결정 변수를 이진수로 제한하고, 특수한 구조를 갖도록 하여, 일반적인 정수 계획법보다 간단히 해를 구할 수 있다. 모의 실험을 통해 최적으로 알려진 water filling 알고리즘과 비슷한 성능을 보이면서도 정수의 비트 수를 가짐으로써 구현이 가능하며 기존의 알고리즘보다 좋은 성능을 가짐을 확인하였다.

ABSTRACT

In this paper, an optimal and practical integer bit loading algorithm is proposed. By using Integer Programming, the proposed algorithm assigns a number of bits to different subchannels so that the data rate can be maximized for a given energy. Furthermore, since the bit loading problem requires the integer variable be binary, the optimal solution can be obtained more easily than the general integer programming problem. The results of computer simulation demonstrate that the proposed algorithm not only offers significant implementational advantages over the well-known water filling method but also shows better performance than other algorithms that rely on rounding to integer rates.

Ⅰ. Introduction

In recent years, the concept of multitone transmission has attracted a lot of interest as a means to increase the data rate on a channel under given requirements such as fixed transmitter power budget and equal probability of error on all subchannels. Two most common forms of multicarrier modulation (MCM) are orthogonal frequency-division multiplexing (OFDM) and discrete multitone modulation (DMT). In Europe, OFDM has been used in applications such as digital audio broadcasting (DAB) while DMT has been selected by the American National Standardization Institute (ANSI) and the European Telecommunications Standard Institute as the standard for transmission over asymmetric digital subscriber lines (ADSL), a technique for providing high speed data services from a central office to a customer's premises.

Unlike OFDM which assigns the same number of bits to each subchannel, DMT systems assign more bits to subchannel with higher signal-to-noise levels. The scheme used to assign energy and bits, known as bit loading algorithm, is an important aspect of the design of a DMT system. The problem of distributing the available energy among a set of parallel AWGN channels so as to

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maximize the overall bit rate has been long solved and the solution is given by the so called water filling distribution[1]. Although it will yield an optimal solution, bit rate can be any real number. Hence, this distribution is not well suited for practical data transmission. A number of algorithms have been proposed to solve the discrete loading problem in practice. They first find the optimal solution assuming continuous variable subchannel rate and then the rates are rounded to integer rates. These methods are computationally efficient but suboptimal.

In this paper, we propose a practical and efficient DMT loading algorithm by using integer programming(IP). Convergence to the optimal solution maximizing the bit rate is guaranteed. The rest of the paper is organized as follows. First the structure of a DMT system is presented and existing bit loading algorithms are discussed in Section Ⅱ. Section Ⅲ describes the proposed bit loading algorithm and Section Ⅳ presents computer simulation results, which indicate the improvements in the data rate and signal to noise ratio (SNR) improvements. Section Ⅴ concludes the paper.

Ⅱ. System Description

1. The Basic System.

The block diagrams of a basic DMT transmitter and receiver pair are illustrated in Fig 1 and Fig 2, respectively. Serial input data, which may actually consist of several channels, along with any additional control and operations information at a total rate of $\frac{B_{tot}}{T}$ bits per second are first buffered and grouped into B_{tot} bits for each block multicarrier symbol, with a block symbol period of T. In the case of an ANSI standard compliant ADSL system, $T=250$ μsec. The B_{tot} bits are appropriately encoded, converted to a parallel form, and modulated by N separate carriers, with b_n bits modulated by the nth carrier. The modulation process is accomplished by a N point IFFT operation. In the case of an ANSI standard compliant ADSL system, $N=256$.

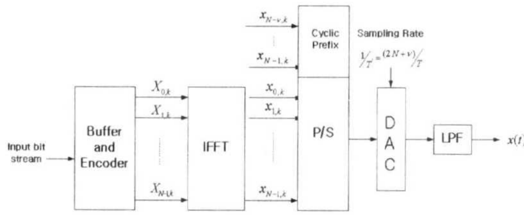

Fig. 1 The DMT Transmitter

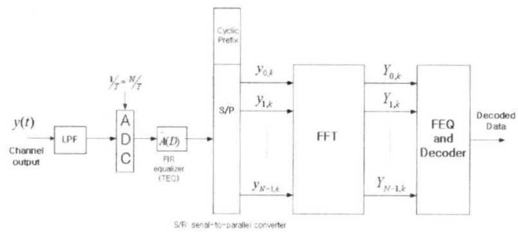

Fig. 2 The DMT Receiver

As a direct consequence of finite FFT/IFFT size, a number of practical, implementational issues for the DMT system need to be addressed. The first of which is interblock interference (IBI). In a finite-length DMT system, the effective length of the channel impulse response, or the channel constraint length ν, will cause IBI, as the tail of the previous block multicarrier symbol will corrupt the beginning of the current block multicarrier symbol. As a result, the subchannels are not strictly independent of each other in the frequency domain. To mitigate the effect of IBI, a technique known as cyclic prefix, or cyclic extension[2] is applied to the modulated block multicarrier symbol. The length of such cyclic prefix will be the same as the channel contraint length ν. The cyclic prefix procedure is a simple wrapping around the current block symbol so that to the discrete-time equivalent channel the input data sequence appears periodic. Due to the cyclic nature of the DFT and IDFT vectors the input data sequence appears periodic to the channel. In particular, for the periodic input data sequence, the multiplication in the frequency domain due to the linear filtering operation corresponds to the time domain circular convolution.

Referring back to Fig. 1 and 2, the parallel outputs of the IFFT are converted back to serial form with the appropriate cyclic prefix attached before passing through a Digital-to-Analog Converter (DAC) that operates at a sampling rate of $\frac{1}{T'} = \frac{2N+\nu}{T}$. The resulting analog waveform is lowpass filtered and sent through a D.C. isolating transformer to produce an analog transmit line signal. At the receiver end, the received analog signal is passed through a D.C. isolating transformer, low-pass filtered, and converted back to digital form by an Analog-to-Digital Converter (ADC) operating at the same sampling rate of $\frac{1}{T'}$. The resulting digital received sequence is passed through a FIR time-domain equalizer (TEQ) to limit the effective memory of the channel. The purpose of TEQ is not to flatten the channel completely as in a traditional linear or decision feedback equalizer, but rather it is to reduce the length of cyclic prefix to a more manageable number, without sacrificing performance significantly. In other words, this time domain equalizer, when concatenated with the channel, will produce a new target channel with a much smaller effective constraint length. The cyclic prefix is then stripped before the received sequence is passed to the FFT demodulator, which converts the sequence back to parallel frequency domain signals.

The phases of the received FFT outputs, $Y_{i,k}$'s in Fig. 2, are unlikely to be exactly the same as those of the transmitted 2-dimensional symbols at the input to the IFFT at the transmitter, or $X_{i,k}$'s in Fig 1. This phase distortion is the result of a (possibly varying) phase offset between the sampling clocks in the transmitter and the receiver of a practical transceiver system. Furthermore, even if the receiver sampling time can be adjusted perfectly, one still has to cope with the phase responses of the channel and the concatenated time domain equalizer. To compensate for this phase distortion, a set of "frequency domain equalizers" (FEQ) that consists of N 1-tap complex Least-Mean-Square (LMS) adaptive filters is used following the FFT block before the received signal is decoded. These 1-tap equalizers offer the additional benefit of received signal scaling before decoding. From a practical standpoint, it is convenient to use these 1-tap equalizers to adjust the gain of each FFT output so that a signal decision element can be used for all subchannels regardless of the different subchannels attenuation. Lastly, a decision element will decode the frequency domain signals and serially outputs the resulting data stream.

2. Existing Loading Algorithm

One crucial aspect in a well designed DMT-system is an accurate and efficient bit loading algorithm that will maximize the transport capacity of any given loop. Bit loading algorithm determines the proper number of bits and energy for each and every subchannel in a parallel set of subchannel. One example of a loading algorithm is the optimum water filling algorhim of [1] that solves a set of linear equations with boundary constraints. The solution of these water filling equations for large N may produce the rate that have fractional part or be very small. Such small or fractional number can complicate encoder and decoder implementation. Alternative suboptimal loading algorithms allow the computation of bit distributions that are more amenable to implementation.

A number of algorithms, all of which are suboptimal compared to water filling, have been proposed to solve the discrete loading problem in practice: Hughes-Hartogs[3], Chow[4], Fischer[5] and Leke[6].

The Hughes-Hartogs algorithm generates a table of incremental energies required to transmit one additional bit on each of the subchannels. Then at each step, one more bit is added to the subchannel that requires the least incremental energy. Whereas this technique can be used to solve both data rate and margin maximization, the algorithm requires an intensive amount of sorting.

The extensive sorting needed renders the algorithm impractical for applications where the number of bits per DMT symbol as well as the number of subchannels used are large.

Chow's algorithm exploits the fact that the difference between the optimal water-filling distribution and the flat-energy distribution is minimal. Thus, the same amount of energy is assigned to the channels turned on:

$$e_n = \frac{E_{tot}}{N_{on}},$$

where e_n is the energy allocated to the nth subchannel, E_{tot} total energy budget and N_{on} number of subchannels turned on. And the number of bits in each subchannel is given by:

$$b_n = \frac{1}{2}\log_2\left(1+\frac{e_n \cdot g_n}{\Gamma_n}\right), \qquad (1)$$

where g_n represents the subchannel signal to noise ratio when the transmitter applies unit energy to that subchannel (for multitone, $g_n = \frac{|H_n|^2}{\sigma_n^2}$). Γ_n is the SNR gap which is a function of the coding scheme of choice, the target probability of symbol error, $P_{e,n}$ and the desired noise margin $\gamma_{m,n}$, coding gain $\gamma_{c,n}$ and is given by

$$\Gamma_n = 10\log_{10}\left(\frac{\left[Q^{-1}\left(\frac{P_{e,n}}{N_{e,n}}\right)\right]^2}{3}\right) + \gamma_{m,n} - \gamma_{c,n}, \qquad (2)$$

where $N_{e,n}$ is the number of nearest neighbors of an input signal constellation for the nth subchannel. Conceptually, the SNR gap measures how far the performance of a communication system is from the channel capacity. With powerful coding techniques, the SNR gap can be reduced. Furthermore, for an uncoded, zero-margin system with a BER of 10^{-7}, the SNR gap is approximately 9.8 dB.

Chow's algorithm attempts to maximize the margin in a suboptimal fashion that relies on rounding to integer rates and this procedure is repeated, with a different set of subchannls turned off at each step, until the optimal margin is achieved and the sum of the bits in each subchannel equals target bit rates.

The goal of the algorithm proposed by Fischer is to minimize the probability of error on each subchannel and again relies on rounding. The minimum is achieved when all subchannels have the same probability of error. Setting equal the error probabilities, Fischer arrives at a set of iterative equations which also lead to a flat-energy distribution.

The algorithm in [6] assigns the energy to different subchannels in order to maximize the data rate for a given margin while Chow and Fischer are aimed mainly at maximizing margin at a target data rate. This algorithm computes a bit distribution by rounding the approximate water filling results and they suffer from its drawbacks as well. This method requires moderate amounts of sorting, in addition to mathematical computation.

Ⅲ. Proposed Bit Loading Algorithm

1. Problem Statement

The two types of loading algorithm are the Bit Rate Maximization Porblem (BRMP) and the Margin Maximization Problem(MMP).

Bit Rate Maximization Problem(BRMP): In this problem, we are given the subchannel gain to noise ratios, g_n, and a fixed amount of energy E_{tot}. The goal is to distribute the energy among the subchannels such that the overall bit rate can be maximized

$$\max\ B = \sum_{n=1}^{N} b_n = \frac{1}{2}\log_2\left(1+\frac{e_n \cdot g_n}{\Gamma_n}\right) \qquad (3)$$

subject to

$$\sum_{n=1}^{N} e_n \le E_{tot}. \qquad (4)$$

Margin Maximization Problem(MMP): In this problem, we are again given the subchannel gain

to noise ratios, g_n, but this time we have to transmit a fixed number of bits per symble B_{tot}, The goal is to determine the bit allocation that requires the least amount of energy, leaving the maximum amount of energy for system margin.
The MMP is stated as follows:

$$\min \ E = \sum_{n=1}^{N} e_n \tag{5}$$

subject to

$$\frac{1}{2}\sum_{n=1}^{N} \log_2\left(1+\frac{e_n \cdot g_n}{\Gamma}\right) = B_{tot}. \tag{6}$$

2. Integer Programming Formulation

Integer programming (IP) deals with constrained optimization problems in which some or all of the variables are restricted to be integers. The integer programming problem can be written in the general form:

$$(IP) \quad \max \quad \boldsymbol{cx}$$

subject to:

$$\boldsymbol{Ax} \leq \boldsymbol{b}$$

$$\boldsymbol{x} \geq 0, \quad \boldsymbol{x} \text{ integer-valude}$$

where $\boldsymbol{A}$ is $m \times n$ matrix of the coefficient of constraints, $\boldsymbol{b}$ is $m \times 1$ vector of rightsides, $\boldsymbol{c}$ is $n \times 1$ vector of objective coefficient and $\boldsymbol{x}$ is $n \times 1$ vector of decision variables.

To formulate the bit rate maximization problem, we define the binary variable x_{ij} as,

$$x_{ij} = \begin{cases} 1 & \text{if } j \text{ bits are allocated to the } i \text{ th subchannel} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

and from (1) we obtain the energy e_{ij}, when j bits are allocated to the ith subchannel, as

$$e_{ij} = \frac{\Gamma}{g_i}(2^{2j}-1). \tag{8}$$

Using these notations, then (3), (4), (5), (6) becomes, respectively, as follows:

$$\max \quad R = \sum_{i=1}^{N}\sum_{j=1}^{M} j \cdot x_{ij}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} e_{ij} \cdot x_{ij} \leq E_{tot}$$

$$\min \quad E = \sum_{i=1}^{N}\sum_{j=1}^{M} e_{ij} \cdot x_{ij}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} j \cdot x_{ij} = B_{tot}$$

Finally, we obtain the following IP formulation for the bit rate maximization problem

$$\max \quad R = \sum_{i=1}^{N}\sum_{j=1}^{M} j \cdot x_{ij} \tag{9}$$

subject to:

$$\sum_{j=1}^{M} x_{ij} \leq 1 \quad \text{for } i = 1, 2, \cdots, N \tag{10}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} e_{ij} \cdot x_{ij} \leq E_{tot} \tag{11}$$

$$x_{ij} \in \{0, 1\}, \tag{12}$$

where N is the number of subchannel and M the maximum number of bits which are allocated to subchannel. Note that (10) ensures that each subchannel has only one value for the number of the allocated bits.

Similarly, we can obtain the IP formulation for the margin maximization problem:

$$\min \quad E = \sum_{i=1}^{N}\sum_{j=1}^{M} e_{ij} \cdot x_{ij} \tag{13}$$

subject to:

$$\sum_{j=1}^{M} x_{ij} \leq 1 \text{ for } i = 1, 2, \cdots, N \tag{14}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} j \cdot x_{ij} = B_{tot} \tag{15}$$

$$x_{ij} \in \{0, 1\} \tag{16}$$

The maximum margin is then

$$\gamma_{\max} = \frac{E_{tot}}{E^*}. \tag{13}$$

3. How to solve the IP problem

Two types of algorithms have been developed to solve Integer Programming problem. One is cutting plane method and the other is the branch and bound method. Cutting plane method is exact algorithm for Integer Programming problems. Cutting plane algorithm for general integer programming problems were first proposed by R. E. Gomory in [7]. Gomory's cutting plane algorithm solves an integer program by solving the LP relaxation to optimality, generating a cutting plane from a row of the tableau if necessary, adding this additional constraint to the relaxation, solving the new relaxation, and repeating until the solution to the relaxation is integer. It was shown in [8] that, if a cutting plane is always generated from the first possible row, then Gomory's cutting plane algorithm will solve an integer program in a finite number of iterations. Gomory's method is the most well known and often used for general cuts, but this generality sacrifices fast achievement of solution. Thus, the Gomory's algorithm seems to be unsuitalbe for the proposed IP model which contains a large number of binary variables.

The branch and bound method is a quasi-enumerative approach to problem solving that has been applied to a wide variety of combinatorial problems. The basic idea of branch and bound is to partiton a given problem into a number of subproblems. This process of partitioning is usually called branching and its purpose is to establish subproblems that are easier to solve than the original problem because of their smaller size or more amenable structure. In the proposed IP formulation, the rules for brancing, bounding and fathoming have been simplified because each integer variable can only take on the values of zero or one. We move from one node of the search tree to a node on the next lower level by fixing an additional variable to either zero or one, as shown in Fig. 3

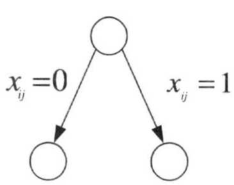

Fig. 3 Branching with binary variables

At a particular node, those variables whose values have been specified by the branching process are referred to as fixed variables. The remaining variables whose values have not yet been specified at that node are called free variables. In general, after a variable fixed to zero or one, the size of free variables is reduced by one. But the proposed IP formulation reduces the size of the free variables by 1 to M if the value of fixed variable is one.

IV. Simulation Results and Discussion

We now apply the proposed loading algorithm to the carrier serving area (CSA) high bit rate digital subscriber lines (HDSL) standard test Loops. CSA is an identifiable subset of the current subscriber loop population in the U.S. The CSA consists of mainly 24 -and 26-gauge twisted pair, with lengths on 24-gauge wire to 12kft and on 26-gauge wire to 9kft. CSA loops can also contain bridge taps of limited length; a complete specification of the CSA appears in [9]. We consider a set of representative CSA channels, provided by Bell communication Research [10], that have the respective loop cable arrangements illustrated in Fig. 4

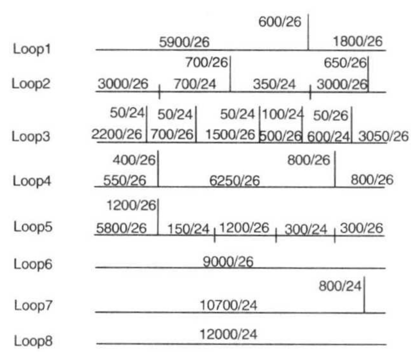

Fig. 4 Configuration of the 8 CSA HDSL Loops under study (length(in ft.)/gauge)

To illustrate the operation of this algorithm, in Fig. 5 we plot the channel impulse response of HDSL canonical test loop8. Fig. 6 shows the subchannel gain to noise ratios, g_n, with 2 mW of input power in the presence AWGN. The basic DMT system that is used for simulation has a sampling rate of 2.5 MHz, and an FFT size of 512, resulting in a multicarrier symbol rate of 4kHz. We will focus the simulation efforts on the bit rate maximization problem for an uncoded, zero-margin system with a BER of 10^{-7}. In fact, we will further restrict the integer bit constellation algorithm to have at least 1 bit and at most 6 bits per each subchannel. The proposed bit distributions are shown in Fig. 7.

Using the CSA test loops in Fig. 4, the optimal bit rate maximization is compared to the water filling algorithm and the one in [6]. The algorithm in [6] attempts to maximize the bit rate in a suboptimal sense that relies on rounding to integer rates. Data rates caculated for the eight loops are shown in Fig. 8. As expected, the data rate of proposed scheme is lower than that of water filling algorithm but higher than data rate of rounding method.

Finally, Fig. 9 shows the SNR performance of DMT systems with different types of loading algorithm. From Fig. 9, it can be observed that the proposed loading algorithm has about 1 dB gain over the algorithm in [6].

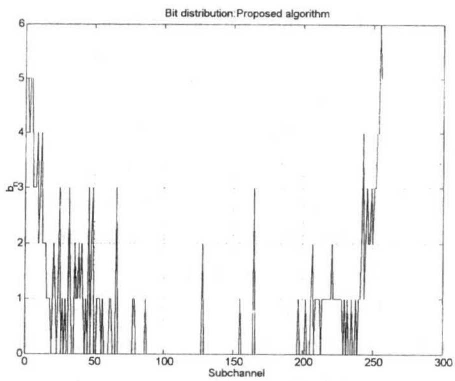

Fig. 7 Loop 8 bit distribution with proposed algorithm

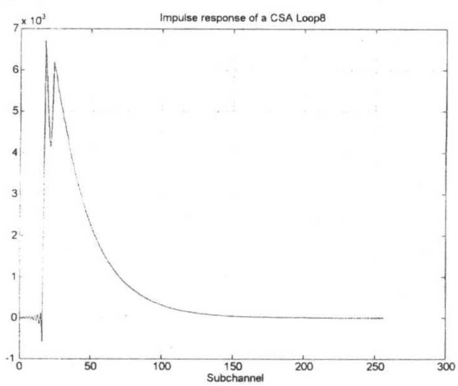

Fig. 5 Channel impulse response of Loop8

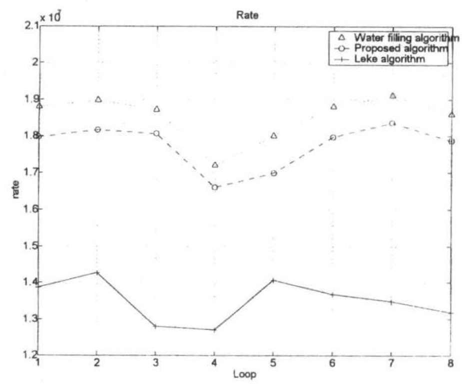

Fig. 8 Bit rate for the various CSA Loops

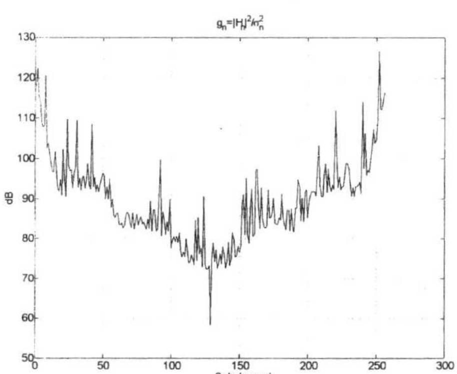

Fig. 6 g_n of Loop8

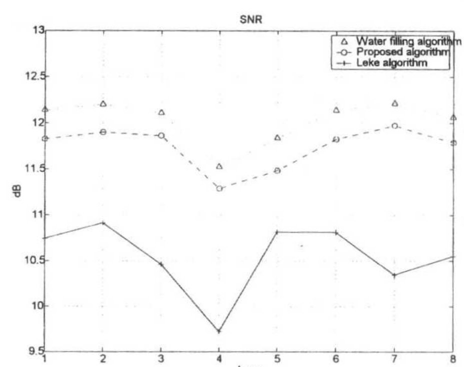

Fig. 9 SNR for versus the CSA Loops

V. Conclusion

In this thesis, we have presented a practical DMT loading algorithm for high speed data transmission over channels with severe ISI, as in the ADSL transmission environment. The proposed loading algorithm is more suitable for practical data transmission than the water filling algorithm because it result in integer bit constellation by using the Integer Programming. Another interesting feature of the algorithm presented here is that it could be formulated as a binary integer programming. In addition, the proposed algorithm can be readily modified to solve the margin maximization problem. We showed, through the computer simulation, that the performance of the proposed algorithm with integer number of bits per symbol is nearly as good as the optimal water filling solution. Futhermore, the proposed method has higher date rate and SNR than that of previous algorithm which relies on rounding.

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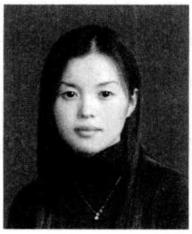

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