

Sensitivity of random phase error and frequency offset on uplink performance of MC-CDMA

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ABSTRACT

Performance degradation of MC-CDMA with frequency offset and random phase error is analyzed for the uplink in frequency selective slowly Rayleigh fading channel. The frequency offset denotes the frequency difference between oscillator of transmitter and oscillator of receiver, and random phase error means the phase difference between received carrier and carrier estimated by PLL loop of receiver. In this research, the good approximation methods(LLN, CLT) are adopted to derive the optimum average BER of system by means of analytic approach, and linear approximation is also applied to estimate the statistic of random phase error. From the analysis results, it is concluded that firstly, the smaller frequency offset, the greater degradation effect due to random phase error. Secondly, if the average loop SNR of PLL system is much larger than E_b/N_0 , the effect of random phase error can be neglected regardless of combining techniques(EGC, MRC) and approximation methods as well as frequency offset. Specially, it is shown that when an average loop SNR is 20dB larger than E_b/N_0 , the performance difference compared with case of perfect phase estimation is within 1dB. Thirdly, MRC method outperforms EGC method in uplink.

I. 서론

The high data rate is necessarily required to achieve multimedia services and those applications. However, the high input data rate causes the increase of chip rate to obtain the same processing gain(PG) in direct sequence - code division multiple access (DS-CDMA). Then, the bandwidth of transmitted signal, which is greater than the coherence bandwidth of channel, has a wideband spectrum, and this signal experiences frequency selective multipath fading. Generally, since the frequency selective multipath fading channel causes severe inter-chip interference (ICI) as well as inter-symbol interference(ISI) in DS-CDMA, the received signal is greatly distorted. Therefore, a conventional DS-CDMA scheme does not seem to be suitable for

multimedia services that require high data rate transmission.

Recently, the multicarrier - CDMA (MC-CDMA) systems of several kinds, which combine OFDM modulation with CDMA multiple access technique, have been proposed to solve this problem[1-4]. These systems have robustness against fading because each subcarrier does not experience time dispersion and high bandwidth efficiency due to orthogonality between subcarriers. Also, this scheme can naturally achieve the frequency diversity with simple one tap equalization. In [1], it is shown that MC-CDMA outperforms conventional DS-CDMA, which uses complex RAKE receiver, in multipath fading channel.

However, the frequency offset, which is caused by the Doppler effect of channel and the non-identical oscillators property between trans-

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mitter and receiver, dramatically degrades the performance of system. In [5-6], the bit error rate(BER) of MC-CDMA with frequency offset is analytically derived in downlink fading channel, and the degradation of system due to frequency offset is evaluated on downlink performance. However, these papers are analyzed in which the perfect phase synchronization is assumed. Generally, this assumption is not guaranteed in real environment, since achieving perfect phase synchronization is very difficult in multipath fading channel. On the other hand, another parameter yielding the performance degradation of system is phase noise due to unstable local oscillator in transmitter or receiver. The phase noise is modeled as random process with zero mean[7-8] and shifts the phase of carrier. The impact of phase noise on MC-CDMA performance is evaluated in [7], but this result is merely analyzed in additive white Gaussian noise(AWGN) in spite of the assumption that this derivation procedure can be applied in fading channel by semi-analytical approach. In [8], the effect of phase jitter due to phase noise on the performance of MC-CDMA is only studied under AWGN environment. Then, the phase jitter is represented as the phase difference between carrier of transmitter and phase locked carrier of receiver.

In this study, the random phase error is introduced to investigate the sensitivity of imperfect phase synchronization as well as frequency offset in multipath fading channel. The random phase error is caused by imperfect phase estimation, which is called to partially coherent reception and given as the phase difference between received carrier and carrier estimated by synchronizer of receiver. Then, the phase of received carrier is composed of the sum of phase shift due to phase noise, phase rotation due to channel's phase response and initial phase of transmitter. Generally, the probability density function (pdf) of random phase error is derived from the imperfect phase estimation of phase locked loop (PLL) used for phase synchronization

and well known as Tikhonov distribution, when a first(or second) order PLL loop is in lock[9-11]. The effect of partially coherent reception on performance of DS-CDMA with RAKE receiver is evaluated in [10], and the sensitivity of phase error on PSK system with equal gain diversity is studied by Prabhu[11].

This paper analyzes the performance degradation of MC-CDMA with frequency offset and random phase error in uplink Rayleigh fading channel. The remainder of this paper is organized as follows. The system and channel model are described in section II. In section III, the optimum average BER of MC-CDMA with frequency offset and phase error is approximately derived by analytic approach. The numerical results are represented in section IV and the conclusion about this research is given in section V.

II. MC-CDMA system and channel model

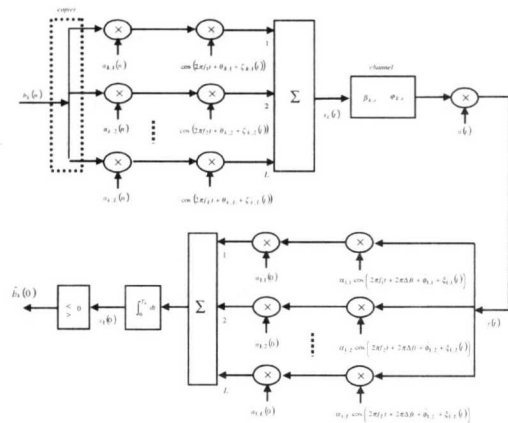


Fig. 1 MC-CDMA Transmitter and Receiver

In this research, the system model of reference [2] is adopted to analyze the sensitivity of frequency offset and random phase error. In fig. 1, the input symbols, $b_k(n)$, are assumed to be binary antipodal signal of n th bit interval in k th user and take on values of -1 and $+1$ with equal probability. A single input data is copied into L parallel branches. Each subchannel of

parallel stream is multiplied by one chip, $a_{k,i}(n)$, of orthogonal sequences that are considered as a PN code or Hadamard-Walsh code with L length and then BPSK modulated to a sub-carrier. It is noted that the chip interval is the same as the bit duration, T_b and the spacing between subcarriers uses a reciprocal of chip interval, $1/T_b$ to support high bandwidth efficiency by orthogonality. Finally, the transmitted signal is composed of the sum of these parallel branches and represented as

$$s_k(t) = \sum_{n=-\infty}^{\infty} \sum_{i=1}^L b_k(n) a_{k,i}(n) P_{T_b}(t-nt) \times \cos(2\pi f_i t + \theta_{k,i} + \zeta_{k,i}(t)), \quad P_{T_b}(t) = \begin{cases} 1, 0 \leq t \leq T_b \\ 0, \text{otherwise} \end{cases} \quad (1)$$

where f_i is i th subcarrier's local frequency given as $f_1 + (i-1)/T_b$. Also, the phase, $\theta_{k,i}$ for the k th user's i th subcarrier is uniformly distributed over $[0, 2\pi]$ and is independent and identically distributed (i.i.d) for all k and i . $\zeta_{k,i}(t)$ denotes phase shift due to phase noise at the transmitter and is assumed to be i.i.d random process for all k and i . Specially, in this study, we may regard $\zeta_{k,i}(t)$ as constant random variables over at least one chip interval, i.e., $\zeta_{k,i}(t) \cong \zeta_{k,i}$ although the phase noise is modeled as time varying random process[7].

The frequency selective slowly Rayleigh fading channel is considered to analyze the performance of system. Here, it is assumed that each subcarrier is subject to flat fading, if the reciprocal of chip interval is less than the coherence bandwidth of channel. Also, the fading characteristic of each subcarrier is assumed to be i.i.d and nearly constant over at least one chip interval by slow variation of channel. Under such a condition, the amplitude responses of channel are defined as Rayleigh random variables for all k and i as below

$$f(\beta_{k,i}) = \frac{\beta_{k,i}}{\sigma_{k,i}^2} \exp\left[-\frac{\beta_{k,i}^2}{2\sigma_{k,i}^2}\right] \quad (2)$$

and the phase responses, $\phi_{k,i}$ are assumed to be

uniform random variables for all k and i . Here, it is noted that the local mean power for the k th user's i th channel, $P_{k,i}$, is $0.5E[\beta_{k,i}^2] = \sigma_{k,i}^2$ where $E[\cdot]$ is expectation operator, and the local mean powers of all subchannels are equal because of i.i.d fading characteristic of subchannel. Therefore, $P_{k,i}$ is rewritten to P_k and the total mean power of k th user is defined as $P_{tot,k} = LP_k$. Also, the average bit energy, E_b of each user can be written as $P_{tot,k}T_b$. Assuming K active synchronous MC-CDMA users in uplink, the received signal is given as

$$r(t) = \sum_{n=-\infty}^{\infty} \sum_{k=1}^K \sum_{i=1}^L \beta_{k,i} b_k(n) a_{k,i}(n) \times P_{T_b}(t-nt) \cos(2\pi f_i t + \phi_{k,i}) + n(t), \quad \phi_{k,i} = \theta_{k,i} + \zeta_{k,i} + \phi_{k,i} \quad (3)$$

where $n(t)$ denotes AWGN with zero mean and two-sided power spectral density, $N_0/2$, and each element of $\phi_{k,i}$ is mutually independent.

III. Performance analysis

When the received signal passes demodulation and integral block, respectively, the decision variable for i th user's j th data is given as

$$v_i(0) = \frac{2}{T_b} \int_0^{T_b} r(t) \sum_{i=1}^L a_{1,i}(0) \times \cos(2\pi f_i t + 2\pi \Delta f t + \phi_{1,i} + \zeta_{1,i}(t)) a_{1,i} dt, \quad \phi_{1,i} = \phi_{1,i} + x_{1,i} \quad (4)$$

where $\alpha_{1,i}$ is diversity gain, and Δf denotes frequency offset. Also, $\zeta_{1,i}(t)$ denotes phase shift, which is caused by phase noise on the i th user's i th PLL loop, at the receiver. Here, when uplink is considered, the phase shift, $\zeta_{1,i}(t)$, may be neglected because the oscillator of base station is usually more stable than that of mobile[7]. The imperfectly estimated phase, $\phi_{1,i}$ is divided into two terms, i.e., a desired phase, $\phi_{1,i}$, and a random phase error, $x_{1,i}$. The random phase error, $x_{1,i}$ is composed of initial phase offset ($\Delta \theta_{1,i}$), phase rotation offset ($\Delta \phi_{1,i}$) and phase shift offset

($\Delta \zeta_{1,i}$), and each element of $x_{1,i}$ is mutually independent.

$$\begin{aligned} x_{1,i} &= \phi_{1,i} - \phi_{1,i} \\ &= (\theta_{1,i} - \theta_{1,i}) + (\phi_{1,i} - \phi_{1,i}) + (\zeta_{1,i} - \zeta_{1,i}) \\ &= \Delta \theta_{1,i} + \Delta \phi_{1,i} + \Delta \zeta_{1,i} \end{aligned} \quad (5)$$

where $x_{1,i}$ assumed to be i.i.d for all PLL loops and treated as being constant value over at least one chip interval. Then, its pdf is referred as Tikhonov distribution[9-11].

$$f(x_{1,i}) = \frac{\exp[-\gamma_{1,i} \cos(x_{1,i})]}{2\pi I_0(\gamma_{1,i})}, \quad \text{for } |x_{1,i}| < \pi \quad (6)$$

where $I_0(\cdot)$ is zeroth-order modified Bessel function of the first kind. Also, $\gamma_{1,i}$ denotes instantaneous loop SNR on the 1th user's i th PLL system, and its pdf[10] is given as

$$f(\gamma_{1,i}) = \frac{1}{m_L} \exp\left[-\frac{\gamma_{1,i}}{m_L}\right] \quad (7)$$

where m_L is an average loop SNR commonly applied for all PLL loops and is proportional to E_b/N_0 , i.e., $m_L(\text{dB}) = E_b/N_0(\text{dB}) + x(\text{dB})$ where x is zero or positive integer.

The decision variable can be divided into the five independent terms due to frequency offset and random phase error.

$$v_1(0) = D + I_{MAI} + I_{ICN} + I_{ICL} + \eta \quad (8)$$

• Desired component is

$$D = b_1(0) \left[\frac{\sin \pi \epsilon}{\pi \epsilon} \right] \times \sum_{i=1}^L \beta_{1,i} \alpha_{1,i} [\cos \pi \epsilon \cos x_{1,i} - \sin \pi \epsilon \sin x_{1,i}]$$

• Interference components are

$$\begin{aligned} I_{MAI} &= \sum_{k=2}^K \sum_{i=1}^L \beta_{k,i} \alpha_{1,i} b_k(0) a_{k,i}(0) a_{1,i}(0) \times \\ &\quad \left[\frac{\sin \pi \epsilon}{\pi \epsilon} \right] \cos(\pi \epsilon + \phi_{1,i} - \phi_{k,i}), \\ I_{ICN} &= \sum_{i=1}^L \sum_{j=1, i \neq j}^L \beta_{1,j} \alpha_{1,i} b_1(0) a_{1,j}(0) a_{1,i}(0) \times \\ &\quad \left[\frac{\sin \pi \epsilon}{\pi(i-j+\epsilon)} \right] \cos(\pi \epsilon + \phi_{1,i} - \phi_{1,j}), \\ I_{ICL} &= \sum_{k=2}^K \sum_{i=1}^L \sum_{j=1, i \neq j}^L \beta_{k,j} \alpha_{1,i} b_k(0) a_{k,j}(0) a_{1,i}(0) \times \end{aligned}$$

$$\left[\frac{\sin \pi \epsilon}{\pi(i-j+\epsilon)} \right] \cos(\pi \epsilon + \phi_{1,i} - \phi_{k,i}).$$

• Noise component is

$$\begin{aligned} \eta &= \frac{2}{T_b} \int_0^{T_b} n(t) \sum_{i=1}^L \alpha_{1,i} a_{1,i}(0) \times \\ &\quad \cos(2\pi f t + 2\pi \Delta f t + \phi_{1,i}) dt. \end{aligned}$$

where ϵ denotes frequency offset normalized by bit duration and has the range of $-0.5 \leq \epsilon \leq 0.5$. In (8), the interference and noise components is considered as mutually independent Gaussian random variables with zero mean[5-6]. Hence, the total variance of these terms is given as

$$\sigma_{total}^2 = \text{var}(I_{MAI}) + \text{var}(I_{ICN}) + \text{var}(I_{ICL}) + \text{var}(I_\eta) \quad (9)$$

$$\begin{aligned} \cdot \text{var}(I_{MAI}) &= \frac{1}{2} \sum_{k=2}^K \sum_{i=1}^L E[(\beta_{k,i} \alpha_{1,i})^2] \times \\ &\quad [\sin \pi \epsilon / \pi \epsilon]^2, \\ \cdot \text{var}(I_{ICN}) &= \frac{1}{2} \sum_{i=1}^L \sum_{j=1, i \neq j}^L E[(\beta_{1,j} \alpha_{1,i})^2] \times \\ &\quad [\sin \pi \epsilon / \pi(i-j+\epsilon)]^2, \\ \cdot \text{var}(I_{ICL}) &= \frac{1}{2} \sum_{k=2}^K \sum_{i=1}^L \sum_{j=1, i \neq j}^L E[(\beta_{k,j} \alpha_{1,i})^2] \times \\ &\quad [\sin \pi \epsilon / \pi(i-j+\epsilon)]^2, \\ \cdot \text{var}(I_\eta) &= \frac{N_0}{T_b} \sum_{i=1}^L E[(\alpha_{1,i})^2]. \end{aligned}$$

Therefore, assuming '1' data bit is transmitted, the conditional BER in AWGN can be represented as

$$\begin{aligned} P(\Psi) &= \frac{1}{2} \text{erfc} \left(\sqrt{\frac{[\sin \pi \epsilon / \pi \epsilon]^2 \Psi^2}{\sigma_{total}^2}} \right), \\ \Psi &= \sum_{i=1}^L \beta_{1,i} \alpha_{1,i} [\cos \pi \epsilon \cos x_{1,i} - \sin \pi \epsilon \sin x_{1,i}] \end{aligned} \quad (10)$$

where $\text{erfc}(\cdot)$ denotes complementary error function. In (10), in order to obtain the optimum BER of system for multipath fading channel, the random variables, i.e., $\beta_{1,i}$ and $x_{1,i}$, should be exactly evaluated. We will start from the estimation of random phase error to achieve it. As discussed in section I, the several papers have provided good analysis methods to evaluate the effect of random phase error[9-11]. Specially, in [10], the estimation method, called to linear approximation, is proposed to evaluate the performance of partially coherent DS-CDMA with

rake receiver. Then, the probability of error obtained by this method provides very exact error bound in frequency selective multipath fading channel. Therefore, this method is used to evaluate performance degradation of MC-CDMA with random phase error. When the linear approximation, by replacing each term($\cos x_{1,i}$, $\sin x_{1,i}$) with its expectation value, is applied in (10), Ψ can be rewritten as [appendix A]

$$\Psi = \cos \pi \varepsilon \sum_{i=1}^L [\beta_{1,i} \alpha_{1,i}] I(\gamma_{1,i}) \quad (11)$$

where $I(\gamma_{1,i}) \equiv I_1(\gamma_{1,i})/I_0(\gamma_{1,i})$ denotes the loss factor of system caused by random phase error and is converged to value of one with increasing $\gamma_{1,i}$. Here, the threshold value, γ_{th} , which is minimum loop SNR commonly required for all PLL loops, is introduced to obtain optimum BER[10]. From this parameter, a conditional average BER in AWGN is easily derived as follows [appendix B].

$$P(\beta, \gamma_{th}) = \sum_{p=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{[\sin \pi \varepsilon / \pi \varepsilon]^2 [\cos \pi \varepsilon]^2 \beta^2 F^2(\gamma_{th})}{\sigma_{total}^2 |_{L=p}}} \right) \times P_p(\gamma_{th}), P(\beta, \gamma_{th}) |_{p=0} = 0.5 P_0(\gamma_{th}) \quad (12)$$

where β and $P_p(\gamma_{th})$ denote the sum of Rayleigh amplitudes and the event probability with binomial distribution, respectively. Next, the pdf of random variable, β , will be estimated to obtain the probability of error in fading channel as follows.

$$P(\gamma_{th}) = \int_{-\infty}^{\infty} P(\beta, \gamma_{th}) f(\beta) d\beta \quad (13)$$

Historically, Finding pdf of β has been regarded a difficult problem. Hence, the law of large number (LLN) and central limit theorem (CLT) methods [2], providing very exact approximation on a large number of L , are applied to determine pdf of β where the gain factor, $\alpha_{1,i}$, is 1 for equal gain combining (EGC) and $\beta_{1,i}$ for maximal ratio combining (MRC).

A. Error probability for EGC

When the variable, β , is approximated as $pE(\beta_{1,i})$ by LLN method or as Gaussian random variable with mean, $\sqrt{0.5\pi p^2 P_1}$, and variance, $pP_1(2-0.5\pi)$, by CLT method, the conditional average BER in multipath fading channel is given as

$$P_{E,LLN}(\gamma_{th}) \cong \sum_{p=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{0.5\pi Q_1 [\cos \pi \varepsilon]^2 F^2(\gamma_{th})}{\frac{(K-1)}{p} P_r Q_1 + \frac{1}{p^2} [1 + (K-1)P_r] Q_2 + \frac{N_0}{E_b}}} \right) \times P_p(\gamma_{th}) \quad (14)$$

$$P_{E,CLT}(\gamma_{th}) \cong \sum_{p=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{0.5\pi Q_1 [\cos \pi \varepsilon]^2 F^2(\gamma_{th})}{\frac{(2-0.5\pi)}{p} P_r Q_1 + \frac{(K-1)}{p} P_r Q_1 + \frac{1}{p^2} [1 + (K-1)P_r] Q_2 + \frac{N_0}{E_b}}} \right) \times P_p(\gamma_{th}) \quad (15)$$

where $P_r = P_{tot,k}/P_{tot,1}$ is the power ratio of interference user to desired user, $Q_1 = [\sin \pi \varepsilon / \pi \varepsilon]^2$ and $Q_2 = \sum_{i=1}^p \sum_{j=1, j \neq i}^p [\sin \pi \varepsilon / \pi(i-j+\varepsilon)]^2$. In eqn. (14) and (15), since the error probability is only the function of γ_{th} , a conditional average BER greatly depends on γ_{th} . Therefore, by calculating (14) and (15) for various values of γ_{th} and selecting minimum value, an optimum average BER is finally obtained as follows.

$$P_{opt,E,LLN} = \min[P_{E,LLN}(\gamma_{th})] \quad (16)$$

$$P_{opt,E,CLT} = \min[P_{E,CLT}(\gamma_{th})] \quad (17)$$

B. Error probability for MRC

If an identical process is applied as the subsection A, the optimum average BER for MRC is given as

$$P_{opt,M,LLN} = \min[P_{M,LLN}(\gamma_{th})] \quad (18)$$

$$P_{opt,M,CLT} = \min[P_{M,CLT}(\gamma_{th})] \quad (19)$$

where

$$P_{M,LLN}(\gamma_{th}) \cong \sum_{\rho=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{2Q_1[\cos \pi \epsilon]^2 F^2(\gamma_{th})}{\frac{(K-1)}{\rho} P_r Q_1 + \frac{1}{\rho^2} [1 + (K-1)P_r] Q_2 + \frac{N_0}{E_b}}} \right) \times P_{\rho}(\gamma_{th}) \quad (20)$$

and

$$P_{M,CLT}(\gamma_{th}) \cong \sum_{\rho=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{2Q_1[\cos \pi \epsilon]^2 F^2(\gamma_{th})}{\frac{2}{\rho} + \frac{(K-1)}{\rho} P_r Q_1 + \frac{1}{\rho^2} [1 + (K-1)P_r] Q_2 + \frac{N_0}{E_b}}} \right) \times P_{\rho}(\gamma_{th}) \quad (21)$$

IV. Numerical results

The system with $L=128$, $K=16$, $P_r=0$ is considered to evaluate the sensitivity of frequency offset and random phase error. Also, BER curve (no asterisk line) for perfect phase estimation is given in order to compare the performance of system.

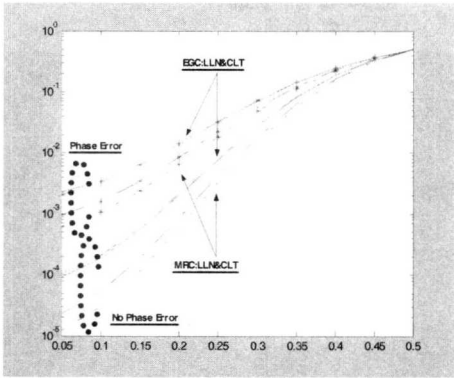


Fig. 2 BER vs. Frequency Offset ($E_b/N_0=10dB$, $m_L=10dB$)

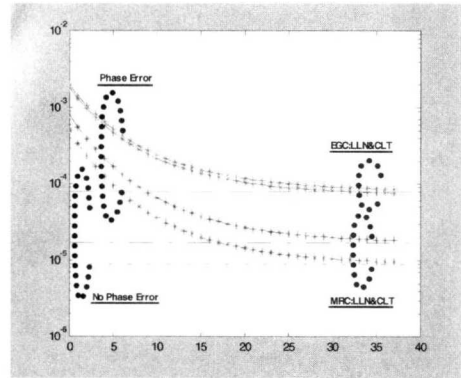


Fig. 4 BER vs. Frequency Offset ($E_b/N_0=10dB$, $\epsilon=0$)

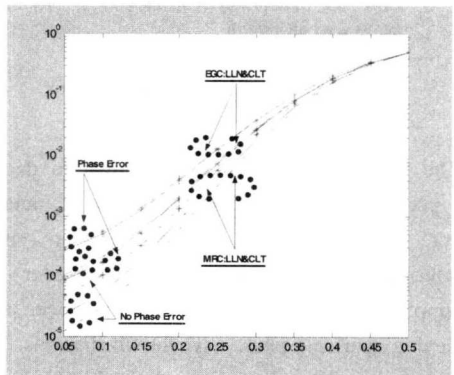


Fig. 3 BER vs. Frequency Offset ($E_b/N_0=10dB$, $m_L=20dB$)

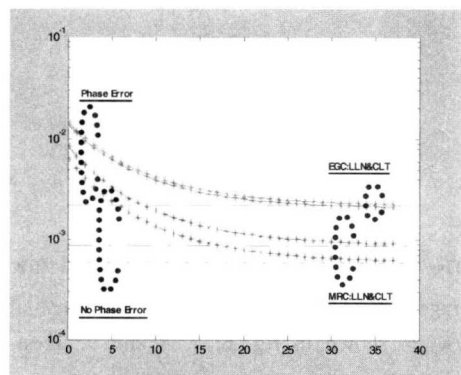


Fig. 5 BER vs. Frequency Offset ($E_b/N_0=10dB$, $\epsilon=0.2$)

Fig. 2 shows BER versus frequency offset for $E_b/N_0=10dB$ and $m_L=10dB$. From this figure, it is clear that MRC performs better than EGC, and the performance difference of BER obtained by LLN and CLT is very slight. Also, the performance of system is sharply degraded by increasing ϵ . However, fig. 3 shows that the performance degradation is improved, if the average loop SNR is increased by $20dB$. Therefore, if an average loop SNR is approaching ∞ , the performance curve will be identical to BER curve obtained by assuming perfect phase synchronization. From fig. 2 and 3, it is noted that the smaller frequency offset, the greater degradation effect due to random phase error. Also, the improvement effect being obtained by increasing average loop SNR is almost neglected, if $\epsilon > 0.35$.

Fig. 4 and 5 show BER versus average loop SNR above $E_b/N_0=10dB$ for $\epsilon=0$ and $\epsilon=0.2$. From this figures, it is seen that the effect of random phase error can be almost neglected regardless of combining techniques and approximation methods as well as frequency offset, if an average loop SNR is 35dB larger than $E_b/N_0=10dB$. This means the fact that MC-CDMA system is more affected by frequency offset than random phase error, assuming that an average loop SNR is sufficiently larger than E_b/N_0 .

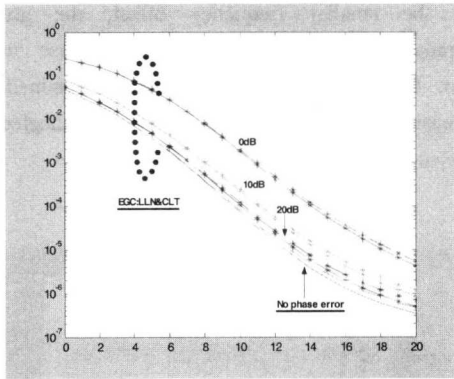


Fig. 6 BER vs. Frequency Offset ($\epsilon=0$, m_L above $E_b/N_0=0, 10, 20dB$)

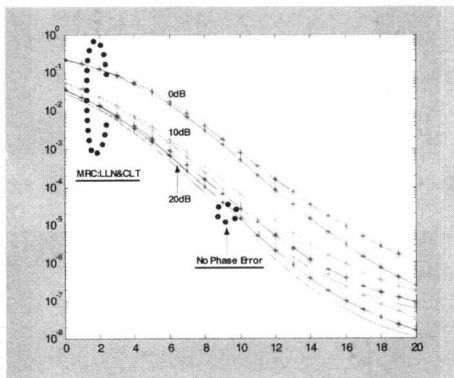


Fig. 7 BER vs. Frequency Offset ($\epsilon=0$, m_L above $E_b/N_0=0, 10, 20dB$)

The probability of error versus E_b/N_0 for the average loop SNR of 0dB, 10dB and 20dB above E_b/N_0 is shown in fig. 6, 7, 8 and 9. Then, the both EGC and MRC are considered, and ϵ is fixed at 0 and 0.3. If an average loop SNR is

20dB larger than E_b/N_0 , the performance difference compared with case of perfect phase estimation is within 1dB regardless of combining techniques and approximation methods as well as frequency offset. Also, if an average loop SNR equals E_b/N_0 , the performance difference is some 3dB.

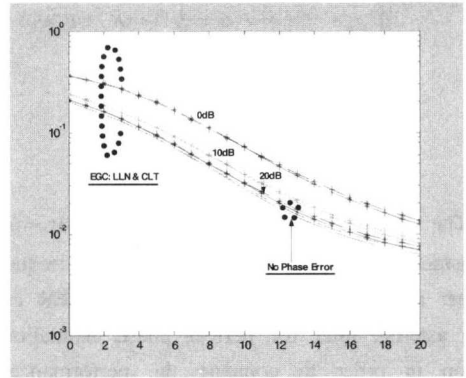


Fig. 8 BER vs. Frequency Offset ($\epsilon=0.3$, m_L above $E_b/N_0=0, 10, 20dB$)

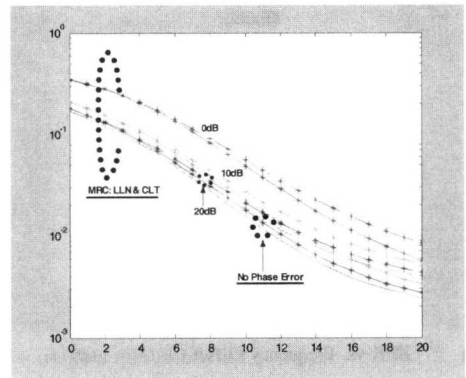


Fig. 9 BER vs. Frequency Offset ($\epsilon=0.3$, m_L above $E_b/N_0=0, 10, 20dB$)

V. Conclusion

The frequency offset is well known to degrade the performance of MC-CDMA system, and its effect is analyzed under perfect phase synchronization in many papers. However, this assumption is not guaranteed in practice, since achieving perfect phase synchronization is very difficult in multipath fading channel. Hence, the random phase error due to imperfect phase

estimation is introduced in this research.

This paper evaluates the effects of two parameters on uplink performance of MC-CDMA system with frequency offset and random phase error. Then, a frequency selective slowly Rayleigh fading is considered to be the channel condition of uplink.

To analysis system, the frequency offset is represented as the value normalized by bit duration, and impact of random phase error is given as the function of average loop SNR which is proportional to E_b/N_0 . Numerical results indicate that firstly, when the frequency offset is small, the random phase error greatly affects the performance of system. However, the larger frequency offset, the smaller degradation effect due to random phase error. Secondly, if the average loop SNR of PLL system is much larger than E_b/N_0 , the effect of random phase error can be neglected regardless of combining techniques (EGC, MRC) and approximation methods(LLN, CLT) as well as frequency offset. Thirdly, the system using MRC has lower error floor than the system using EGC, and the BER derived by LLN nearly equals the BER derived by CLT.

APPENDIX A

Proof of (11)

When applying linear approximation to (10), the parameter, Ψ , is given as

$$\Psi = \sum_{i=1}^L \beta_{1,i} \alpha_{1,i} \{ \cos \pi \epsilon E[\cos x_{1,i}] - \sin \pi \epsilon E[\sin x_{1,i}] \} \quad (A1)$$

Here, the second term, $\sin \pi \epsilon E[\sin x_{1,i}]$, may be neglected, since $E[\cos x_{1,i}] \gg E[\sin x_{1,i}]$. Hence, assuming $E[\cos x_{1,i}] = I(\gamma_{1,i})[10]$, the parameter, Ψ , is rewritten as $\cos \pi \epsilon \sum_{i=1}^L [\beta_{1,i} \alpha_{1,i}] I(\gamma_{1,i})$. From table 1, we can easily verify this simplicity.

Table 1. Comparison of $E[\cos x_{1,i}]$ and $E[\sin x_{1,i}]$

$\gamma_{1,i}$ [dB]	1	5	10	15	20
$E[\cos x_{1,i}]$	0.5307	0.8213	0.9486	0.9841	0.9950
$E[\sin x_{1,i}]$	-0.2250×10^{-10}	0.3578×10^{-10}	-0.0903×10^{-10}	0.0634×10^{-10}	-0.0221×10^{-10}

APPENDIX B

Proof of (12)

To derive (12), let us define the bound parameter, $I(\gamma_{th})$, as follows.

$$I(\gamma_{th}) = \begin{cases} 0 & \gamma_{th} \geq \gamma_{1,i} \\ I(\gamma_{1,i}) & \gamma_{th} < \gamma_{1,i} \end{cases}, 0(dB) \leq \gamma_{th}(dB) \leq m_L(dB) \quad (B1)$$

If all PLL loop SNRs, $\gamma_{1,i} | i=1,2,\dots,L$, are larger than threshold value, γ_{th} , the conditional BER of (10), after applying linear approximation and bound condition, is simply given as

$$P(\beta, \gamma_{th}) = 0.5 \operatorname{erfc} \left(\sqrt{\frac{[\sin \pi \epsilon / \pi \epsilon]^2 [\cos \pi \epsilon]^2 (\beta_L)^2 I^2(\gamma_{th})}{\sigma_{total}^2}} \right),$$

$$\text{for } \beta_L = \sum_{i=1}^L [\beta_{1,i} \alpha_{1,i}] \quad (B2)$$

However, this assumption is not guaranteed in practical, since one or more PLL loop SNRs can be below threshold value. Hence, we will consider the following all cases.

- i) Case 1 : threshold value < all PLL loop SNR's ($\gamma_{1,i} | i=1,2,\dots,L$)
- ii) Case 2 : threshold value > one or more PLL loop SNR's($\gamma_{1,i}$)
- iii) Case 3 : threshold value > all PLL loop SNR's ($\gamma_{1,i} | i=1,2,\dots,L$)

From these conditions, the event probability occurring by each case is represented as the binomial distribution with index value, p .

$$P_p(\gamma_{th}) = \binom{L}{p} \left[\exp\left(-\frac{\gamma_{th}}{m_L}\right) \right]^p \left[1 - \exp\left(-\frac{\gamma_{th}}{m_L}\right) \right]^{L-p},$$

$$p = 0, 1, 2, \dots, L \quad (B3)$$

where $\binom{L}{p}$ is binomial coefficient. Also, $[\exp(-\gamma_{th}/m_L)]$ denotes the event probability when $\gamma_{1,i}$ is larger than threshold value and is obtained by integrating (7) from γ_{th} to ∞ . Therefore, the conditional average BER in AWGN

can be represented as the sum of these values after multiplying the error probability by the event probability of each case.

$$P(\beta, \gamma_{th}) = \sum_{p=0}^L 0.5 \operatorname{erfc} \left(\sqrt{\frac{[\sin \pi \epsilon / \pi \epsilon]^2 [\cos \pi \epsilon]^2 \beta^2 \tilde{I}^2(\gamma_{th})}{\sigma_{total}^2 L = p}} \right) \times P_p(\gamma_{th}),$$

$$\beta = \sum_{i=1}^L [\beta_{1,i} \alpha_{1,i}] \quad (B4)$$

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