Low Complexity Decoder for Space-Time Turbo Codes

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ABSTRACT

By combining the space-time diversity technique and iterative turbo codes, space-time turbo codes(STTCs) are able to provide powerful error correction capability. However, the multi-path transmission and iterative decoding structure of STTCs make the decoder very complex. In this paper, we propose a low complexity decoder, which can be used to decode STTCs as well as general iterative codes such as turbo codes. The efficient implementation of the backward recursion and the log-likelihood ratio(LLR) update in the proposed algorithm improves the computational efficiency. In addition, if we approximate the calculation of the joint LLR by using the approximate ratio(AR) algorithm, the computational complexity can be reduced even further. A complexity analysis and computer simulations over the Rayleigh fading channel show that the proposed algorithm necessitates less than 40% of the additions required by the conventional Max-Log-MAP algorithm, while providing the same overall performance.

Key Words : Space-time, turbo code, low complexity, decoder, Max-Log-MAP

Ⅰ. Introduction

The quality of continuous media transmitted over a band-limited wireless channel suffers heavily from the instability and error-proneness[1], which results from the mobility, fading and fundamental limitations of power and available spectrum. Various diversity techniques have been introduced in order to provide diversity gains over band-limited wireless channels. By combining the space diversity and time diversity techniques, space-time codes can effectively overcome the limitation of the band-limited wireless channel[2,3]. In particular, they can mitigate the effect of the severe magnitude fluctuation and phase rotation that occurs in the multi-path fading channel, by employing multiple transmitters and receivers that receive replicas of the same transmitted symbol through independent fading paths. Tarokh et al. proposed the space-time trellis code, by combining channel coding, modulation and the diversity scheme[2]. They also proposed the space-time block codes, which can be more easily combined with other error correction codes than the space-time trellis codes[3]. By combining the space-time block codes with turbo codes which show near-optimum performance over the additive white Gaussian noise(AWGN) channels, space-time turbo codes(STTCs) show superior error correcting capability over the multi-path fading channel[4,5]. In particular, STTCs with full antenna diversity constitute bandwidth efficient error correction codes, in which the data and the parity bits are transmitted simultaneously through multiple antennas[4]. However, the multi-path transmission and iterative decoding structure of STTCs increase their decoding complexity.

An efficient algorithm for decoding general iterative codes was proposed in[6], and the first version of algorithms for decoding STTCs was presented in [11]. This algorithm considerably reduces the computational complexity, while providing the same performance as that of the conventional Max-Log-MAP algorithm. In this paper, we

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present a modified version of this efficient algorithm, which can be used to decode STTCs as well as general iterative codes such as turbo codes. The proposed algorithm is implemented more efficiently by modifying the backward recursion and log-likelihood ratio updates of the conventional Max-Log-MAP algorithm. It can also be considered as the parallel implementation of the modified soft-output Viterbi algorithm(SOVA). The decoding complexity of STTCs can be reduced much more, if we approximate the calculation of the joint LLR by using the approximate ratio(AR) algorithm. A complexity analysis and computer simulations conducted over the Rayleigh fading channel show the efficiency of the proposed algorithm.

This paper is organized as follows. Section 2 briefly describes the STTC and the decoding algorithm for STTCs with full antenna diversity. In Section 3, the low complexity decoder for STTCs is presented and the approximate ratio(AR) algorithm is proposed. In Section 4, the computational complexity of the proposed algorithm is analyzed and the simulation results are presented. Finally, concluding remarks are given in Section 5.

Ⅱ. Space-time turbo code

Su et al. proposed STTCs with full antenna diversity[4], in which the bandwidth is efficiently utilized by transmitting the data and parity bits simultaneously through two or three transmit antennas. In this paper, we assume that three transmit antennas and two receive antennas are used. The structure of the encoder, which is shown in Fig. 1, is the same as that used for turbo codes, except for the deinterleaver of the second parity bit, which is required when considering the decoding process, since three symbols are transmitted simultaneously through three transmit antennas instead of being time multiplexed. Let us assume that c_k^1, c_k^2 and c_k^3 are the bipolar signals corresponding to the input and two parity bits with $c_k^j \in \{-1,+1\}$ and x_k^1, x_k^2, x_k^3 are the transmitted signals produced by the BPSK modulation. If the fading amplitude for each transmitted path is h_k^{ij} and the additive noise is n_k^i, then the demodulated signal, y_k^j, for two receive antennas, is given by

$$y_k^i = h_k^{i1} \cdot x_k^1 + h_k^{i2} \cdot x_k^2 + h_k^{i3} \cdot x_k^3 + n_k^i, \ (i=1,2). \quad (1)$$

The decoder for the STTCs is also similar to that of the turbo codes. Two decoders for convolutional codes are used for the iterative decoding algorithm. The signals received by the two antennas are used for space diversity and the decoding performance depends on the number of iterations.

The maximum a posteriori(MAP) algorithm is known to be the optimum solution for decoding concatenated codes, such as STTCs and turbo codes. The MAP algorithm can be implemented efficiently by means of the BCJR algorithm, which can be applied to decoding STTCs as follows[7,8].

The log-likelihood ratio(LLR), which is used as a measure of reliability, is given by

$$L(c_k^1) = \ln\left(\frac{P(c_k^1=+1\,|\,\underline{y})}{P(c_k^1=-1\,|\,\underline{y})}\right) = \ln\left(\frac{\sum\limits_{(s',s)\Rightarrow c_k=+1} \alpha_{k-1}(s')\cdot\gamma_k(s',s)\cdot\beta_k(s)}{\sum\limits_{(s',s)\Rightarrow c_k=-1} \alpha_{k-1}(s')\cdot\gamma_k(s',s)\cdot\beta_k(s)}\right) \quad (2)$$

where s' and s are the starting and ending states of each stage in the trellis, respectively. $\alpha_k(s')$ and $\beta_k(s)$ are calculated by using forward recursion and backward recursion, respectively. $\gamma_k(s',s)$ can be calculated by using the following probabilities,

$$\gamma_k(s',s) = P(S_k = s \wedge \underline{y_k}\,|\,S_{k-1}=s') = P(s\,|\,s')\cdot P(\underline{y_k}\,|\,s'\wedge s) \quad (3)$$

where $\underline{y_k}$ is the demodulated signal vector and $\wedge$ is used to represent the joint probability. $P(s|s')$ is equal to $P(c_k^1)$, which is the a-priori information provided by the previous decoder. The value of $P(\underline{y_k}|s'\wedge s)$ for the first decoder for convolutional codes can be calculated by using the received signal and the channel information. If the input data and parity data for the branch from state s' to state s are a and b, respectively, then the probability, $P(\underline{y_k}|s'\wedge s)$, for STTCs can be calculated using the following equation.

$$P(\underline{y_k}\,|\,s'\wedge s) = P(\underline{y_k}\,|\,c_k^1 = a, c_k^2 = b, c_k^3 = +1)\cdot P(c_k^3 = +1\,|\,c_k^1 = a) + P(\underline{y_k}\,|\,c_k^1 = a, c_k^2 = b, c_k^3 = -1)\cdot P(c_k^3 = -1\,|\,c_k^1 = a). \quad (4)$$

By using formulas (3) and (4), $\gamma_k(s', s)$ can be calculated as follows

$$\gamma_k(s', s) = P(\underline{y_k}\,|\,c_k^1 = a, c_k^2 = b, c_k^3 = +1)\cdot P(c_k^1 = a, c_k^3 = +1) + P(\underline{y_k}\,|\,c_k^1 = a, c_k^2 = b, c_k^3 = -1)\cdot P(c_k^1 = a, c_k^3 = -1). \quad (5)$$

By taking logarithms of both sides of equation 5, the MAP algorithm is converted into the Log-MAP algorithm in which the multiplication and the exponential calculation can be avoided. In the Log-MAP algorithm, $\Gamma_k(s', s)$, which is the log value of $\gamma_k(s', s)$, can be calculated as

$$\Gamma_k(s', s) = \mathrm{M}\Big(L_{ap}^{(a,+1)}(c_k^1 = a, c_k^3 = +1) - \frac{1}{N_0} d^2_{c_k^1=a, c_k^2=b, c_k^3=+1},\; L_{ap}^{(a,-1)}(c_k^1 = a, c_k^3 = -1) - \frac{1}{N_0} d^2_{c_k^1=a, c_k^2=b, c_k^3=-1}\Big) + K \quad (6)$$

where K is a constant which will be cancelled and $\mathbf{M(\cdot,\cdot)}$ is the Jacobian logarithm given by

$$\mathrm{M}(X_m, X_n) \equiv \log(e^{X_m} + e^{X_n}) = MAX(X_m, X_n) + \log(1 + e^{-|X_m - X_n|}). \quad (7)$$

And $d^2_{c_k^1=a, c_k^2=b, c_k^3=e}$ can be calculated as

$$d^2_{c_k^1=a, c_k^2=b, c_k^3=e} = \sum_{i=0}^{1} (y_k^i - h_k^{i1}\cdot x_k^1 - h_k^{i2}\cdot x_k^2 - h_k^{i3}\cdot x_k^3)^2 \,\big|_{c_k^1=a, c_k^2=b, c_k^3=e}, \quad (8)$$

where *a, b* and *e* are *+1* or *-1*. In formula (6), $L_{ap}^{(a,b)}(c_k^1 = a, c_k^3 = b)$ is an a-priori joint LLR given by the other decoder for convolutional codes. The joint LLR is given by

$$L^{(a,b)}(c_k^1, c_k^i) = \ln\frac{P(c_k^1 = a, c_k^i = b)}{P(c_k^1 = -1, c_k^i = -1)} = \ln\left(\frac{\sum_{(s',s)\Rightarrow(c_k^1, c_k^i)=(a,b)} \alpha_{k-1}(s')\cdot\gamma_k(s', s)\cdot\beta_k(s)}{\sum_{(s',s)\Rightarrow(c_k^1, c_k^i)=(-1,-1)} \alpha_{k-1}(s')\cdot\gamma_k(s', s)\cdot\beta_k(s)}\right). \quad (9)$$

The LLR can be calculated from the joint LLR as follows.

$$L^{(a)}(c_k^1) = \ln\frac{P(c_k^1 = a)}{P(c_k^1 = -1)} = \ln\frac{P(c_k^1 = a, c_k^i = +1) + P(c_k^1 = a, c_k^i = -1)}{P(c_k^1 = -1, c_k^i = +1) + P(c_k^1 = -1, c_k^i = -1)}. \quad (10)$$

The logarithm of the joint probability in the Log-MAP algorithm is given by

$$\ln P(c_k^1 = a, c_k^i = b) = \mathrm{M}^*_{(s',s)\Rightarrow(c_k^1, c_k^i)=(a,b)}(A_{k-1}(s') + \Gamma_k(s', s) + B_k(s)), \quad (11)$$

where $A_{k-1}(s')$, $\Gamma_k(s', s)$ and $B_k(s)$ are the logarithms of $\alpha_k(s')$, $\gamma_k(s', s)$ and $\beta_k(s)$, respectively, and $\mathbf{M}^*_{\text{cond.}}$ can be calculated using the Jacobian logarithm described in (7) repetitively.

The extrinsic information, which is passed to the other decoder for convolutional codes as the a priori probability, can be obtained by calculating the difference between the joint LLR and the a-priori information, as given by

$$L_{ext}^{(a,b)}(c_k^1, c_k^2) = L^{(a,b)}(c_k^1, c_k^2) - L_{ap}(c_k^1), \quad (12)$$

where $L_{ap}(c_k^1)$ is derived using $L_{ap}^{(a,b)}(c_k^1, c_k^3)$, which is the joint LLR of the a priori probabilities provided by the previous decoder. In the Log-MAP algorithm, the forward recursion and backward recursion can be implemented by

$$A_k(s) = \mathrm{M}\big((\Gamma_k(s', s) + A_{k-1}(s'))_{c_k^1=+1}, (\Gamma_k(s', s) + A_{k-1}(s'))_{c_k^1=-1}\big), \quad k = 1, \cdots, K \quad (13)$$

and

$$B_{k-1}(s) = \mathrm{M}\big((\Gamma_k(s', s) + B_k(s))_{c_k^1=+1}, (\Gamma_k(s', s) + B_k(s))_{c_k^1=-1}\big), \quad k = K, \cdots, 1. \quad (14)$$

By omitting the last log term, i.e. $\log(1+e^{-|X_m - X_n|})$, in the calculation of the Jacobian logarithm of formula (7), the Log-MAP algorithm is converted into the Max-Log-MAP algorithm, in which the forward recursion, backward recursion and joint log-likelihood ratio are calculated more efficiently, at the expense of a slight degradation in performance.

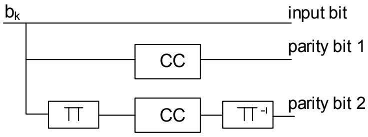

Fig. 1. Encoder for the STTC with full antenna diversity

Ⅲ. Low complexity decoder for STTC

3.1 Low-complexity decoding algorithm

In order to decode STTCs, the LLR or the joint LLR should be calculated by considering the multiple paths formed by the multiple transmitters and receivers. Moreover, the performance of the iterative decoder generally depends on the number of iterations required for decoding. Thus, it is essential to reduce the computational complexity needed for decoding STTCs. We propose a low complexity algorithm for decoding STTCs, which provides the same performance as that of the conventional Max-Log-MAP algorithm, and an approximate algorithm for decoding STTCs is given in the next sub-section. The proposed algorithm is described below. The forward recursion of the proposed algorithm is the same as that of the conventional Max-Log-MAP algorithm, except for storing the difference metric. The difference metric, $\Delta_k(s_m)$, is stored as

$$\Delta_k(s_m) = |(\Gamma_k(s'_i, s_m) + A_{k-1}(s'_i))_{c_k^1=+1} - (\Gamma_k(s'_j, s_m) + A_{k-1}(s'_j))_{c_k^1=-1}|. \quad (15)$$

Notice that the difference metric is used as the reliability value in the SOVA[8]. In the backward recursion, the path metric, $D_k^{cond.}(s_m)$, is assigned to each branch. It is the accumulated value of the difference metrics, as given by

$$D_k^{c_k^1=a}(s_m) = \begin{cases} (B'_k(s_m))_{c_k^1=a}, & \text{if the path is selected at forward recursion at state } s \text{ of stage } k \\ (B'_k(s_m) + \Delta_k(s_m))_{c_k^1=a}, & \text{if the path is not selected at forward recursion at state } s \text{ of stage } k \end{cases} \quad (16)$$

where the new backward metric, $B'_k(s_i)$, is the minimum value of the path metric in the backward recursion, as given by

$$B'_{k-1}(s'_i) = \min(D_k^{c_k^1=-1}(s_m), D_k^{c_k^1=+1}(s_n)). \quad (17)$$

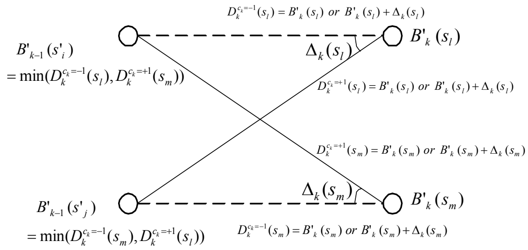

Fig. 2. Backward recursion for the proposed algorithm

The new backward recursion in the proposed algorithm is illustrated in Fig. 2. The value of the path metric represents the difference in reliability between the maximum likelihood path and the paths which include the selected branch at stage *k*. This value is accumulated with the minimum differences recursively, as can be seen in formulas (16) and (17). The value of the path metric for the maximum likelihood path is zero, while that for the other paths is always greater than zero. The initial value of $B'_k(s_i)$ is zero or infinity if the trellis is terminated. If the trellis is not terminated, the value of $B'_k(s_i)$ for the state on the estimated maximum likelihood path can be set to zero. The LLR is calculated directly as

$$L(c_k^1) = \min_m(D_k^{c_k^1=-1}(s_m)) - \min_n(D_k^{c_k^1=+1}(s_n)). \quad (18)$$

The joint LLR, $L^{(a,b)}(c_k^1, c_k^i)$, can be calculated as

$$L^{(a,b)}(c_k^1, c_k^i) = \min_m(D_k^{c_k^1=-1, c_k^i=-1}(s_m)) - \min_n(D_k^{c_k^1=a, c_k^i=b}(s_n)). \quad (19)$$

Notice that the minimum value of the path metric in formulas (18) and (19) is the minimum value among the relative differences between the maximum likelihood path and those paths which include the branch satisfying each condition at stage *k*.

The proposed algorithm can also be compared with the soft-output Viterbi algorithm(SOVA), which shows inferior performance with reduced complexity. While the proposed algorithm considers the difference metrics for all states at each stage, the SOVA only considers the difference metric along the maximum likelihood path. The proposed algorithm can be considered as the parallel implementation of the modified SOVA. Notice that the modified SOVA provides the same performance as the Max-Log-MAP algorithm[9].

Since the number of additions involving real numbers in the backward recursion is half of that required for the Max-Log-MAP algorithm, and no addition is required to calculate the log-likelihood ratio, the computational complexity of the proposed algorithm is significantly reduced, as demonstrated in the analysis of Section 4.

3.2 Approximating the Joint Ratio algorithm

By approximating $p(c_k^1=a, c_k^3=+1)$ (or $p(c_k^1=a, c_k^3=-1)$ as $0.5 \cdot p(c_k^1=a)$ in formula (5), we can obtain a simpler decoding method, which we refer to as the approximate ratio (AR) algorithm for convenience. The log value of $p(c_k^1)$ is given by

$$\ln(p(c_k^1)) = \frac{c_k^1}{2} \times L_0(c_k^1) + K', \qquad (20)$$

where *K'* is a constant which will be canceled out. Thus, in the AR decoding method, which can be considered as an approximation of the joint ratio(JR) decoding method where the joint LLR should be calculated, the following formula can be used to calculate the log values of the branch metric

$$\Gamma_k(s',s) = \frac{c_k^1}{2} \times L_0(c_k^1) + \mathrm{M}^{*_2}\left(-\frac{1}{N_0} d^2_{c_k^1=a, c_k^2=b, c_k^3=+1}, -\frac{1}{N_0} d^2_{c_k^1=a, c_k^2=b, c_k^3=-1}\right) \qquad (21)$$

The AR algorithm shows the same performance as the independent ratio (IR) method proposed in [10], although these two algorithms are derived using different approaches. In the AR decoding method, it is not necessary to calculate the joint Log-likelihood ratio. Since the calculation of the branch metric is simpler and only the log-like lihood ratio, $L(c_k^1)(=ln(P(c_k^1=a)/P(c_k^1=-1)))$, need to be calculated, the decoding complexity of the AR method is less than that of the JR method. However, the performance of the AR algorithm is inferior to that of the JR algorithm as is shown in the next Section.

Ⅳ. Complexity and performance analysis

The computational complexities of the conventional and proposed algorithms are compared by analyzing the number of operations involving real numbers. The forward recursion for the proposed algorithm is same as that of the conventional Max-Log-MAP algorithm, except for storing the difference metric. In the backward recursion, the number of additions of the proposed algorithm is half of that of the Max-Log-MAP algorithm, since additions are needed for only a half of the paths, as can be seen in equation (16). In calculating the LLR, the additions in formula (11) of the conventional algorithm are no longer needed in the proposed algorithm. Moreover, if the path is the maximum likelihood path, then the minimum value for the path metric, $min(P_k^{cond.}(S'))$, is known to be zero, without the need for calculation, when using the proposed algorithm. The numbers of operations involving real numbers are compared in Tables 1 and 2. The number of additions required by the proposed algorithm is less than 40% of that needed by the conventional Max-Log-MAP algorithm, in the case of both the JR and the AR decoding methods. The AR decoding method requires less computation than the JR decoding method, since the LLR can be calculated directly in the AR decoding method.

In Figs. 3 and 4, the BER performance over an uncorrelated Rayleigh fading channel is presented. STTCs with two $(1,5_{8/78})$ recursive convolutional codes and the S-random interleaver were used for the simulations. The BER for the proposed algorithm is the same as that for the conventional Max-Log-MAP algorithm in all cases. As can be seen in Fig. 3, the JR decoding method shows a lower BER than the AR decoding method. Fig. 4 shows that the performance of the STTCs improves significantly as the number of iterations increases.

Table 1. The number of operations involving real numbers (Joint Ratio method: calculating $L^{(a,b)}(c_k^1, c_k^i)$ for one stage with 4 states)

Operation	Function	Conventional	Proposed	Ratio
Additions (involving real numbers)	Forward	4X2	4X2	0.375
	Backward	4X2	4	
	LLR	4X2+4X2	0	
	Total	32	12	
Comparisons (involving real numbers)	Forward	4	4	0.941
	Backward	4	4	
	LLR	4X2+1	4+3+1	
	Total	17	16	

Table 2. The number of operations involving real numbers (Approximate Ratio method: calculating $L(c_k^1)$ for one stage with 4 states)

Operation	Function	Conventional	Proposed	Ratio
Additions (involving real numbers)	Forward	4X2	4X2	0.375
	Backward	4X2	4	
	LLR	4X2+4X2	0	
	Total	32	12	
Comparisons (involving real numbers)	Forward	4	4	0.765
	Backward	4	4	
	LLR	4X2+1	4+1	
	Total	17	13	

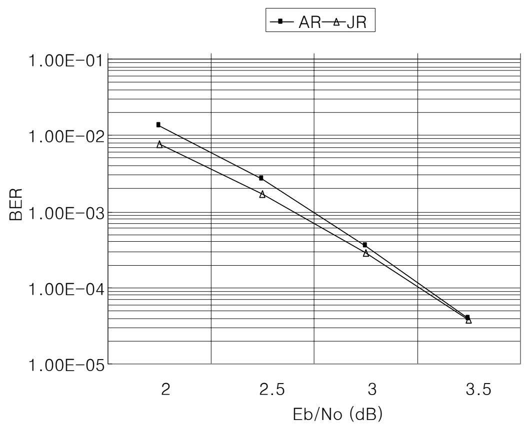

Fig. 3. BER performance of the proposed algorithm as a function of E_b/N_0 (JR decoding method or AR decoding method, number of turbo iterations: 4, block size: 512)

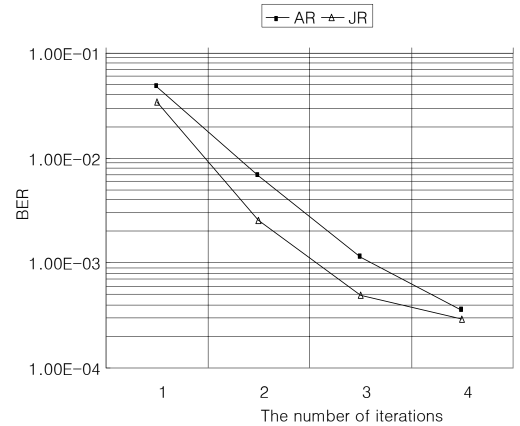

Fig. 4. BER performance of the proposed algorithm as a function of the number of iterations (JR decoding method or AR decoding method, E_b/N_0:3.0dB, block size: 512)

Ⅴ. Conclusion

An efficient algorithm for decoding STTCs with full antenna diversity was presented. By efficiently implementing the backward recursion and log-likelihood ratio updates of the Max-Log-MAP algorithm, the number of additions and comparisons involving real numbers was significantly reduced. An approximate algorithm for decoding STTCs was also proposed. It was shown that the proposed algorithm requires less than 40% of the additions needed by the conventional algorithm, while providing the same overall performance. Thus, we believe that the proposed algorithm can be used as a fast algorithm for decoding STTCs.

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