레일리 페이딩 채널에서 MRC 결합 기법 적용과 수학적 접근을 통한 BER 성능 분석

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General Expression for BER with MRC Reception over Rayleigh Fading Paths

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요 약

본 논문은 레일리 페이딩 채널에서 확률밀도함수와 최대비 결합기 출력의 최대모멘트 생성 함수를 일반적이고 간결한 수식적 접근을 통해 보여주었다. 또한, 이를 이용하여 *M*-PSK, *M*-PAM, *M*-QAM의 closed form BER을 증명하였다. 마지막으로 다양한 모의 실험을 통해 증명한 수식이 모의 실험 결과와 정확하게 일치함을 알 수 있다.

Key Words : BER, outage probability, MRC, pdf, Rayleigh fading channels

ABSTRACT

This paper provides a general and compact expression for the probability density function (pdf) and the moment-generating function (MGF) of the maximal ratio combiner output over Rayleigh fading channels. It is then used to derive closed form expression bit error rate (BER) for *M*-PSK, *M*-PAM and *M*-QAM, respectively. A variety of simulations is performed and shows that they match exactly with analytic ones.

Ⅰ. Introduction

Diversity is an effective technique used in wireless communication systems to combat the performance degradation caused by fading. It can alleviate the deleterious effect of fading by means of multiple reception of the same information bearing signals. Recently, communication systems in which spatial diversity is achieved by multiple communication nodes collaborating together to form a virtual antenna array have been proposed [1]-[7]. This form, called cooperative communication, was exploited to overcome some scenarios where wireless mobiles may be unable to support multiple antennas due to size or other constraints[1]. So, it allows single-antenna mobiles to gain several benefits of transmit diversity.

When evaluating the performance of Decode-and-Forward (DF) relaying system [3]-[7] or repetition coding [8] on Rayleigh fading channels using Maximal Combining Ratio (MRC) combining technique [9] at the destination, we usually deal with the problem of finding the expression for pdf of a sum of independent exponential random variables. For simplicity, some previous analysis [3]-[7] always assumed that the

※이 논문은 2007년도 정부(과학기술부)의 재원으로 한국과학재단의 지원을 받아 수행된 연구임(No. R01-2007-000-20400-0)
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논문번호 : KICS2008-05-211 접수일자 : 2008년 5월 11일, 최종논문접수일자 : 2008년 8월 18일

Rayleigh fading channels among nodes are independent and identically distributed (i.i.d.) or independent but not identically distributed (i.n.d.). However, in real scenarios, the condition of i.i.d. and i.n.d. among channels is not always happened and considering the most general case in which only some or all channels whose expected values of channel powers are distinct is more generalized and appropriate. Although, [10] suggested the way to calculate pdf expression for the case where are some channel powers of the same mean and the remaining of different means. However, this case is really not the most general case and also be a special case of the case we will study. In this paper, beside the general formula for pdf is established, the exact closed form expression of BERs for *M*-PSK, *M*-PAM and *M*-QAM as well as the outage probability over Rayleigh fading channels are done. In addition, the closed-form BER expression of 2-*P* Rx transmit diversity [11] as well as the outage probability of repetition coding [8] are derived as typical applications for the derived expressions.

The rest of this paper is organized as follows. In section II, we introduce the model under study. Section III shows the formulas allowing for evaluation of the average SNR, MGF, outage probability, and average BER for *M*-PSK, *M*-PAM and *M*-QAM modulation scheme. Some application scenarios whose performance can be evaluated by using the derived expressions will be performed in Section IV. Section V, we contrast the simulations and the results yielded by theory. Finally, the paper is closed in section VI.

Ⅱ. System model

We consider a diversity reception system for *M*-PSK, *M*-PAM and *M*-QAM over *N* independent Rayleigh fading channels. The complex baseband equivalent signals received over the *i*-th channel can be written as

$$r_i(t) = h_i(t)s(t) + n_i(t), i = 1,2,...N \tag{1}$$

where $h_i(t) = \alpha_i e^{j\phi_i}$ is a zero-mean complex Gaussian random variable with a Rayleigh-distributed amplitude α_i and a uniformly distributed phase angle ϕ_i. $s(t)$ is the complex baseband transmitted signal. $n_i(t)$ is a zero-mean complex Gaussian random variable representing the AWGN with variance N_0 which is the one-sided power spectral density in V^2/Hz. Due to Rayleigh fading, the channel powers, denoted by $|h_i|^2 = \alpha_i^2$, where $i = 1,...,N$ are independent and exponential random variables whose means are λ_i.

We assume matched filter detection and perfect channel estimation for the systems. The average error rates of modulation schemes in slow and flat Rayleigh fading channels can be derived by averaging the error rate for the AWGN channel over the pdf of the SNR in Rayleigh fading.

$$P(\epsilon) = \int_0^\infty P(\epsilon|\gamma) f_\gamma(\gamma) d\gamma \tag{2}$$

where $P(\epsilon|\gamma)$ is the error rate conditioned on γ, $f_\gamma(\gamma)$ is the pdf of the instantaneous SNR per bit γ. Let us define γ_i denotes the instantaneous SNR of each path received by the destination with their expected values $\overline{\gamma_i} = (E_b/N_0)\lambda_i$. For MRC, γ is defined as

$$\gamma = (E_b/N_0)\sum_{i=1}^{N}\alpha_i^2 \tag{3}$$

where E_b is the average energy per bit defined as with E_s is the average symbol energy. $E_b = E_s/\log_2(M)$

Ⅲ. Derivation and Analysis

In this section, important performance criteria for the MRC system operating over Rayleigh fading channels for the most general case will be studied.

3.1 pdf formulation

In order to evaluate exactly the performance of a MRC reception system over Rayleigh fading channels, the probability density function of a sum of independent exponential random variables (r.v.'s) must be known. Depending on values of $\overline{\gamma_i}$, we can classify them into three distinguished cases:

a) $\overline{\gamma_1}=\cdots=\overline{\gamma_i}=\cdots=\overline{\gamma_N}=\overline{\gamma}$ for all *i*, this case is called as i.i.d. case which was considered in [12]-[13] and [15].

b) $\overline{\gamma_i}$ are distinct for all *i*, this case is called as i.n.d. case which was considered in [12].

c) Otherwise, some equalities among the $\overline{\gamma_i}$. Note that two above cases and the case mentioned in [10] are also special cases of case (c).

For ease of analysis, we sort and renumber the $\overline{\gamma_i}$ in ascending order as

$$\begin{aligned}&\overline{\gamma_1}=\cdots=\overline{\gamma_{r_1}}=\beta_1<\overline{\gamma_{r_1+1}}=\cdots=\overline{\gamma_{r_1+r_2}}=\beta_2<\cdots\\&<\overline{\gamma_{r_1+r_2+\cdots+r_{K-1}+1}}=\cdots=\overline{\gamma_{r_1+r_2+\cdots+r_K}}=\beta_K\end{aligned} \tag{4a}$$

For convenience, we rewrite (4a) as

$$\begin{aligned}\overline{\gamma_1}=\cdots=\overline{\gamma_{r_1}}&=\beta_1\\ \overline{\gamma_{r_1+1}}=\cdots=\overline{\gamma_{r_1+r_2}}&=\beta_2\\ &\vdots\\ \overline{\gamma_{r_1+\cdots+r_{K-1}+1}}=\cdots=\overline{\gamma_{r_1+\cdots+r_K}}&=\beta_K\end{aligned} \tag{4b}$$

where

- $\beta_1<\cdots<\beta_k<\cdots<\beta_K$.
- $\sum_{k=1}^{K} r_k=N$, r_k is a positive integer.

Because all Rayleigh channels are independent; the Laplace transform of the Rayleigh-distributed pdf can be evaluated in closed form with the result [16]:

$$\begin{aligned}M_\gamma(s)&=\prod_{i=1}^{N}M_{\gamma_i}(s)=\prod_{i=1}^{N}\left(\frac{1}{1-s\overline{\gamma_i}}\right)\\&=\left(\frac{1}{1-s\beta_1}\right)^{r_1}\cdots\left(\frac{1}{1-s\beta_K}\right)^{r_K}\\&=\prod_{k=1}^{K}\left(\frac{1}{1-s\beta_k}\right)^{r_k}\end{aligned} \tag{5}$$

where $M_{\gamma_i}(s)$ is the moment generating function (MGF) of the instantaneous fading γ_i given by

$$M_{\gamma_i}(s)=\frac{1}{1-s\overline{\gamma_i}} \tag{6}$$

Using the partial-fraction expansion [17] for the MGF, (5) can be shown that

$$M_\gamma(s)=\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[\frac{A_{k,n}}{(1-s\beta_k)^n}\right] \tag{7}$$

where

$$A_{k,n}=\frac{1}{(r_k-n)!}\left\{\frac{\partial^{(r_k-n)}}{\partial s^{(r_k-n)}}\left[(1-s\beta_k)^{r_k}M_\gamma(s)\right]\right\}\Big|_{s=\frac{1}{\beta_k}} \tag{8}$$

In addition, for convenience, the coefficients $A_{k,n}$ can be obtained more easily by solving the system of *N* equations which is established by randomly choosing *N* distinct values of *s* but not equal to any β_k. Denoting *N* values of *s* as B_u with $u=1,\ldots,N$, we can obtain the linear system of equations as

$$\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[\frac{A_{k,n}}{(1-B_u\beta_k)^n}\right]=\prod_{i=1}^{N}\left(\frac{1}{1-B_u\overline{\gamma_i}}\right) \tag{9}$$

Thus $A=[A_{1,1}\cdots A_{k,n}\cdots A_{K,r_K}]^T$ is obtained by $A=C^{-1}D$ where $[.]^T$ is a transpose operator, C is a $N\times N$ matrix whose elements are $C_{uv}=1/(1-B_u\beta_k)^n$ with $v=n+\sum_{l=1}^{k-1}r_l$, $D=[D_1\cdots D_u\cdots D_N]^T$ with $D_u=\prod_{i=1}^{N}(1-B_u\overline{\gamma_i})^{-1}$ and $u,v=1,\ldots,N$.

Finally, the pdf of γ is determined by the inverse Laplace transform of $M_\gamma(s)$ as follows [13], [17]:

$$f_\gamma(\gamma)=\left[\sum_{k=1}^{K}\sum_{n=1}^{r_k}A_{k,n}\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}\right]U(\gamma) \tag{10}$$

where U(.) is the unit-step function and $\Gamma(n)=\int_0^\infty t^{n-1}e^{-t}dt$.

3.2 Average output SNR

The average SNR per bit of the maximal ratio combiner output can be easily obtained from the first derivative of $M_\gamma(s)$ evaluated at $s=0$ [16, p. 4, eq. (1.2)]. Differentiating (7) with respect to s and evaluating the result at $s=0$, we obtain:

$$\overline{\gamma_{MRC}}=\frac{dM_\gamma(s)}{ds}\Big|_{s=0}=\sum_{k=1}^{K}\sum_{n=1}^{r_k}A_{k,n}n\beta_k \tag{11}$$

As a check, consider the i.i.d. case, i.e., $\overline{\gamma_1}=\cdots=\overline{\gamma_N}=\overline{\gamma}$. Then, from (11) we have:

$$\overline{\gamma_{MRC}} = N\bar{\gamma} \tag{12}$$

which agrees with [16, p. 332, eq. (9.55)].

3.3 The outage probability

P_{out} is defined as the probability that the MRC output SNR falls below a certain predetermined threshold SNR γ_{th} and hence can be obtained by integrating the pdf of γ:

$$P_{out} = \int_0^{\gamma_{th}} f_\gamma(\gamma) d\gamma \tag{13}$$

Solving (13) gives:

$$P_{out} = \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left\{A_{k,n}\left[1-e^{-\frac{\gamma_{th}}{\beta_k}}\sum_{u=0}^{n-1}\frac{\left(\gamma_{th}/\beta_k\right)^u}{u!}\right]\right\} \tag{14}$$

Note that for $\overline{\gamma_1} = \cdots = \overline{\gamma_N} = \bar{\gamma}$, (14) becomes:

$$P_{out} = 1 - e^{-\frac{\gamma_{th}}{\bar{\gamma}}}\sum_{u=0}^{N-1}\frac{(\gamma_{th}/\bar{\gamma})^u}{u!} \tag{15}$$

which is in agreement with the previous known result [15, p. 283, eq. (6.25)] as expected.

3.4 Bit Error Rate

In this section, we derive the exact bit error probability of *M*-PSK, *M*-PAM and *M*-QAM for the most general case of $\overline{\gamma_i}$. It is assumed that the bit-symbol mappings follow a Gray code. To obtain the BER of *M*-PSK, *M*-PAM or *M*-QAM with MRC reception over Rayleigh fading channels, the derivation methods mentioned in [12], [16] are employed.

3.4.1 M-PSK

To obtain the BER of *M*-PSK with MRC on Rayleigh fading channels, we proceed analogous to [12]

$$P_b^{PSK}(\epsilon) = \frac{1}{\log_2 M}\sum_{m=1}^{M} e_m \Pr\{\theta \in \Theta_m\} \tag{16}$$

where $\Theta_m = [\theta_L^m, \theta_U^m] = [(2m-3)\pi/M; \ (2m-1)\pi/M]$ for $m = 1,...,M$ and e_m is the number of bit errors in the decision region Θ_m. With no loss of generality, it is assumed that $\phi = 0$, the probability $\Pr\{\theta \in \Theta_m\}$ is

$$\begin{aligned}\Pr\{\theta \in \Theta_m\} &= \int_{\theta_L^m}^{\theta_U^m}\int_0^\infty f_\theta(\theta|\phi,\gamma) f_\gamma(\gamma) d\gamma d\theta \\ &= \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left\{A_{k,n}\left[\int_{\theta_L^m}^{\theta_U^m}\int_0^\infty f_\theta(\theta|\phi,\gamma)\frac{1}{\Gamma(n)}\frac{\gamma^{n-1}}{\beta_k^n}e^{-\frac{\gamma}{\beta_k}}d\gamma d\theta\right]\right\} \\ &= \sum_{k=1}^{K}\sum_{n=1}^{r_k}\{A_{k,n} I_n[\theta_U^m, \theta_L^m; \beta_k]\end{aligned} \tag{17}$$

where $f_\theta(\theta|\phi,\gamma)$ is defined by [12, eq. (9b)] and using the analysis in [12], $I_n\left[\theta_U^m, \theta_L^m; \beta_k\right]$ can be derived as follows:

$$I_n\left[\theta_U^m, \theta_L^m; \beta_k\right]) = \frac{\theta_U^m - \theta_L^m}{2\pi} + \frac{1}{2}\rho_U^m\left\{\begin{array}{c}\left(\frac{1}{2} + \frac{\tan^{-1}(\alpha_U^m)}{\pi}\right)\sum_{p=0}^{n-1}\binom{2p}{p}\frac{1}{\left[((\mu_U^m)^2+1\right]^p} + \\ \frac{\sin(\tan^{-1}(\alpha_U^m))}{\pi}\sum_{p=1}^{n-1}\sum_{q=1}^{p}\frac{T_{qp}}{\left[4((\mu_U^m)^2+1\right]^p}cos^{2(p-q)+1}(\tan^{-1}(\alpha_U^m))\end{array}\right\}$$

$$-\frac{1}{2}\rho_L^m \left\{ \begin{array}{l} \left(\frac{1}{2}+\frac{\tan^{-1}(\alpha_L^m)}{\pi}\right)\sum_{p=0}^{n-1}\binom{2p}{p}\frac{1}{\left[((\mu_L^m)^2+1\right]^p}+ \\ \frac{\sin(\tan^{-1}(\alpha_L^m))}{\pi}\sum_{p=1}^{n-1}\sum_{q=1}^{p}\frac{T_{qp}}{\left[4((\mu_L^m)^2+1\right]^p}cos^{2(p-q)+1}(\tan^{-1}(\alpha_L^m)) \end{array} \right\} \tag{18a}$$

$$\mu_U^m = \sqrt{\log_2(M)\beta_k}\, sin(\theta_U^m), \mu_L^m = \sqrt{\log_2(M)\beta_k}\, sin(\theta_L^m) \tag{18b}$$

$$\alpha_U^m = \sqrt{\log_2(M)\beta_k}\, cos(\theta_U^m)/\sqrt{(\mu_U^m)^2+1}\,,\ \alpha_L^m = \sqrt{\log_2(M)\beta_k}\, cos(\theta_L^m)/\sqrt{(\mu_L^m)^2+1} \tag{18c}$$

$$\rho_U^m = \mu_U^m/\sqrt{(\mu_U^m)^2+1}\,, \rho_L^m = \mu_L^m/\sqrt{(\mu_L^m)^2+1} \tag{18d}$$

$$T_{qp} = \frac{\binom{2p}{p}}{\binom{2(p-q)}{p-q}4^q[2(p-q)+1]} \tag{18e}$$

3.4.2 *M*-PAM

For *M*-PAM in which $M=2^m$ with $m=1,2,\ldots$, the BER in the AWGN channel is given in [18] as

$$P_b^{PAM}(\epsilon|\gamma) = \frac{1}{M\log_2 M}\sum_{u=1}^{\log_2 M}\sum_{v=0}^{(1-2^{-u})M-1}\Xi_v^u erfc\left(\sqrt{\Psi_v\gamma}\right)] \tag{19a}$$

$$\Xi_v^u = (-1)^{\left\lfloor \frac{v2^{u-1}}{M}\right\rfloor}\left(2^{u-1}-\left\lfloor \frac{v2^{u-1}}{M}+\frac{1}{2}\right\rfloor\right) \tag{19b}$$

$$\Psi_v = (2v+1)^2 3(\log_2 M)/(M^2-1) \tag{19c}$$

where γ denotes the SNR per bit, $\lfloor x \rfloor$ denotes the largest integer to x and erfc(.) is the complimentary err or function.

In order to obtain the average BER over Rayleigh fading channels, we take the expectation with respect to the channel and use the results obtained in [Appendix]. Then we have:

$$\begin{aligned} P_b^{PAM}(\epsilon) &= \int_0^\infty \left[\frac{1}{M\log_2 M}\sum_{u=1}^{\log_2 M}\sum_{v=0}^{(1-2^{-u})M-1}\Xi_v^u erfc\left(\sqrt{\Psi_v\gamma}\right)\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}\right]\right]d\gamma \\ &= \frac{1}{M\log_2 M}\sum_{u=1}^{\log_2 M}\sum_{v=0}^{(1-2^{-u})M-1}\Xi_v^u\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}H[\Psi_v,\beta_k,n]\right] \\ &= \frac{1}{M\log_2 M}\sum_{u=1}^{\log_2 M}\sum_{v=0}^{(1-2^{-u})M-1}\Xi_v^u\sum_{k=1}^{K}\sum_{n=1}^{r_k}A_{k,n}\left[1-\sqrt{\frac{\Psi_v\beta_k}{1+\Psi_v\beta_k}}\sum_{j=0}^{n-1}\binom{2j}{j}\frac{1}{[4(1+\Psi_v\beta_k]^j}\right] \end{aligned} \tag{20}$$

3.4.3 *M*-QAM

It is straightforward to find BER of a rectangular or square QAM if we treat it as two independent PAM constellations [14], [18]. Consider two independent PAM constellations: I-ary PAM for the in-phase component and J-ary PAM for the quadrate component, where $M=2^m=I\times J$ with $m=1,2,\ldots$. The exact average BER of *M*-QAM in an AGWN channel is given by [18]

$$P_b^{QAM}(\epsilon|\gamma) = \frac{1}{\log_2(I.J)}\left(\begin{array}{l}\frac{1}{I}\sum_{u=1}^{\log_2 I}\left[\sum_{l_1=0}^{(1-2^{-u})I-1}\left[\Gamma_I(u,l_1)erfc\left(\sqrt{\Lambda_u\gamma}\right)\right]\right]+ \\ \frac{1}{J}\sum_{v=1}^{\log_2 J}\left[\sum_{l_2=0}^{(1-2^{-v})J-1}\left[\Gamma_J(v,l_2)erfc\left(\sqrt{\Lambda_v\gamma}\right)\right]\right]\end{array}\right) \tag{21a}$$

where

$$\Gamma_X(a,b) = (-1)^{\left\lfloor \frac{b2^{a-1}}{X} \right\rfloor}\left(2^{a-1} - \left\lfloor \frac{b2^{a-1}}{X} + \frac{1}{2} \right\rfloor\right), \quad \Lambda_c = \frac{(2c+1)^2 3\log_2(I.J)}{I^2+J^2-2} \tag{21b}$$

For this case, it is similar to the case of M-PAM, from (2), (10) & (21), we have:

$$P_b^{QAM}(\epsilon) = \int_0^\infty \left\{ \frac{1}{\log_2(I.J)} \begin{pmatrix} \frac{1}{I}\sum_{u=1}^{\log_2 I}\left[\sum_{l_1=0}^{(1-2^{-u})I-1}\left[\Gamma_I(u,l_1)erfc\left(\sqrt{\Lambda_u\gamma}\right)\right]\right] \\ \frac{1}{J}\sum_{v=1}^{\log_2 J}\left[\sum_{l_2=0}^{(1-2^{-v})J-1}\left[\Gamma_J(v,l_2)erfc\left(\sqrt{\Lambda_v\gamma}\right)\right]\right] \end{pmatrix} \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}\right] \right\} d\gamma$$

$$= \frac{1}{\log_2(I.J)} \begin{pmatrix} \frac{1}{I}\sum_{u=1}^{\log_2 I}\left[\sum_{l_1=0}^{(1-2^{-u})I-1}\left[\Gamma_I(u,l_1)\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}\int_0^\infty erfc\left(\sqrt{\Lambda_u\gamma}\right)\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}d\gamma\right]\right]\right] \\ \frac{1}{J}\sum_{v=1}^{\log_2 J}\left[\sum_{l_2=0}^{(1-2^{-v})J-1}\left[\Gamma_J(v,l_2)\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}\int_0^\infty erfc\left(\sqrt{\Lambda_v\gamma}\right)\frac{\gamma^{n-1}e^{-\gamma/\beta_k}}{\Gamma(n)\beta_k^n}d\gamma\right]\right]\right] \end{pmatrix} \tag{22}$$

By using the result in [Appendix], we can rewrite (22) as follow

$$P_b^{QAM}(\epsilon) = \frac{1}{\log_2(I.J)} \begin{pmatrix} \frac{1}{I}\sum_{u=1}^{\log_2 I}\left[\sum_{l_1=0}^{(1-2^{-u})I-1}\left[\Gamma_I(u,l_1)\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}H[\Gamma_u,\beta_k,n]\right]\right]\right] \\ \frac{1}{J}\sum_{v=1}^{\log_2 J}\left[\sum_{l_2=0}^{(1-2^{-v})J-1}\left[\Gamma_J(v,l_2)\sum_{k=1}^{K}\sum_{n=1}^{r_k}\left[A_{k,n}H[\Gamma_v,\beta_k,n]\right]\right]\right] \end{pmatrix}$$

$$= \frac{1}{\log_2(I.J)} \begin{pmatrix} \frac{1}{I}\sum_{u=1}^{\log_2 I}\left[\sum_{l_1=0}^{(1-2^{-u})I-1}\left[\Gamma_I(u,l_1)\sum_{k=1}^{K}\sum_{n=1}^{r_k}A_{k,n}\left[1-\sqrt{\frac{\Lambda_u\beta_k}{1+\Lambda_u\beta_k}}\sum_{j_1=0}^{n-1}\binom{2j_1}{j_1}\frac{1}{[4(1+\Lambda_u\beta_k)]^{j_1}}\right]\right]\right] \\ \frac{1}{J}\sum_{v=1}^{\log_2 J}\left[\sum_{l_2=0}^{(1-2^{-v})J-1}\left[\Gamma_J(v,l_2)\sum_{k=1}^{K}\sum_{n=1}^{r_k}A_{k,n}\left[1-\sqrt{\frac{\Lambda_v\beta_k}{1+\Lambda_v\beta_k}}\sum_{j_2=0}^{n-1}\binom{2j_2}{j_2}\frac{1}{[4(1+\Lambda_v\beta_k)]^{j_2}}\right]\right]\right] \end{pmatrix} \tag{23}$$

Ⅳ. Applications

In this section, some application scenarios whose performance can be evaluated by using our derived expressions will be performed for illustrative purpose.

4.1 Repetition Coding

The repetition coding problem was described in [8]. In here, by using our derived formula of pdf, the exact closed-form expression of outage probability instead of limiting analysis of repetition coding system over Rayleigh fading can be obtained. In particular, we must calculate the following expression [8, eq. (9)]:

$$P_o = \Pr\left[\sum_{i=1}^{N} SNR|h_i|^2 < 2^{NR}-1\right] \tag{24}$$

where N denotes the number of blocks, h_i captures the effects of path-loss and multi-path fading; R is pre-specified transmission rate. If we let $\gamma_i = SNR|h_i|^2, \gamma = \sum_{i=1}^{N}\gamma_i$ with $\overline{\gamma_i} = \lambda_i SNR, \gamma_{th} = 2^{NR}-1$ and applying the result given by (15), we have:

$$P_o = \sum_{k=1}^{K}\sum_{n=1}^{r_k}\left\{A_{k,n}\left[1-e^{-\frac{\gamma_{th}}{\beta_k}}\sum_{u=0}^{n-1}\frac{(\gamma_{th}/\beta_k)^u}{u!}\right]\right\} \tag{25}$$

4.2 Alamouti code

Another typical application is to establish the exact closed-form BER expression of the Alamouti code [19] for 2 transmit antennas (Tx's) and P receive ones (Rx's).

Alamouti code for two Tx's is represented by a transmission matrix

$$\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \tag{26}$$

where s_1 and s_2 are two consecutive *M*-PSK, *M*-PAM or *M*-QAM modulated symbols; (.)* denotes complex conjugate operator.

The signal transmission on two Tx's is processed as follows. At the first time slot, s_1 and s_2 are simultaneously sent on antenna 1 and 2, respectively. Then, $-s_2^*$ and s_1^* continue to be transmitted on antenna 1 and 2 at the second time slot.

Channel model: The flat fading channel is usually assumed for most spatial diversity systems in which path gains $h_{t,i}$ from Tx *t* to Rx *i* are modeled as samples of independent zero-mean complex Gaussian random variables (ZMCGRVs) with variances $2\lambda_{t,i}$, and are constant during two-symbol durations but change over longer intervals where $t=1,2$ and $i=1,...,P$.

Receiver: Consider the case of *P* Rx's. The received signal is a superposition of signals from two Tx's attenuated by flat fading and corrupted by noise given by

$$r_{1,i} = s_1 h_{1,i} + s_2 h_{2,i} + n_{1,i} \tag{27a}$$

$$r_{2,i} = -s_2^* h_{1,i} + s_1^* h_{2,i} + n_{2,i} \tag{27b}$$

where $r_{1,i}$ and $r_{2,i}$ are the received signals at the 1st and 2nd time-durations of Rx *i* $n_{1,i}$ and $n_{2,i}$ are independent zero-mean complex Gaussian random variables with variance λ.

Assuming coherent detection, maximum likelihood (ML) decoding can be achieved based only on linear processing at the receiver [20]. As a result, the symbols s_1 and s_2 are estimated by

$$s'_1 = \sum_{i=1}^{P} \left[r_{1,i} h_{1,i}^* + (r_{2,i})^* h_{2,i} \right] \tag{28a}$$

$$s'_2 = \sum_{i=1}^{P} \left[r_{1,i} h_{2,i}^* - (r_{2,i})^* h_{1,i} \right] \tag{28b}$$

Substituting $r_{1,i}$ and $r_{2,i}$ from (27) into (28), we obtain

$$s'_1 = \sum_{i=1}^{P} \left(|h_{1,i}|^2 + |h_{2,i}|^2 \right) s_1 + n_1 \tag{29a}$$

$$s'_2 = \sum_{i=1}^{P} \left(|h_{1,i}|^2 + |h_{2,i}|^2 \right) s_2 + n_2 \tag{29b}$$

where

$$n_1 = \sum_{i=1}^{P} n_{1,i} h_{1,i}^* + n_{2,i}^* h_{2,i} \tag{30a}$$

$$n_2 = \sum_{i=1}^{P} n_{1,i} h_{2,i}^* - n_{2,i}^* h_{1,i} \tag{30b}$$

Let

$$\Omega = \sum_{i=1}^{P} \left(|h_{1,i}|^2 + |h_{2,i}|^2 \right) = \sum_{u=1}^{N=2P} |h_u|^2 \tag{31}$$

where $|h_u|^2 = |h_{t,i}|^2$ are exponentially distributed r.v.'s with $u=(t-1)P+i$.

From (30), we find that n_1 and n_2 are ZMCGRVs with the identical variance $\Omega\lambda$, given the channel realizations. Rewrite (29) in the following form

$$s'_1 = \Omega s_1 + n_1 \tag{32a}$$

$$s'_2 = \Omega s_2 + n_2 \tag{32b}$$

Because s_1 and s_2 are attenuated and corrupted by the same fading and noisy level, their probability of error is equal. As a result, BER of s_1 is sufficient to evaluate the performance of the system. Applying the results given by (16), (20) and (23), we can easily obtain the bit average BERs of the Alamouti code for 2 transmit antennas (Tx's) and P receive ones (Rx's) for *M*-PSK, *M*-PAM, *M*-QAM, respectively.

Ⅴ. Numerical Results

In this section, some examples of the average BER of M-PSK, M-PAM and M-QAM over Rayleigh fading channel paths are given. Results computed using our theoretical analysis and Monte Carlo simulation are compared. Let us denote ω as a vector consisting of value of λ_i of each path, ω is appropriately chosen for the illustrative purpose, i.e. $\omega = [0.5\ 0.5\ 1\ 1\ 1\ 1.5]$, it is straightforward to see that $[\beta_1\ \beta_2\ \beta_3] = [0.5\ 1\ 1.5]$ and $[r_1\ r_2\ r_3] = [2\ 2\ 1]$. From Fig. 1 and Fig. 3, we study the average BER performance for different levels of *M*-PSK, *M*-PAM and *M*-QAM modulation, respectively. Note that with Gray

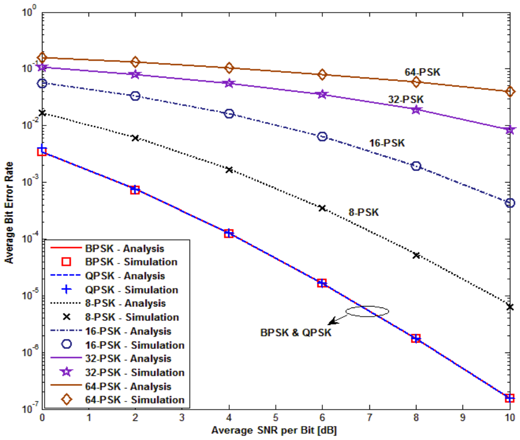

그림 1. 레일레이 페이딩에서 M-PSK에 대한 BER성능
Fig. 1. Exact BER for *M*-PSK over Rayleigh fading paths

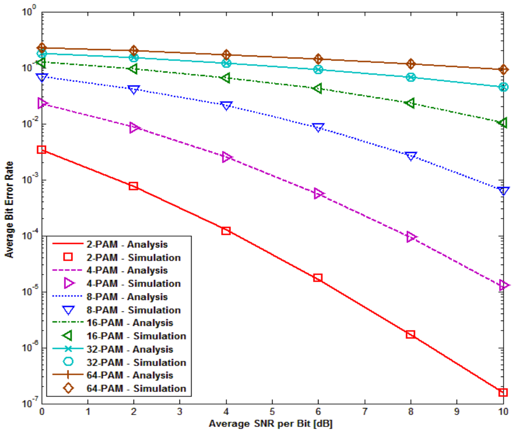

그림 2. 레일레이 페이딩에서 M-PAM에 대한 BER성능
Fig. 2. Exact BER for *M*-PAM over Rayleigh fading paths

code used for bit-symbol mappings, average BER of BPSK is same with that of QPSK and 4-QAM. In addition, it can be seen that, our analytical results and the simulation results are in excellent agreement.

Fig. 4 shows the outage probability of repetition coding for different values of N where a certain combination of λ_i is chosen correspondingly to each N for the illustration. In particular, we selected ω as $\omega=[1]$, $\omega=[0.5\ 1]$, $\omega=[0.5\,1\,1]$, $\omega=[0.5\,1\,1\,1]$, $\omega=[0.5\ 0.5\ 1\ 1\ 2]$ for N = 1, 2, 3, 4, 5, respectively. It can be observed from the figure that the analytical results match tightly the simulation results. Moreover, it is realized that the diversity order

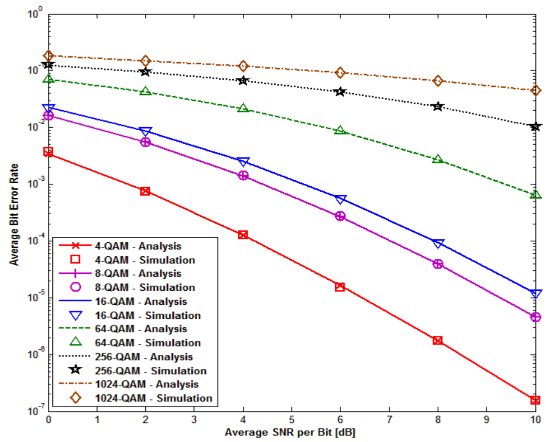

그림 3. 레일레이 페이딩에서 M-QAM에 대한 BER성능
Fig. 3. BER for *M*-QAM over Rayleigh fading paths

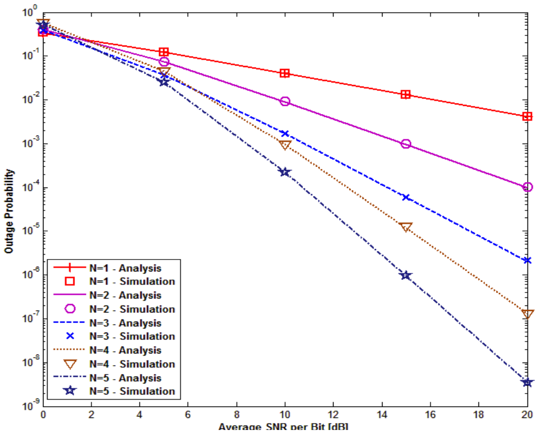

그림 4. $R=0.5$bps/Hz 에서 반복 부호화의 Outage 확률
Fig. 4. Outage Probability of repetition coding for $R=0.5$ bps/Hz

increases according to the number of blocks N which is consistent with what was found in [8].

In Fig. 5 and Fig. 6, we study the average BER of the Alamouti code for 2 transmit antennas (Tx's) and 6 receive ones (Rx's) for *M*-PSK, *M*-QAM over Rayleigh fading paths whose expected values of channel powers are given in Table 1.

It can be seen that there is no difference between the theoretical formulas and simulation results. In addition, the more receive antennas are deployed, the better performance is obtained. It also noticed that all results match with that reported in [20].

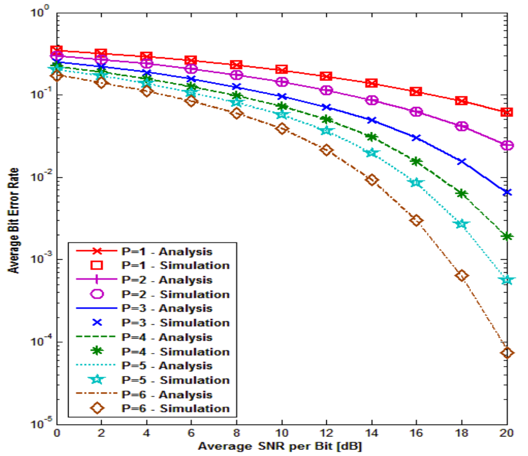

그림 5. 1024-QAM에서 Alamouti 부호의 평균 BER성능
Fig. 5. Average BER of Alamouti code for 1024-QAM with different values of *P*

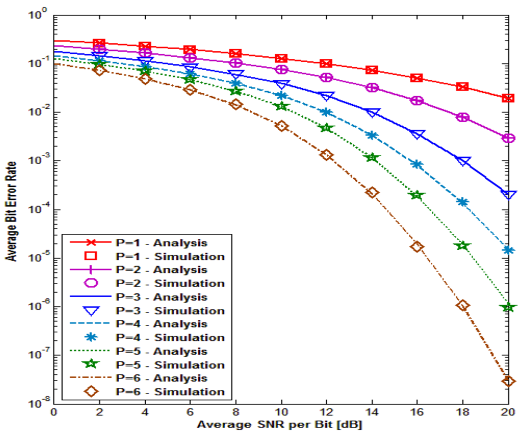

그림 6. 32-PSK에서 Alamouti 부호의 평균 BER성능
Fig. 6. Average BER of Alamouti code for 32-PSK with different values of *P*

Table 1

$\lambda_{t,i}$	$i=1,\ldots,6$					
$\lambda_{1,i}$	0.5	0.5	1	1	1	2
$\lambda_{2,i}$	0.5	0.5	1	1	1	2

Ⅵ. Conclusion

In this paper, the average BER for *M*-PSK, *M*-PAM, *M*-QAM and outage probability of MRC over slow and frequency non-selective fading channels were analyzed. Their validity was demonstrated by a variety of Monte-Carlo simulations. The expressions are general and offer a convenient way to evaluate any system which exploits MRC technique.

Appendix

The purpose of this appendix is to evaluate the integral used in (19) and (22):

$$H_1[a,b,n]=\int_0^{\infty} erfc(\sqrt{a\gamma})\frac{\gamma^{n-1}e^{-\frac{\gamma}{b}}}{\Gamma(n)b^n}d\gamma \qquad (32)$$

where $erfc(x)=\frac{2}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{\sin^2\theta}\right)d\theta$ is defined in [16, p. 121, eq. (4A.6)]. Interchange the order of integration and apply the result in [16, p. 149, eq. (5A.4a)], we obtain

$$\begin{aligned} &H_1[a,b,n]\\ &=\frac{2}{\pi}\int_0^{\pi/2}\left[\int_0^{\infty}\exp\left(-\frac{a}{\sin^2\theta}\right)\frac{\gamma^{n-1}e^{-\gamma/b}}{\Gamma(n)b^n}\right]d\theta\\ &=\frac{2}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+ab}\right)^n d\theta \qquad (33)\\ &=1-\sqrt{\frac{ab}{1+ab}}\sum_{u=0}^{n-1}\binom{2u}{u}\frac{1}{[4(1+ab)]^u} \end{aligned}$$

References

〔1〕 A. Nosratinia, A. Hedayat, and T. E. unter, "Cooperative Communication in Wireless Networks", *IEEE Communications Magazine*, Vol.42,10, pp.74-80, Oct. 2004.

〔2〕 J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior", *IEEE Trans. on Inform. Theory*, Vol.50,12,3062－3080, Dec. 2004.

〔3〕 A. K. Sadek, W. Su, and K. J. R. Liu, "A class of cooperative communication protocols for multi-node wireless networks", *2005 IEEE 6th Workshop on Signal Processing Advances in Wireless Communications*, pp.560-564, June 2-8, 2005.

〔4〕 In-Ho Lee, Dongwoo Kim, "BER Analysis for Decode-and-Forward Relaying in Dissimilar

Rayleigh Fading Channels", *IEEE Communications Letters*, Vol.11, No.1, Jan 2007.

〔5〕 Norman C. Beaulieu, Jeremiah Hu, " A Closed-Form Expression for the Outage Probability of Decode-and-Forward relaying in Dissimilar Rayleigh Fading channels", *IEEE Communication letters*, Vol.10, No.12, Dec 2006.

〔6〕 Abdulkareem Adinoyi, Halim Yanikomeroglu, "Cooperative Relaying in Multi-Antenna Fixed Relay Networks," *IEEE Trans. On Wireless Communications*, Vol.6, No.2, Feb 2007.

〔7〕 Ahmed K. Sadek, Weifeng Su, K. J. Ray Liu, "Multinode Cooperative Communications in Wireless Networks," *IEEE Trans. on Signal Processing*, Vol.55, No.1, Jan 2007.

〔8〕 J. N. Laneman, "Limiting analysis of outage probabilities for diversity schemes in fading channels", *GLOBECOM 2003*, Vol.3,p.1242 – 1246, 1-5 Dec. 2003.

〔9〕 D. G. Brennan, "Linear Diversity Combining Techniques", *PROCEEDINGS OF THE IEEE*, Vol.91, No.2, Feb. 2003.

〔10〕 Ho Van Khuong, Hyung Yun Kong, "Genral Expression for pdf of a sum of independent Exponential Random Varialbles", *IEEE Communications Letters*, Vol.10, No.3, Mar 2006.

〔11〕 S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE Trans. on Commun.*, Vol.16,8, pp.1451-1458, Oct. 1998.

〔12〕 S. Chennakeshu and J. B. Anderson, "Error rates for Rayleigh fading multichannel reception of MPSK signals," page 74-80, *IEEE Trans. Commun.*, Vol.COM 43, February/March/April 1995, pp.338-346.

〔13〕 S. M. Ross, "Introduction to Probability Models", *Academic Press, Ninth Edition, 2007.*

〔14〕 John Proakis,"Digital Communication - Fourth Edition," New York: *McGrawHill, 2001*

〔15〕 Gordon L. Stuber, "Principles of Mobile Communication - Second Edition", *Kluwer Academic Publishers, 2002.*

〔16〕 Marvin K. Simon,"Digital Communication over Fading channels - Second Edition," John Wiley & Sons, Inc., *Hoboken, New Jersey, 2005.*

〔17〕 M. J. Roberts, "Signals and Systems", *McGraw Hill, 2003.*

〔18〕 Kyongkuk Cho, Dongewon Yoon, "On the general BER expression of one-and-two dimensional amplitude modulations," *IEEE Trans. on Comm,*. Vol.50, No.7, July 2002.

〔19〕 S. M. Alamouti, "A simple transmit diversity technique for wireless communications", *IEEE Trans. on Commun.,* Vol.16,8, pp.1451-1458, Oct. 1998.

〔20〕 V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block coding for wireless communications: performance results", *IEEE Trans. on Commun.,*Vol.17, Issue 3, pp.451-460,1999.

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