

2-홉 무선 네트워크를 위한 하이브리드 복호 후 전달 기법의 성능 분석

정회원 공형운*

Performance Analysis of Hybrid Decode-and-Forward Schemes for 2-hop Wireless Network

Hyung-Yun Kong* *Regular Member*

요 약

본 논문에서는 하이브리드 복호 후 전달(HDF)기법을 이용하는 2홉 무선 통신 네트워크의 BER(Bit Error Rate) 성능을 분석한다. 기존의 HDF 기법은 릴레이에서 복호된 데이터의 전달유무를 수신되는 신호 대 잡음비(SNR)를 기준으로 한다. 반면 본 논문에서는 SNR 기준 방식 대신 대수 우도비(Log Likelihood Ratio, LLR) 분석 기법을 제안한다. 제안한 새로운 HDF 기법의 근사적 BER 표현식을 유도하며, Monte-Carlo 시뮬레이션으로 검증한다. 또한 HDF 기법을 위한 최적의 임계값을 계산한다. 다양한 수식으로 인한 결론은 본 논문에서 제안하는 LLR기반 HDF가 기존의 SNR기반 HDF보다 레일리 페이딩채널과 AWGN환경에서 어떠한 임계값과 어떠한 릴레이 위치에도 뛰어난 성능을 보임을 증명한다.

Key Words : HDF, LLR, Rayleigh Fading, AWGN

ABSTRACT

This paper analyses BER (Bit Error Rate) performance of 2-hop wireless communications networks with hybrid decode-and-forward (HDF) relays. The conventional HDF method is usually based on the receive signal-to-noise ratio (SNR) for the relay to decide whether to forward the decoded data in order to obviate the erroneous detection at the relay. In contrast, we propose a new solution of using log-likelihood ratio (LLR) as an efficient alternative to SNR. The approximate BER expressions of different HDF schemes are also derived and verified by Monte-Carlo simulations. In addition, we compute the optimum thresholds for HDF schemes. A variety of numerical results demonstrate that the new LLR-based HDF significantly outperforms the SNR-based HDF for any threshold level and relay location under flat Rayleigh fading channel plus AWGN (Additive White Gaussian Noise).

I. Introduction

Signal fading due to multi-path propagation is a serious problem in wireless communications. Using a diversified signal in which information

related to the same data appears in multiple time instances, frequencies, or antennas that are independently faded can reduce considerably this effect of the channel^[1]. Among well-known diversity techniques, the spatial diversity has

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* 울산대학교 전기전자 정보시스템 공학부 무선통신 연구실(hkong@mail.ulsan.ac.kr)

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received a great deal of attention in recent years because of the feasibility of deploying multiple antennas at both transmitter and receiver^[2]. However, when wireless mobiles may not be able to support multiple antennas due to size and power limitations, or other constraints^[3], the spatial diversity is not exploited. To overcome this restriction, a new technique, called cooperative communications, was born which allows single-antenna terminals to gain some benefits of transmit diversity^{[3]-[18]}. The main idea is that in a multi-user network, two or more users share their information and transmit jointly as a virtual antenna array^{[4]-[6]}. This enables them to obtain higher diversity than they could have individually. The way the users share information is by tuning into each other's transmitted signals and by processing information that they overhear. Since the inter-user channel is noisy and faded, this overheard information is not perfect. Hence, one has to carefully study the possible signaling strategies that can exploit the benefits of cooperative communications at most.

Some typical cooperative communications schemes (or signaling strategies) for 2-hop wireless networks including a source (S), a relay (R) and a destination (D) were proposed in [7]: fixed amplify-and-forward relaying (AF), fixed decode-and-forward relaying (DF) and selection relaying¹⁾ (HDF). For AF, R just amplifies and retransmits its received signal to D . Since the signal processing at R is very simple, AF is extensively studied in the literature [8]-[13] and its approximate BER expressions are also derived in [9]-[13] for different kinds of shadow-fading environments. However, AF causes the noise enhancement at R and requires more complexity at D due to the demand for the channel state information (CSI) of all links in the network through which the source signals can reach D for maximum ratio combining (MRC) at D [12]-[13]. DF can reduce the disadvantages of AF at D by allowing R to always decode, encode and forward the source information [7], [9] and [14]. Yet, if R decodes unsuccessfully, DF introduces decoding

error at R that can adversely effect the eventual detection at D . Additionally, there has been no its closed-form BER expression so far even though [9] investigated this problem, the final result is only the approximate conditional BER [9, (9)].

To overcome the drawback of DF, [7] and [15]-[18] proposed HDF where R must make an independent decision on whether to decode and forward the source information or not. Different performance criteria for making decision were also mentioned. In [15]-[17], CRC (Cyclic-Redundancy-Check) is used and the approximate BER expression for HDF is also derived. However, this expression is based on a relatively impractical assumption that there exists an ideal CRC code, i.e., the result of CRC can tell us exactly that the recovered information is correct or incorrect. In addition, using CRC causes the waste of transmission bandwidth due to redundant information insertion. A more commonly alternative criterion without any loss of spectral efficiency is SNR or the square amplitude of path gain in [3], [7] and [18]. We refer SNR-based HDF to as HDF-SNR where only the received signals with quality exceeding the preset threshold are decoded and retransmitted to D . Its analytical BER expression has not been established yet although some efforts in computing its BER were made in [18], the final result is only the upper bound.

It is shown that HDF-SNR outperforms DF^[3]. Nevertheless since HDF-SNR only relies on fading level to decide the retransmission without accounting for the noisy level, it reflects partially the characteristic of the received signal. In this paper, LLR instead of SNR is used to assess the quality of the received signal more reliably in that both noise and fading are taken into account. This proposed method is named HDF-LLR. In fact, LLR is mentioned extensively in the literature, e.g.^{[19]-[20]}, especially in [21]-[22] where

1) Selection relaying in [7] also appears under another terminology in [3] as Hybrid Decode-and-Forward. So, we use a unique notation HDF to refer to both terminologies in the sequel.

it is employed for the optimum receive antenna selection and its advantage over SNR is proved mathematically. Therefore, our work here is considered as the application extension of LLR in a different scenario: cooperative communications. However, our additional contributions include: 1) deriving the theoretical BER expressions for HDF-LLR and HDF-SNR to evaluate the performance without time-consuming simulations. These expressions also generalize that of DF when the BER threshold equals 1; 2) calculating the optimum decoding thresholds for HDF-SNR and HDF-LLR. It is noted that by the empirical rules, [18] proposed the optimum decoding threshold for HDF-SNR without showing any justification. In this paper, based on the approximate BER expression for HDF-SNR, we can quickly find the optimum decoding threshold in a very appealing form.

The rest of this paper is organized as follows. Section 2 describes the decode-and-forward schemes. Then we derive their approximate BER expressions in section 3. Numerical and simulation results are presented in section 4 to verify their validity and finally, the paper is concluded in section 5. The proofs and finding the optimum decoding thresholds are relegated to Appendices 1-4.

II. LLR-based HDF

Consider a 2-hop wireless network consisting of single-antenna terminals: a source (S), a relay (R) and a destination (D). Assuming that the channels between terminals experience independent fast and frequency-flat Rayleigh fading, i.e., they are constant during one-symbol period but change independently to the next. As a result if we denote α_{ij} as the path gain between transmitter i and receiver j , its amplitude $|\alpha_{ij}|$ follows the Rayleigh distribution with *pdf*(probability density function) given by

$$f_{|\alpha_{ij}|}(r) = \frac{2r}{\lambda_{ij}} \exp\left(-\frac{r^2}{\lambda_{ij}}\right) U(r) \quad (1)$$

and its phase is uniformly distributed over $[0, 2\pi]$. In (1), $\lambda_{ij} = E[|\alpha_{ij}|^2]$ is the average fading power, and $E[\cdot]$ and $U(\cdot)$ denote the expectation operator and the unit-step function, respectively.

Although [9] and [11] investigated the BER performance of AF and DF in a composite environment including Rayleigh fading, path loss and lognormal shadowing, they are unsuccessful in obtaining the compact BER expressions due to the complexity of lognormal distribution. Therefore, we ignore the lognormal shadowing term for convenience of analysis. To incorporate the path loss onto BER performance analysis, we use the model, which is commonly discussed in the literature (e.g. [18] and [23]), where the variance of α_{ij} is given by $\lambda_{ij} = (d_{SD}/d_{ij})^\eta$ with d_{ij} being the distance between transmitter i and receiver j , and η being the path loss exponent.

For convenience of presentation, we utilize discrete-time complex equivalent base-band models to express all the signals. In addition, we assume perfect channel-state information at all the respective receivers but not at the transmitters. Moreover, the BER analysis is only based on BPSK (Binary Phase Shift Keying) modulation which was also examined in [21]-[22].

The general decode-and-forward schemes (fixed or hybrid) consist of two phases.

2.1 First phase

In the first phase, S broadcasts a BPSK-modulated symbol a and so, the signals received at R and D are given by

$$y_{SR} = \alpha_{SR} \sqrt{E_S} \alpha + n_{SR} \quad (2)$$

$$y_{SD} = \alpha_{SD} \sqrt{E_S} \alpha + n_{SD} \quad (3)$$

where y_{ij} denotes a signal received at the

terminal j from the terminal i , n_{ij} a zero-mean unit-variance complex additive noise sample at the terminal j , E_i the average symbol energy (ASE) of the terminal i .

There are two conventional ways to process the received signal at R according to the decode-and-forward scheme:

Method 1

For DF, R always recovers the original data by maximum likelihood (ML) decoding as

$$\hat{a} = \text{sign}(y_R) \quad (4)$$

where $\text{sign}(\cdot)$ is a signum function and

$$y_R = \text{Re}(\alpha_{SR}^* y_{SR}) = |\alpha_{SR}|^2 \sqrt{E_S} a + n_R \quad (5)$$

with $n_R = \text{Re}(\alpha_{SR}^* n_{SR})$ is a Gaussian r.v. with zero-mean and variance $|\alpha_{SR}|^2/2$, given channel realization $\text{Re}(\cdot)$ is a real part.

Although DF is simple, if the detection at R is unsuccessful, the cooperation can be detrimental to the eventual detection of the symbols at D .

Method 2

This method forces R to evaluate the quality of the received signal and check whether it satisfies the preset requirement. If this is the case, the relay detects and forwards the restored data to D . Otherwise, it keeps silent in the second phase. Therefore, the problem of error retransmission induced by R in DF can be mitigated. So far, only the signal-to-noise ratio or the square amplitude of path gain $|\alpha_{SR}|^2$ is commonly used in assessing the reliability of a signal in HDF scheme.

The instantaneous BER at R for BPSK transmission is computed from (5) as

$$P_{e-SR} = Q\left(\sqrt{2E_S|\alpha_{SR}|^2}\right) \quad (6)$$

where $Q(\cdot)$ is a Q-function[24].

Since P_{e-SR} can also be calculated in term of LLR in [21], the preset requirement is adopted according to the error probability P_{eT} .

For HDF-SNR, the condition for R to transmit \hat{a} in (4) in the second phase is

$$P_{e-SR} \leq P_{eT} \quad \text{or} \quad |\alpha_{SR}|^2 \geq T_\alpha \quad (7)$$

where

$$T_\alpha = (\text{erf}^{-1}(1-2P_{eT}))^2/E_S$$

$$P_{eT} = Q(\sqrt{2E_S T_\alpha}) = \frac{1}{2}(1 - \text{erf}(\sqrt{E_S T_\alpha})) \quad (8)^2$$

(7) shows that the retransmission of R only depends on instantaneous fading level regardless of instantaneous noisy level in (5). Therefore, it reflects partially the characteristic of the received signal and thus in several cases the condition in (7) does not guarantee for R to detect the signal reliably at a priorly desired degree. For example if $a = +1$ is transmitted, then the large negative values of n_R can still cause the sum in (5) to be negative (equivalently, wrong decision is made) even though $|\alpha_{SR}|^2$ is extremely larger than T_α . As a consequence, we propose using LLR in [19]-[22] to account for the noise term in (5).

(5) yields the conditional probability density

$$p(y_R|a, |\alpha_{SR}|^2 \sqrt{E_S}) = \frac{\exp\left(-\frac{(y_R - |\alpha_{SR}|^2 \sqrt{E_S} a)^2}{2|\alpha_{SR}|^2/2}\right)}{\sqrt{2\pi|\alpha_{SR}|^2/2}}$$

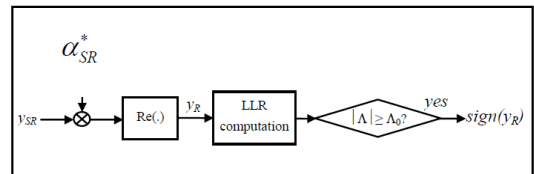


그림. 1 HDF-LLR 방식에서 중계기에서의 신호 처리 과정
Fig. 1 Signal processing at the relay for HDF-LLR

2) $\text{erf}^{-1}(\cdot)$ is the inverse error function easily calculated by Matlab software: $\text{erf}(\cdot)$ denotes the error function.

Assuming that $a = -1$ or $+1$ is equally likely, its a-posteriori LLR is given by

$$\begin{aligned} \Lambda &= \ln \frac{p(a=1|y_R, |\alpha_{SD}|^2 \sqrt{E_S})}{p(a=-1|y_R, |\alpha_{SD}|^2 \sqrt{E_S})} \\ &= \ln \frac{p(y_R|a=1, |\alpha_{SD}|^2 \sqrt{E_S})}{p(y_R|a=-1, |\alpha_{SD}|^2 \sqrt{E_S})} \\ &= \ln \frac{1}{\sqrt{2\pi|\alpha_{SD}|^2/2}} \exp\left(-\frac{(y_R - |\alpha_{SD}|^2 \sqrt{E_S}(1))^2}{2|\alpha_{SD}|^2/2}\right) \\ &= \ln \frac{1}{\sqrt{2\pi|\alpha_{SD}|^2/2}} \exp\left(-\frac{(y_R - |\alpha_{SD}|^2 \sqrt{E_S}(-1))^2}{2|\alpha_{SD}|^2/2}\right) \\ &= 4y_R \sqrt{E_S} \end{aligned} \quad (9)$$

It is well-known that the sign of Λ is the hard decision value, and its magnitude is a good measure of the reliability of symbols. Moreover, BER in term of Λ is also derived as [21]

$$P_{e-SR} = \frac{1}{1 + e^{|\Lambda|}} \quad (10)$$

For HDF-LLR, R sends $\hat{\alpha}$ in (4) in the second phase when

$$P_{e-SR} = \frac{1}{1 + e^{|\Lambda|}} \leq P_{eT}$$

or

$$|\Lambda| \geq \ln\left(\frac{1}{P_{eT}} - 1\right) = A_0 \quad (11)$$

From (9)-(11), we realize that the proposed method is different from HDF-SNR in that it accounts for both fading and noise terms in (5) and Λ provides the reliability information of the maximum a-posteriori probability decision which minimizes the error probability. Therefore, it is expected that it will result in a better performance than HDF-SNR.

2.2 Second phase

In the second phase, that the relay to send \hat{a} in (4) to D or not depends on the signal processing way in the first phase. Assuming that R assists S in data transmission, the signal arriving at D is of the form

$$y_{RD} = \alpha_{RD} \sqrt{E_R} \hat{a} + n_{RD} \quad (12)$$

For a fair comparison, it is essential that the total consumed energy of the cooperative system does not exceed that of corresponding direct transmission system [18]. Therefore, complying this energy constraint requires $E_S = E_R = E_T/2$ where E_T is total ASE of the system which is also the ASE of the source in case of direct transmission.

Now D combines the received signals from both phases based on MRC [25] and then detects the transmitted signal a as follows

$$\bar{a} = \text{sign}\left(\text{Re}\left(\alpha_{SD}^* y_{SD} + \alpha_{RD}^* y_{RD}\right)\right) \quad (13)$$

Using (3) and (12) and the fact that $E_S = E_R$ to rewrite (13) as

$$\begin{aligned} \bar{a} &= \text{sign}\left(\sqrt{E_S}\left(|\alpha_{SD}|^2 a + |\alpha_{RD}|^2 \hat{\alpha}\right) + n\right) \\ &= \text{sign}\left(\sqrt{E_S}\left(|\alpha_{SD}|^2 + \epsilon |\alpha_{RD}|^2\right) \alpha + n\right) \end{aligned} \quad (14)$$

Here $n = \text{Re}\left(\alpha_{SD}^* n_{SD} + \alpha_{RD}^* n_{RD}\right)$ is a Gaussian r.v. with zero-mean and variance $(|\alpha_{SD}|^2 + |\alpha_{RD}|^2)/2$, given channel realizations; $\epsilon = -1$ means that the relay made the wrong decision on the symbol a and otherwise, $\epsilon = 1$.

III. BER Performance Analysis

Since both HDF-SNR and HDF-LLR converge to DF when $P_{eT} = 1$, their approximate BER expressions derived in this section can generalize that of DF. This is another contribution of the paper besides proposing the application of LLR to replace SNR in HDF.

3.1 For HDF-SNR

There are two events causing the erroneous signal detection at D : 1) (7) not satisfied and an error happened in direct transmission, 2) (7) satisfied and an error still happened in the cooperative transmission from S and R to D . The

signal restoration at D is based on (13) for case 2) but for case 1), the term y_{RD} there is considered as zero.³⁾ Since these events are mutually exclusive, the average BER of HDF-SNR is given by

$$\overline{P_{e_HDF_SNR}} = \overline{P_{e_SNR1}} + \overline{P_{e_SNR2}} \quad (15)$$

where the averaging is taken over the distributions of path gains and

$$\overline{P_{e_SNR1}} = \overline{P_{e_SD}} \Pr [|\alpha_{SR}|^2 < T_\alpha] \quad (16)$$

$$\overline{P_{e_SNR2}} = \Pr [\overline{a} = 1 | a = -1, |\alpha_{SR}|^2 \geq T_\alpha] \quad (17)$$

The derivation of (17) is relied on the ML detection and the condition in (7) satisfied. Specifically, from (14) we rewrite (17) as

$$\begin{aligned} \overline{P_{e_SNR2}} &= \Pr [-\sqrt{E_S}(|\alpha_{SD}|^2 + |\alpha_{RD}|^2) + n > 0] \Pr [\epsilon = 1 | |\alpha_{SR}|^2 \geq T_\alpha] \\ &+ \Pr [-\sqrt{E_S}(|\alpha_{SD}|^2 + |\alpha_{RD}|^2) + n > 0] \Pr [\epsilon = -1 | |\alpha_{SR}|^2 \geq T_\alpha] \\ &= \overline{P_{e1}}(1 - \overline{P_{SR_SNR}}) + \overline{P_{e2}}\overline{P_{SR_SNR}} \end{aligned} \quad (18)$$

where P_{e1} is the probability that an error happens in the cooperative transmission from S and R to D given that R decoded correctly; P_{e2} is the probability that an error happens in the cooperative transmission from S and R to D given that R decoded unsuccessfully.

The terms in (16) and (18) are derived in the Appendix 2.

3.2 For HDF-LLR

The above procedure of deriving (15) can be used to find the average BER of HDF-LLR by replacing $|\alpha_{SR}|^2$ with $|A|$ and T_α with A_0 . In particular, it is given by

$$\overline{P_{e_HDF_LLR}} = \overline{P_{e_LLR1}} + \overline{P_{e_LLR2}} \quad (19)$$

where

$$\overline{P_{e_LLR1}} = \overline{P_{e_SD}} \Pr [|A| < A_0] \quad (20)$$

$$\overline{P_{e_LLR2}} = \overline{P_{e1}}(1 - \overline{P_{SR_LLR}}) + \overline{P_{e2}}\overline{P_{SR_LLR}} \quad (21)$$

The terms in (20)-(21) are shown in the Appendix 2. In addition, (15) and (19) are the basis for computing the optimum thresholds to minimize the error probability for HDF-LLR and HDF-SNR. Such computation is performed in the Appendices 3 and 4. Moreover, it is interesting that the optimum thresholds are the same for both HDF-LLR and HDF-SNR (see (A25) and (A26)).

IV. Numerical Results

An asymmetric network geometry is examined where the relay is located on a line between S and D . The direct path length S - D is normalized to be l . We also denote d as the distance between S and R . In all presented results, the path loss exponent $\eta = 3$ is under investigation.

We verify the accuracy of BER expressions in (15) and (19) by comparing with Monte-Carlo simulations. The results are depicted in Fig. 2 for $P_{eT} = 0.1$ and $d = 0.6$. We can see that the simulation results well match the theoretical ones. This shows that the theoretical BER expressions are almost exact. In addition, Fig. 2 demonstrates that no matter which version of HDF is used, the cooperation significantly improves the BER performance in comparison with direct transmission with E_T gain of about 8dB and 11dB at the target BER of 10^{-3} for HDF-SNR and HDF-LLR, respectively. These results are obvious because the cooperation benefits from diversity gain as well as from path-loss reduction.

Since the analysis agrees with simulation, we will use the theoretical formulas in (15) and (19) to prove the potentials of HDF-LLR in enhancing the BER performance in comparison to HDF-SNR in the sequel.

When R is near S , the quantity $|a_{SR}|^2$ is

3) D can detect the presence of the signal from R by measuring the signal strength in the second phase.

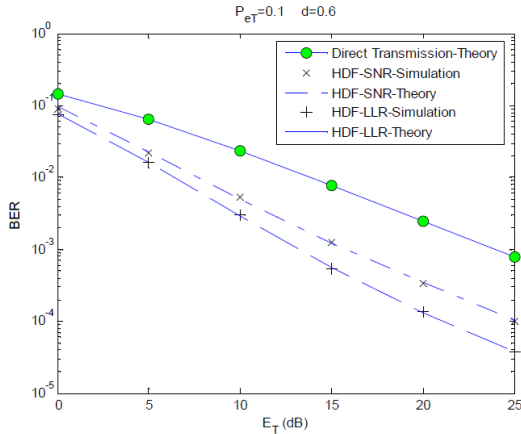


그림 2. 수식적 결과와 시뮬레이션 결과의 BER 비교
Fig. 2 BER comparison between theory and simulation

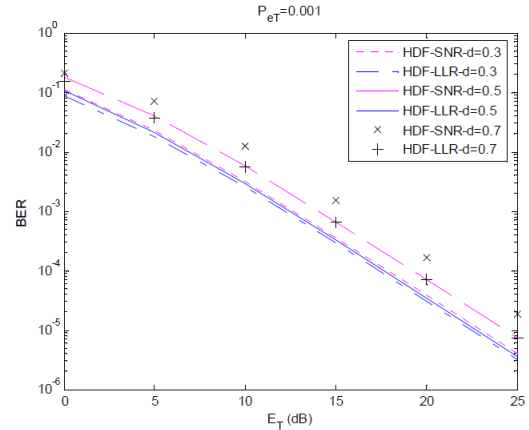


그림 3. d값에 따른 HDF-SNR과 HDF-LLR의 BER 비교
Fig. 3 BER comparison between HDF-SNR and HDF-LLR via d

usually large due to small path loss, leading to the first term in (5) to dominate the remaining term. Therefore in such case, HDF-SNR and HDF-LLR obtain the relatively same performance. This is illustrated in Fig. 3 for $d=0.3$. Moreover as d increases, the above property is no longer correct and now, the noise term n_R dramatically affects the sign of the expression (5). As a result, HDF-SNR performs worse for $d=0.5$ and 0.7 . The performance degradation also occurs similarly for HDF-LLR. However, since HDF-LLR evaluates the reliability of the received signal based on y_R , not on $|\alpha_{SR}|^2$ and n_R individually, its performance is superior to HDF-SNR. Specifically, HDF-LLR achieves a total energy gain of 2dB over HDF-SNR for $d=0.5$ and 0.7 over the whole range of E_T .

The influence of the threshold on BER performance of HDF-SNR and HDF-LLR is depicted in Fig. 4 for $d=0.8$. It is well-known that if the threshold (T_α or Λ_0) is so large, then the diversity order is reduced since the probability that the relay retransmits the source data is small. Also, the small threshold increases the percentage of incorrect detection at the relay and thus decaying the performance of the receiver. This remark is once again shown in Fig. 4 for both HDF-SNR and HDF-LLR. Moreover, we expect

their optimum thresholds. Based on (A25) or (A26) which is derived in the Appendix 3 or 4, we find the list of the optimum thresholds $P_{eT,opt}$ corresponding to each E_T and $d=0.8$ in Table 1. It is realized that these $P_{eT,opt}$ decrease with respect to the increase of E_T . The BER performances of HDF-LLR and HDF-SNR achieved from the threshold optimization are also plotted in Fig. 4. It is obvious that the optimum BER curves lie below all the others.

Fig. 4 also demonstrates that HDF-LLR performs considerably better than DF for any value of E_T and the investigated threshold. Nevertheless, HDF-SNR is really superior to DF when E_T is high. This is consistent with the remark in section 2 about the performance of evaluating the reliability of the received signal through SNR and LLR. For large values of E_T , HDF-SNR really performs well because of the dominance of the first term in (5) over the second one. Otherwise, the noise term n_R plays an important role and the wrong decision can be made if ignoring it. Therefore, HDF-LLR which considers both terms shows its advantage under any condition of threshold as well as E_T . Moreover, it is realized that HDF-LLR attains better performance than HDF-SNR with the

energy savings of 2dB at any target BER.

V. Conclusion

LLR is an efficient measure for evaluating the reliability of a signal. Its application in HDF illustrates that the LLR-based HDF dramatically outperforms SNR-based HDF and threshold-free decode-and-forward scheme (DF) under any scenario of BER threshold, relay position and transmit power. The approximate BER formulas of the proposed scheme as well as those of the existing schemes such as DF and HDF-SNR, which have not been reported yet, are also derived and confirmed by simulations to verify its superiority to the others. In addition, the mathematical proofs in Appendices 3 and 4 show that the optimum thresholds (P_{eT_opt}) for HDF-LLR and HDF-SNR are identical and must be chosen decreasingly according to the increase of the transmit power.

HDF-LLR is a very simple schemewith high BER performance and thus, should be considered as a promising technical solution for cooperative communications in the future wireless networks to improve the quality of information transmission and extend the coverage area as well.

Appendix 1

This Appendix calculates some useful integrals for use in the sequel. First, applying [26, (7) on page 361 and (4) on page 880] and [27, (2) and (14)], we obtain

$$f_\epsilon(\epsilon, \lambda_1, \lambda_2) = \int_0^\infty g^2 e^{-\lambda_1 g^2 - \lambda_2 \sqrt{\epsilon} g} dg \tag{A1}$$

$$\cong -\frac{\lambda_2 \sqrt{\epsilon}}{4\lambda_1^2} + \sqrt{\frac{\pi}{\lambda_1^5}} \frac{\lambda_2^2 \epsilon / 2 + \lambda_1 \left(\frac{1}{6} + \frac{1}{2} e^{-\frac{\lambda_2^2 \epsilon}{12\lambda_1}} \right)}{4} \tag{A1}$$

$$f(C, \lambda_1, \lambda_2) = \int_0^\infty Q(\sqrt{2C\epsilon}) \left[\int_0^\infty \frac{g^2}{\sqrt{\epsilon}} e^{-\lambda_1 g^2 - \lambda_2 \sqrt{\epsilon} g} dg \right] d\epsilon \tag{A2}$$

$$\cong \frac{1}{2} \int_0^\infty \left[\frac{1}{6} e^{-C\epsilon} + \frac{1}{2} e^{-4C\epsilon/3} \right] \frac{1}{\sqrt{\epsilon}} f_\epsilon(\epsilon, \lambda_1, \lambda_2) d\epsilon$$

where the function $\Phi(\cdot)$ is defined in [26, (1) on page 880].

By substituting (A1) into (A2) and changing the variable $L = \sqrt{\epsilon}$, (A2) is rewritten as

$$f(C, \lambda_1, \lambda_2) = \frac{1}{2} \int_0^\infty \left[\left(\frac{1}{6} e^{-\alpha L^2} + \frac{1}{2} e^{-4\alpha L^2/3} \right) \cdot \left(-\frac{\lambda_2 L}{2\lambda_1^2} + \sqrt{\frac{\pi}{\lambda_1^5}} \frac{\lambda_2^2 L^2 / 2 + \lambda_1 \left(\frac{1}{6} + \frac{1}{2} e^{-\frac{\lambda_2^2 L^2}{12\lambda_1}} \right)}{4} \right) \right] dL \tag{A3}$$

Applying the results in [26, (2)-(3) on page 360], we can compute (A3) as

$$f(C, \lambda_1, \lambda_2) = \frac{1}{2} \left[-\frac{13\lambda_2}{96C\lambda_1^2} \sqrt{\frac{\pi}{\lambda_1^5}} \left(\sqrt{\frac{\pi}{C}} \left(\frac{\lambda_2^2 + 4C\lambda_1}{576C} \right) + \sqrt{\frac{12\lambda_1\pi}{12\lambda_1 C + \lambda_2^2}} \frac{\lambda_1(\lambda_2^2 + 3\lambda_1 C)}{12(12\lambda_1 C + \lambda_2^2)} + \sqrt{\frac{3\pi}{4C}} \frac{3\lambda_2^2 + 16C\lambda_1}{768C} + \sqrt{\frac{12\lambda_1\pi}{16\lambda_1 C + \lambda_2^2}} \frac{\lambda_1(\lambda_2^2 + 4\lambda_1 C)}{4(16\lambda_1 C + \lambda_2^2)} \right) \right] \tag{A4}$$

Appendix 2

This appendix derives the approximate expressions for the terms in section 3. Before deriving these expressions, we introduce new random variables as follows

- Let $x = |\alpha_{SD}|^2, y = |\alpha_{RD}|^2, q = |\alpha_{SR}|^2$. Since α_{ij} are zero-mean complex Gaussian r.v.'s with variance λ_{ij} as mentioned in section 2, x, y and q have exponential distribution with mean values λ_{ij} ; of that is,

$$f_x(x) = \lambda_x e^{-\lambda_x x} U(x); f_y(y) = \lambda_y e^{-\lambda_y y} U(y); f_q(q) = \lambda_q e^{-\lambda_q q} U(q) \tag{A5}$$

where

$$\lambda_x = 1/\lambda_{SD}, \lambda_y = 1/\lambda_{RD}, \lambda_q = 1/\lambda_{SR};$$

$f_x(x), f_y(y)$ and $f_q(q)$ are pdf's of r.v.'s x, y and q, respectively.

• Let $w = x + y$. The *pdf* of w , hence is expressed as

$$f_w(w) = \int_{-\infty}^{\infty} f_x(x)f_y(w-x)dx = \begin{cases} \frac{\lambda_x\lambda_y}{\lambda_x-\lambda_y} [e^{-\lambda_y w} - e^{-\lambda_x w}], & \lambda_x \neq \lambda_y \\ \lambda_x^2 e^{-\lambda_x w} & , \lambda_x = \lambda_y \end{cases} \quad (A6)$$

From section 2.2, we know that n is a Gaussian r.v. with zero-mean and variance $(|\alpha_{SD}|^2 + |\alpha_{RD}|^2)/2 = w/2$, given channel realizations.

• Let $w = x - y$. In the case of $z \geq 0$, we obtain its *pdf* as [28,(6-55)]

$$f_z(z) = \int_0^{\infty} \lambda_x e^{-\lambda_x(z+y)} \lambda_y e^{-\lambda_y y} dy = \frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y} e^{-\lambda_x z} \quad (A7)$$

• Let $v = z^2$. Then, the *pdf* of v in the case of $z \geq 0$ is easily found as [28, (5-22)]

$$f_v(v) = \frac{1}{2\sqrt{v}} f_z(\sqrt{v}) = \frac{\lambda_x \lambda_y}{\lambda_x + \lambda_y} e^{-\lambda_x \sqrt{v}}$$

• Let $u = z^2/w = v/w$. Then, the *pdf* of u is computed as follows[28,(6-60)]

$$f_u(u) = \int_0^{\infty} w \frac{1}{2\sqrt{wu}} \frac{\lambda_x \lambda_y e^{-\lambda \sqrt{wu}}}{\lambda_x + \lambda_y} \frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y} [e^{-\lambda_y w} - e^{-\lambda_x w}] dw \quad (A8)$$

By changing the variable $k = \sqrt{w}$, the above is reduced to

$$f_u(u) = \frac{\lambda_x^2 \lambda_y^2}{\lambda_x^2 - \lambda_y^2} \int_0^{\infty} \frac{k^2}{\sqrt{u}} [e^{-(\lambda_y k^2 + \lambda_x \sqrt{uk})} - e^{-(\lambda_x k^2 + \lambda_y \sqrt{uk})}] dk \quad (A9)$$

5.1 Expressions for $\overline{P_{e1}}$ and $\overline{P_{e2}}$

The expression of $\overline{P_{e1}}$ in (18) is written in the explicit form as

$$\overline{P_{e1}} = \overline{\Pr[-\sqrt{E_s} (|\alpha_{SD}|^2 + |\alpha_{RD}|^2) + n > 0]} = \overline{Q(\sqrt{2E_s w})} \quad (A10)$$

Now we consider separately two cases in (A6).

Case of $\lambda_x = \lambda_y$

This is the case that both paths S-D and R-D have the similar quality to the destination. Hence, we obtain from (A10)

$$\overline{P_{e1}} = \int_0^{\infty} Q(\sqrt{2E_s w}) \lambda_x^2 w e^{-\lambda_x w} dw$$

By changing the variable of the integration $m = E_s w$ and letting $\gamma = E_s/\lambda_x$, the probability of error is derived as follows

$$\overline{P_{e1}} = \int_0^{\infty} Q(\sqrt{2m}) \frac{1}{\gamma^2} m e^{-\frac{m}{\gamma}} dm = \frac{1}{4} \left(1 - \sqrt{\frac{\gamma}{1+\gamma}} \right) \left(2 + \sqrt{\frac{\gamma}{1+\gamma}} \right) \quad (A11)$$

Also in this case, it is easy to realize that

$$\overline{P_{e2}} = \overline{P_r[-\sqrt{E_x} (|\alpha_{SD}|^2 - |\alpha_{RD}|^2) + n > 0]} = 0.5 \quad (A12)$$

Case of $\lambda_x \neq \lambda_y$

The asymmetric scenario happens when fading level of one of the propagation paths to the receiver is different from the other path. In such a case, (A10) is of the form

$$\overline{P_{e1}} = \int_0^{\infty} Q(\sqrt{2E_s w}) \frac{\lambda_x \lambda_y}{\lambda_x - \lambda_y} [e^{-\lambda_y w} - e^{-\lambda_x w}] dw = \frac{\lambda_x}{(\lambda_x - \lambda_y)} \left[1 - \sqrt{\frac{1}{1 + \frac{\lambda_y}{E_s}}} \right] - \frac{\lambda_y}{2(\lambda_x - \lambda_y)} \left[1 - \sqrt{\frac{1}{1 + \frac{\lambda_x}{E_s}}} \right] \quad (A13)$$

With $z = x - y$, we rewrite $\overline{P_{e2}}$ in (18) as (see Fig. 5)

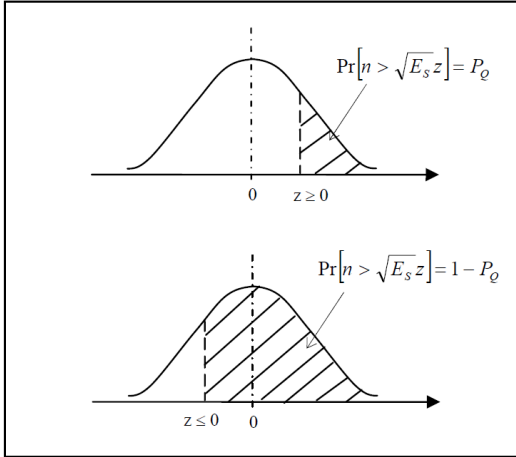


그림 5. n의 통계적 분포와 z의 가능한 두 가지 경우
Fig. 5 Statistical distribution of n and two possible cases of z.

$$\begin{aligned} \overline{P_{e2}} &= \overline{\Pr[-\sqrt{E_s}(|\alpha_{SD}|^2 - |\alpha_{RD}|^2) + n > 0]} \\ &= \overline{\Pr[n > \sqrt{E_s}z]} \\ &= Q\left(\sqrt{\frac{2E_s z^2}{w}}\right) \Pr[z \geq 0] \\ &\quad + \left[1 - Q\left(\sqrt{\frac{2E_s z^2}{w}}\right)\right] \Pr[z \leq 0] \\ &= \overline{P_Q} P_G + (1 - \overline{P_Q})(1 - P_G) \end{aligned} \tag{A14}$$

Consider the case of $z \geq 0$ in the sequel. Then the *pdf* of z is given in (A7). Thus, we have

$$P_G = \Pr[z \geq 0] = \int_0^\infty f_z(z) dz = \frac{\lambda_y}{\lambda_x + \lambda_y} \tag{A15}$$

Moreover, P_Q and $\overline{P_Q}$ are given by

$$P_Q = Q\left(\sqrt{\frac{2E_s z^2}{w}}\right)$$

and

$$\begin{aligned} \overline{P_Q} &= \int_0^\infty Q(\sqrt{2E_s u}) f_u(u) \\ &= \frac{\lambda_x^2 \lambda_y^2}{\lambda_x^2 - \lambda_y^2} [f(E_s, \lambda_y, \lambda_x) - f(E_s, \lambda_x, \lambda_y)] \end{aligned} \tag{A16}$$

Here the *pdf* of u is given in (A9) and the last equality in (A16) is obtained with $f(\bullet, \bullet, \bullet)$

from (A4) in the Appendix 1.

Using (A15) and (A16), we find (A14).

5.2 Expression for $\overline{P_{SR-SNR}}$

Since $\Pr[\epsilon = -1 | |\alpha_{SR}|^2 \geq T_\alpha]$ is the instantaneous error probability of BPSK-modulated symbol sent over Rayleigh fading channel $S-R$ plus AWGN with zero-mean and unit-variance conditioned on $|\alpha_{SR}|^2 \geq T_\alpha$, its average BER is easily established as

$$\begin{aligned} \overline{P_{SR-SNR}} &= \overline{\Pr[\epsilon = -1 | |\alpha_{SR}|^2 \geq T_\alpha]} \\ &= \int_{T_\alpha}^\infty P_{e-SR} f_q(q) dq \\ &= \int_{T_\alpha}^\infty Q(\sqrt{2E_s q}) \lambda_q e^{-q \lambda_q} dq \end{aligned} \tag{A17}$$

where P_{e-SR} in (6) and $f_q(q)$ in (A5). Using [27, (2) and (14)], we can approximate the Q-function in a compact form

$$Q(\sqrt{2x}) = \frac{1}{2} \operatorname{erfc}(\sqrt{x}) \cong \frac{1}{2} \left[\frac{1}{6} e^{-x} + \frac{1}{2} e^{-\frac{4x}{3}} \right]$$

Then we obtain

$$\overline{\lambda P_{SR-SNR}} \cong \frac{\lambda_q}{4} \left[\frac{e^{-(E_s + \lambda_q) T_\alpha}}{3(E_s + \lambda_q)} + \frac{e^{-(\frac{4E_s}{3} + \lambda_q) T_\alpha}}{(4E_s + \lambda_q)} \right] \tag{A18}$$

5.3 Expression for $\overline{P_{e-SD}}$

$\overline{P_{e-SD}}$ denotes the average error probability of BPSK-modulated symbol over Rayleigh fading channel $S-D$ plus AWGN with zero-mean and unit-variance and so, it is easily established as [1]

$$\overline{P_{e-SD}} = \frac{1}{2} \left[1 - \sqrt{\frac{E_s / \lambda_x}{1 + E_s / \lambda_x}} \right] \tag{A19}$$

5.4 Expression for $\Pr[|\alpha_{SR}|^2 < T_\alpha]$

$$\begin{aligned} \Pr[|\alpha_{SR}|^2 < T_\alpha] &= \int_0^{T_\alpha} \lambda_q e^{-q \lambda_q} dq \\ &= 1 - e^{-\lambda_q T_\alpha} \end{aligned} \tag{A20}$$

5.5 Expression for $\Pr[|A| < \Lambda_0]$ and $\overline{P_{SR-LLR}}$

Let $|A|=4h$ with $h = \sqrt{E_S}|y_R|$ from (9). Then the pdf of h is given in [21], (17)

$$f_h(h) = \frac{e^{-2(\sqrt{1+\beta^{-1}})h} + e^{-2(\sqrt{1+\beta^{-1}})h}}{\sqrt{\beta^2 + \beta}} \quad (A21)$$

where $\beta = E[|\alpha_{SR}|^2]E_S = \lambda_{SR}E_S$

Moreover, we have

$$\Pr[|A| < \Lambda_0] = \Pr\left[h \leq \frac{\Lambda_0}{4}\right] = \int_0^{\frac{\Lambda_0}{4}} f_h(h)dh \quad (A22)$$

$$= \frac{1 - e^{-2(\sqrt{1+\beta^{-1}}+1)\frac{\Lambda_0}{4}}}{2(\beta+1 + \sqrt{\beta^2 + \beta})} \frac{1 - e^{-2(\sqrt{1+\beta^{-1}}+1)\frac{\Lambda_0}{4}}}{2(\beta+1 - \sqrt{\beta^2 + \beta})} \quad (A23)$$

Using the instantaneous BER expression in (10), we obtain

$$\begin{aligned} \overline{P_{SR-LLR}} &= \overline{\Pr[e=-1||A| \geq \Lambda_0]} \\ &= \int_{\frac{\Lambda_0}{4}}^{\infty} \frac{1}{1+e^{4h}} f_h(h)dh \quad (A24) \end{aligned}$$

With $f_h(h)$ in (A21), the single-variable integral in (A24) can be easily calculated by a numerical method [24].

Appendix 3

The Appendix 3 is used to calculate the optimum thresholds P_{eT-opt} for HDF-LLR. Rewrite (19) in the explicit form by using (A22) and (A24)

$$\begin{aligned} \overline{P_{e-HDF-LLR}} &= \overline{P_{e-SD}} \int_0^{\frac{\Lambda_0}{4}} f_h(h)dh \\ &+ \overline{P_{e1}} \left(1 - \int_{\frac{\Lambda_0}{4}}^{\infty} \frac{f_h(h)}{1+e^{4h}} dh\right) + \overline{P_{e2}} \int_{\frac{\Lambda_0}{4}}^{\infty} \frac{f_h(h)}{1+e^{4h}} dh \end{aligned}$$

Note that the quantities $\overline{P_{e-SD}}$, $\overline{P_{e1}}$, $\overline{P_{e2}}$, $f_h(h)$ are independent of Λ_0 . Using Leibnitz differentiation rule [28], we can compute the

derivative of $P_{e-HDF-LLR}$ with respect to 0 and find the optimum value of P_{eT} denoted as P_{eT-opt} by setting the derivative expression to zero, we can

$$P_{eT-opt} = \frac{\overline{P_{e-SD}}}{\overline{P_{e2}} - \overline{P_{e1}}} \quad (A25)$$

Appendix 4

This Appendix is used to calculate the optimum thresholds P_{eT-opt} for HDF-SNR. Rewrite (15) in the explicit form by using (A17) and (A20)

$$\begin{aligned} \overline{P_{e-HDF-SNR}} &= \overline{P_{e-SD}} (1 - e^{-\lambda_q T_\alpha}) \\ &+ \overline{P_{e1}} \left(1 - \int_{T_\alpha}^{\infty} Q(\sqrt{2E_s q}) \lambda_q e^{-q\lambda_q} dq\right) \\ &+ \overline{P_{e2}} \int_{T_\alpha}^{\infty} Q(\sqrt{2E_s q}) \lambda_q e^{-q\lambda_q} dq \end{aligned}$$

Using Leibnitz differentiation rule [28], we can compute the derivative of $P_{e-HDF-LLR}$ with respect to T_α and find the optimum value of P_{eT} (or T_α) denoted as P_{eT-opt} (or $T_{\alpha-opt}$) by setting the derivative expression to zero, we can

$$P_{eT-opt} = Q(\sqrt{2E_s T_{\alpha-opt}}) = \frac{\overline{P_{e-SD}}}{\overline{P_{e2}} - \overline{P_{e1}}} \quad (A26)$$

The first equality in (A26) is obtained from (8). The optimum threshold found in (A26) is of a very appealing form and much simpler than that in [18] which is calculated by the experiment and has not been verified yet. In addition, we observe that the optimum thresholds for HDF-SNR and HDF-LLR are identical (compare (A25) with (A26)).

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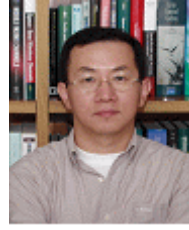
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공형윤 (Hyung-Yun Kong)

정회원



1989년 2월 미국 New York Institute of Technology 전자공학과(학사)

1991년 2월 미국 Polytechnic University 전자공학과(석사)

1996년 2월 미국 Polytechnic University 전자공학과(박사)

1996년~1996년 LG전자 PCS팀장

1996년~1996년 LG전자 회장실 전략 사업단

1996년~현재 울산대학교 전기전자정보시스템 공학부 교수

<관심분야> 모듈레이션, 채널 부호화, 검파 및 추정 기술, 협력통신, 센서 네트워크