Outage Analysis of a Cooperative Multi-hop Wireless Network for Rayleigh Fading Environment

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ABSTRACT

This paper presents an information theoretic outage analysis for physical layer of a cooperative multihop wireless network. Our analysis shows that cooperation by selecting a proper relay at each hop increases the coverage or data rate of the network. In our analysis we consider both symmetric and asymmetric network model. We also investigate the availability of cooperative relay at each hop and show that end-to-end performance of the network depends on the relay selection procedure at each hop. We also verify our analytical results with simulations.

Key Words : Cooperative diversity, outage probability, multi hop network, fading channel, selection relaying

Ⅰ. Introduction

To provide transmit diversity when users cannot support multiple antennas, a cooperative diversity protocol[1] has been proposed. Various cooperative transmission protocols, implementation issues, performance and outage analysis have been studied in the literature[1-5]. So far, the most of the works related to cooperative communication has focused on single-hop networks and a significant research challenges for cooperative multi hop network still exist. In [6] the outage analysis of multi hop diversity protocol has been analyzed. In this protocol each node at multi hop route listens from all of its previous nodes and combined them optimally. This protocol requires signal connectivity between nonadjacent terminals. In [7] routing strategies of multi hop cooperative network has been proposed and confirmed a higher order cooperative diversity through outage analysis. This protocol incorporated cooperative diversity with routing and implementation of such protocol requires global channel state information at each node to select a cooperative route which is very difficult to obtain. In this paper we proposed a cascaded extension of single hop cooperative protocols to a multi hop cooperative protocol.

We develop a closed form end-to-end outage probability expression for the multi hop cooperative protocol in Rayleigh fading environment. As the single hop cooperative protocols the multi hop protocol also improve the end-to-end capacity of the system in terms of outage probability and data rate. Consider a route has been established by the network layer and a proper relay selection technique is used at each hop. We analyze performance of both symmetric and asymmetric network and a significant improvement is achieved over non-cooperative multi hop network. We also investigate the effect of the position and availability of relay at each hop. Throughout this paper we consider selection relaying[1] at relays for simplicity.

Ⅱ. System Model

Let, an m-hop wireless network that selects a route $N_1 + N_2 + N_3 + ... + N_{m+1}$ with N_1=Source

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and N_{M+1} = Destination as shown in Fig.1.

The details of routing protocols are beyond the scope of this paper. Assume that, in each hop there is a relay that assist the transmitting node to forward the information towards the next hop. Therefore, we exploit a cooperative diversity hop by hop. Consider that the nodes are randomly distributed over the area and the channels between two nodes are subjected to flat Rayleigh fading plus AWGN. Each node has a single half duplex radio and a single antenna. The baseband equivalent received signal at node j due to the transmission of node i for symbol n is given by,

$$\gamma_{i,j}(n) = \alpha_{i,j} s(n) + \eta_j(n) \tag{1}$$

where $\eta_j(n)$ is AWGN noise sample with variance $N_0/2$ per dimension at terminal j, $\alpha_{i,j}$ is fading coefficient between node i and j. $s(n)$ is the signal transmitted by node i with unit power. We consider flat Rayleigh fading, hence, $\alpha_{i,j}$ is modeled as independent samples of zero mean complex Gaussian random variable with variance $\sigma^2_{i,j}$ per dimension. The fading coefficients are constant over the channel coherence time. To take path loss into account, we can model the variance of channel coefficient between node i and j as a function of distance between two nodes[8] as,

$$\sigma^2_{i,j} = d_{ij}^{-\beta} \tag{2}$$

where β is so called path loss exponent that varies from 2 to 6 on the basis of channel environment and d_{ij} is the distance between node i and j.

In cooperative protocols, relays must process their received signals; however, current limitations in radio implementation preclude transmitting and receiving at the same time in the same frequency band. Because of considerable attenuation over the wireless channel and insufficient electrical isolation between transmit and receive circuitry make, we adopt time division multi-plexing (TDM) for channel access in this paper. However, the basic idea and operation of our proposed technique does not depend on the specifics of the channel access protocol.

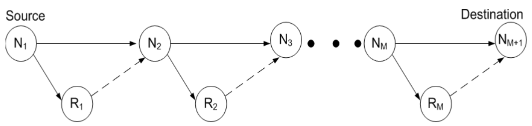

Fig 1. System Model

Ⅲ. Outage Analysis

In this paper, we focus on the end-to-end outage performance of the cooperative multi hop scenario shown in Fig.1. For hop i node N_i transmits data to node N_{i+1} 'with the help of relay R_i. We define the outage probability of hop i between two nodes as,

$$P_{out}(i) = pr[I(N_i, N_{i+1}) < R] \tag{3}$$

where, $I(N_i, N_{i+1})$ is the mutual information between N_i and N_{i+1}, and R is the target rate of the system.

3.1 Direct transmission

In this case, we consider the system fails to find a relay at hop i and a non-cooperative direct transmission (DT) has taken place between node N_i and N_{i+1}. Now the outage probability of direct transmission can be given as,

$$\begin{aligned} p^{DT}_{out}(i) &= \Pr[I(N_i, N_{i+1}) < R] \\ &= \Pr[\log(1+\gamma|\alpha_{N_i,N_{i+1}}|^2) < R] \\ &= \Pr[|\alpha_{N_i,N_{i+1}}|^2 < \frac{2^R-1}{\gamma}] \end{aligned} \tag{4}$$

where, $\gamma = P/N_0$. Now consider $\alpha_{i,j}$ has Rayleigh distribution so $|\alpha_{i,j}|^2$ has exponential distribution with parameter $\sigma^{-2}_{i,j}$[1]. So using the CDF of $|\alpha_{i,j}|^2$ we can easily calculate the outage probability of direct transmission as,

$$P^{DT}_{out} = 1 - \exp(-\sigma^{-2}_{N_i,N_{i+1}} \frac{2^R-1}{\gamma}) \tag{5}$$

3.2 Cooperative Transmission

Assume a selection relaying[1] strategy at each hop, therefore the relay will forward the information when the received SNR at relay satisfies a predefine threshold. In this protocol the relay will forward the received information if it satisfies the predefined rate R otherwise it will be silent. The mutual information between N_i and N_{i+1} can be given as [1],

$$I(N_i,N_{i+1}) = \begin{cases} \frac{1}{2}\log(1+\gamma|\alpha_{N_i,N_{i+1}}|^2); \\ \quad I(N_i,R_i) < R \\ \frac{1}{2}\log(1+\gamma[|\alpha_{N_i,N_{i+1}}|^2+\gamma|\alpha_{R_i,N_{i+1}}|^2]); \\ \quad I(N_i,R_i) \geq R \end{cases} \tag{6}$$

where $I(N_i, R_i)$ is the mutual information between N_i and R_i, and given as,

$$I(N_i,R_i) = \frac{1}{2}\log(1+\gamma|\alpha_{N_i,R_i}|^2) \tag{7}$$

Since the two events in eqn. (6) is mutually exclusive so the outage probability can be written as,

$$\begin{aligned} P_{out}^i &= \Pr[I(N_i,N_{i+1}) < R] \\ &= \Pr[I(N_i,R_i) < R] \times P_1 \\ &\quad + \Pr[I(N_i,R_i) \geq R] \times P_2 \end{aligned} \tag{8}$$

where, $P_1 = P_{out}^{DT}$ and P_2 represent the outage probability of cooperative transmission between node N_i and N_{i+1} with the help of relay R_i and given as,

$$\begin{aligned} P_2 &= \Pr[\frac{1}{2}\log(1+\gamma|\alpha_{N_i,N_{i+1}}|^2+|\alpha_{R_i,N_{i+1}}|^2) < R] \\ &= \Pr[|\alpha_{N_i,N_{i+1}}|^2+|\alpha_{R_i,N_{i+1}}|^2 < \frac{2^{2R}-1}{\gamma} \end{aligned} \tag{9}$$

The CDF of the random variable $|\alpha_{N_i,N_{i+1}}|^2+|\alpha_{R_i,N_{i+1}}|^2$ is given as [1],

$$F(x) = \begin{cases} 1-(1+\sigma^{-2}x)\exp(-\sigma^{-2}x); \\ \quad \text{for}\, \sigma_{N_i,N_{i+1}}^{-2} = \sigma_{R_i,N_{i+1}}^{-2} = \sigma^{-2} \\ 1-[\frac{\sigma_{R_i,N_{i+1}}^{-2}}{\sigma_{R_i,N_{i+1}}^{-2}-\sigma_{N_i,N_{i+1}}^{-2}} exp(\sigma_{N_i,N_{i+1}}^{-2}x) + \\ \frac{\sigma_{N_i,N_{i+1}}^{-2}}{\sigma_{N_i,N_{i+1}}^{-2}-\sigma_{R_i,N_{i+1}}^{-2}} exp(\sigma_{R_i,N_{i+1}}^{-2}x)]; \\ \quad \text{for}\, \sigma_{N_i,N_{i+1}}^{-2} \neq \sigma_{R_i,N_{i+1}}^{-2} \end{cases} \tag{10}$$

Now we can easily rewrite equation (9) using the CDF of (10) as,

$$P_2(\rho) = \begin{cases} 1-(1+\sigma^{-2}\rho)\exp(-\sigma^{-2}\rho); \\ \quad \text{for}\, \sigma_{N_i,N_{i+1}}^{-2} = \sigma_{R_i,N_{i+1}}^{-2} = \sigma^{-2} \\ 1-[\frac{\sigma_{R_i,N_{i+1}}^{-2}}{\sigma_{R_i,N_{i+1}}^{-2}-\sigma_{N_i,N_{i+1}}^{-2}} \exp(\sigma_{N_i,N_{i+1}}^{-2}\rho) + \\ \frac{\sigma_{N_i,N_{i+1}}^{-2}}{\sigma_{N_i,N_{i+1}}^{-2}-\sigma_{R_i,N_{i+1}}^{-2}} \exp(\sigma_{R_i,N_{i+1}}^{-2}\rho)]; \\ \quad \text{for}\, \sigma_{N_i,N_{i+1}}^{-2} = \sigma_{R_i,N_{i+1}}^{-2} = \sigma^{-2} \end{cases} \tag{11}$$

where, $\rho = (2^{2R}-1)/\gamma$. Similarly to equation (5) we can write,

$$\Pr[I(N_i,R_i) < R] = 1-\exp(-\sigma_{N_i,R_i}^{-2}\rho). \tag{12}$$

Therefore,

$$\Pr[I(N_i,R_i) \geq R] = \exp(-\sigma_{N_i,R_i}^{-2}\rho). \tag{13}$$

Now, replacing the values of equation (5), (11), (12) and (13) in equation (8) we can easily calculate the cooperative outage probability of hop I.

3.3 End-to-End outage probability

In this subsection we will derive the end-to-end outage probability of our proposed multi hop cooperative networks. Assume S is a subset of M that has successfully selected a relay and consider all the hopes are independent. So the end-to-end outage probability for an m-hop cooperative ad-hoc network is simple multiplication of outage probability of each hop

$$P_{out} = 1-\prod_{i\in S}^{M}(1-P_{out}^{CT}(i))\prod_{j\in S}^{M}(1-P_{out}^{CT}(j)). \tag{14}$$

Ⅳ. Results and Discussion

In this section we provide some simulation results of the outage probability and verify these results with our developed outage analysis. The analytical results are numerically plotted by using the outage probability equations derived in section III. On the other hand, Monte Carlo simulation is performed to obtain the simulation results. We clearly pointed this issue in the revised paper.

We show the outage probability of cooperative multi hop protocol as a function of average signal to noise ratio (SNR). We compare the outage probability of cooperative multi hop network and non-cooperative multi hop network for various data rates in Fig. 2. For all values of R, the performance of cooperative protocol is better than the non-cooperative one at high SNR region. Fig. 2 shows that the performance gain due to the cooperation is higher at low rate and the performance gain decreases as the rate increases.

In this paper, we simulate the outage probability of the system for a fixed data rate. As shown in fig. 2, the proposed cooperative scheme achieve 14 dB SNR gain over the direct transmission at outage probability of 10-3 for R=0.1. This SNR gain is achieved due the cooperative diversity. If the system operates on a fixed target outage probability then this SNR gain can be easily used to increase the data rate or to increase the network coverage.

The words symmetric and asymmetric are used to represent the network topology. In case of symmetric network, the distance between any two nodes of the network is equal. In contrast, the distances between nodes are unequal for asymmetrical network.

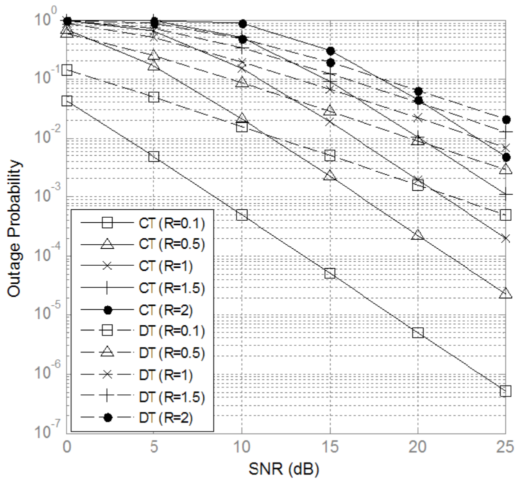

Fig. 2. Outage probability of cooperative and non-cooperative protocol as a function of SNR for symmetric network with different rate.

Fig. 3 shows the outage probability of multi hop cooperative network when a suitable relay has found at each hop. In this figure we consider a symmetrical network i.e., for all hop $d_{N_i,N_{i+1}} = d_{N_i,R_i} = d_{R_i,N_{i+1}} = 1$ and $R=1$. Fig. 3 shows that the end to end outage probability of multi hop cooperative protocol decreases as the number of hops increases. A similar result is also shown in Fig. 4 for an asymmetric network. In this case the distance between all nodes are considered as a uniformly distributed random variables within the range 0.1 to 2.0 with $R=1$. For both case we consider $\beta=2$.

In Fig. 5. we consider a 6 hop network and

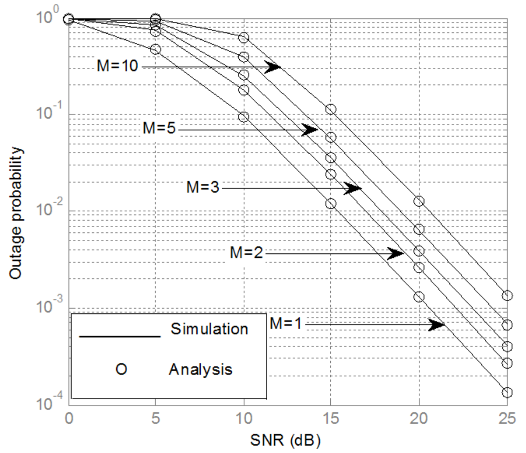

Fig. 3. Outage probability of symmetric network with R=1

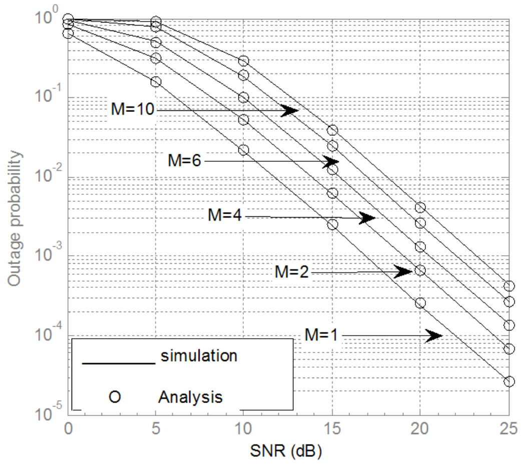

Fig. 4. Outage probability of asymmetric network with R=1

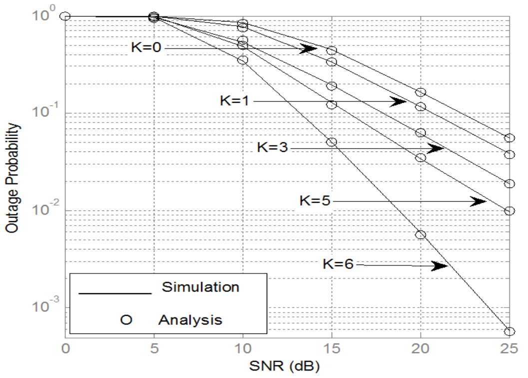

Fig. 5. Outage probability as a function of SNR for asymmetric network with different number of hop that selects a relay.

shows the effect of using a relay in different hops. In this case we consider a strict relay selection policy. We assume $d_{N_i,N_{i+1}}=1$ for all i and $d_{N_i,R_i}, d_{R_i,N_{i+1}}$ are uniformly distributed random variables between 0.1 and 2.0. Therefore we did not consider relays that have the distance greater than 2 from either source or destination. Here $K=|S|$ represents the number of hop that selects a relay for cooperation. When K=0 the network becomes a non cooperative network where in every hop a direct transmission takes place.

As K increases the end-to-end outage performance improves as shown in Fig. 5 The performance difference between all hop select relay (K=6) and only one hop fail to find a relay (K=5) is huge because the only non-cooperative hop act as a bottleneck of the end-to-end performance.

Fig. 6 examines the bottleneck effect on relay selection policy for multi hop cooperative network. We consider, $d_{N_i,N_{i+1}}=1$ for all hops and $d_{N_i,R_i}=d_{R_i,N_{i+1}}=0.5$ except the 3rd hop. We investigate the end-to-end outage probability with various relay position for 3rd hop. For $d_{N_3,R_3}=d_{R_3,N_4}=d$, the end-to-end performance improves as d decreases. The best performance achieve when the relay is equidistant from the two nodes ($d=0.5$). The performance is very close to the best one (i.e., $d=0{,}5$) when we choose a relay with $d=2$ (increase the distance 4 times).

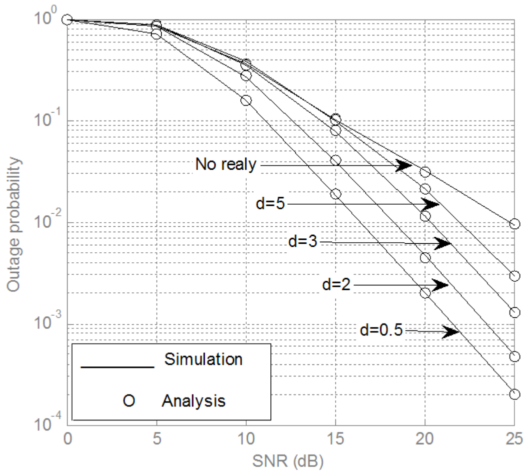

Fig. 6. Outage probability with different relay position.

Interestingly the end-to-end performance is better than no relay in 3rd hop when we consider $d=5$ (increase the distance 10 times). Therefore, a poor relay selection also performs better than direct transmission.

Ⅴ. Conclusion

In this paper we simply extended the cooperative single-hop protocol to a cooperative multi hop protocol by cascading the single-hop networks. We developed the end-to-end outage probability of our proposal for both symmetric and asymmetric networks. We also analyzed the availability of relays at each hop and observed the end-to-end performance when relay is not available for each hop. Analysis and simulation results show that a poor relay selection policy at any hop performs better than no relay selection. In this paper we only analyzed the physical layer performance of cooperative multi hop network. The performance analysis for other higher layer is left for future work.

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