

# A Modified PTS Algorithm for PAPR Reduction in OFDM Signal

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## ABSTRACT

Partial transmit sequence (PTS) algorithm is known as one of the most efficient ways to reduce the peak-to-average power ratio (PAPR) in the orthogonal frequency division multiplexing (OFDM) system. The PTS algorithm, however, requires large numbers of computation to implement. Thus there has been a trade-off between performance of PAPR reduction and computational complexity. In this paper, the performance of PAPR reduction and computation complexity of PTS algorithms are analyzed and compared through computer simulations. Subsequently, a new PTS algorithm is proposed which can be a reasonable method to reduce the PAPR of OFDM when both the performance of PAPR reduction and computational complexity are considered simultaneously.

**Key Words** : PTS, PAPR, OFDM

## I. Introduction

Multi-carrier modulation is gaining popularity with the emerging wireless broadcasting channel, especially for high-rate transmissions with orthogonal frequency division multiplexing (OFDM)<sup>[1,2]</sup>. The benefits of the OFDM are high spectral efficiency, resiliency to radio frequency (RF) interference, and lower multi-path distortion. A major drawback of OFDM on the transmission side, however, is the high peak-to-average power ratio (PAPR)<sup>[3]</sup> of the transmitted signal. The high PAPR not only increases complexity of the analog-to-digital (A/D) and digital-to-analog (D/A) converters, but also reduces the efficiency of the RF power amplifier<sup>[4]</sup>. An OFDM symbol yields a higher PAPR value than that of signal carrier symbols. This necessitates a high power amplifier in the transmitter even in low power mobile communication systems. This obviously increases power consumption and device cost.

In order to reduce the PAPR various methods

have been introduced recently. These include clipping<sup>[5]</sup>, coding<sup>[6]</sup>, selection mapping (SLM)<sup>[3,4]</sup>, and partial transmit sequence (PTS)<sup>[7-12]</sup>. The clipping method deliberately clips the peak amplitude of the OFDM signal to some desired maximum level. This may result in in-band distortion (self-interference) and out-of-band radiation, since the clipping procedure is a nonlinear process. The coding method changes an original information bit pattern to a coded bit pattern with redundant bits so that a coded OFDM signals have a low PAPR. Because it requires lots of redundant bits for the sufficient PAPR reduction, it may be not an efficient method to reduce the PAPR. In the SLM approach, one OFDM signal with the lowest PAPR is selected for transmission at the transmitter from a set of sufficiently different candidate signals which all represent the same data sequence.

With the PTS approach, the transmitter partitions the original data sequence into a number of disjoint sub-blocks and then optimally combines the inverse fast Fourier transforms (IFFTs) of all the sub-blocks

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to generate an OFDM signal with low PAPR for transmission. Unlike the clipping method, the PTS algorithms do not have adverse effects on the signal spectrum, but they require a bank of IFFTs to generate a set of candidate signals. In general, the PTS algorithms can provide good PAPR reduction performance, but each of them may suffer a high computational load due to the need for a bank of IFFTs. On the other side, if the same number of IFFT blocks is used, the PTS algorithm may generate more candidate signals for selection and would perform better than SLM. Thus, in this paper, we will concentrate on the study of the PTS algorithm.

The main drawback of the PTS algorithm is a computational complexity. There are several algorithms in PTS and they can be implemented by an appropriate change of the phase factor. The ordinary PTS (O-PTS) algorithm<sup>[10]</sup> requires a large number of computations to implement it. To reduce the number of computations Cimini and Nelson introduced the sub-optimal PTS (SO-PTS) algorithm<sup>[7]</sup> that needs a smaller number of computations than those of the O-PTS algorithm. The SO-PTS algorithm, however, has a slightly higher PAPR than that of the O-PTS algorithm. Recently, Tsai and Huang introduced the non-uniform PTS(NU-PTS) algorithm<sup>[11]</sup>. Among the PTS algorithms introduced up to the present, it shows the lowest PAPR but requires the largest number of computations. Thus there is a trade-off between the performance of PAPR reduction and the computational complexity.

The performance of PAPR reduction according to the PTS algorithms is analyzed and compared by computer simulations in this paper. As well the numbers of computation according to PTS algorithms are also compared. As a result, a new PTS algorithm to reduce the PAPR of OFDM is proposed and its performance is analyzed in this paper.

## II. OFDM

OFDM in its primary form is considered as a

digital modulation technique, but not a multi-user channel access technique, since it is utilized to transfer one bit stream over one communication channel using one sequence of OFDM symbols. However, OFDM can be combined with multiple accesses, using time, frequency or coding separation of the users. The separation of the sub-carriers is theoretically minimal in a sense that there is a very compact spectral utilization. The attraction of OFDM is mainly due to how the system handles the multipath interference at the receiver.

### 2.1 OFDM system

The OFDM symbol can be given as the sum of numbers of independent symbols that are modulated on to sub-channels of equal bandwidth. Let  $X_k, (k=0, \dots, N-1)$  denote the input data symbol whose period is  $T$ . Then, the complex representation of an OFDM signal is given as:

$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi k \Delta f t}, \quad 0 \leq t \leq NT, \quad (1)$$

where  $N$  is the number of sub-carriers, and  $\Delta f = 1/NT$  is the sub-carrier spacing.

With IFFT and FFT, the OFDM system can be modeled, as shown in Fig. 1. Firstly, the input serial data stream is transformed in parallel by assigning each data word to one carrier in the transmission. Secondly, the data to be transmitted on each carrier are encoded differentially with previous symbols and then mapped into a phase shift keying (PSK) format. Thirdly, we perform IFFT. IFFT is used to find the corresponding time waveform. Finally, the guard interval is inserted to the start of each symbol.

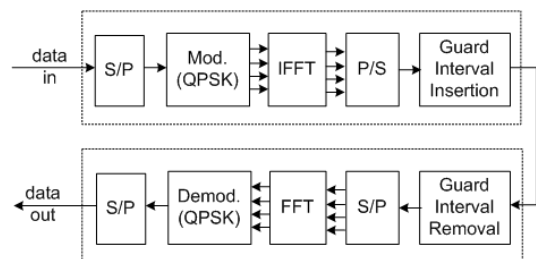


Fig. 1. The OFDM model.

The receiver basically does the reverse operation to the transmitter. The guard period is removed. The FFT of each symbol is then taken to find the original transmitted spectrum. The phase angle of each transmission carrier is then evaluated and converted back to the data word by demodulating the received phase. The data words are then combined back to the same word size as the original data.

### 2.2 PAPR

PAPR<sup>[12]</sup> is a device used to measure the peak power level to the time-average power level in an electrical circuit. PAPR meters are very sensitive to the idle channel noise, nonlinear distortion, and amplitude distortion. The PAPR can be determined by a number of parameters of signal, such as voltage, current, power, frequency, and phase.

The PAPR of the transmitted signal in (1) is defined as

$$PAPR = \frac{\max_{0 \leq t < NT} |x(t)|^2}{\frac{1}{NT} \int_0^{NT} |x(t)|^2 dt}. \quad (2)$$

For simplicity, let us assume that  $X_k=1$  for all the subcarriers. In that scenario, the peak value and mean value of the signal are :

$$\max [x(t)x(t)^*] = N^2, \quad (3)$$

$$E[x(t)x(t)^*] = N. \quad (4)$$

Thus, the PAPR for an OFDM system with N sub-carriers which have all equal modulation is

$$PAPR_{\max} = \frac{N^2}{N} = N. \quad (5)$$

The maximum value of PAPR is proportional to N. A larger PAPR needs a higher power amplifier. This give rises to an increase in the power consumption, as well as the device cost. Thus, PAPR is an important limitation in implementing the OFDM system.

### III. PTS

The block diagram of OFDM with the PTS algorithm is presented in Fig. 2. All of the algorithms described below can be implemented by appropriately changing the ‘*optimization for b*’ block.

In Fig. 2, the input data block  $\mathbf{X}$  is partitioned into  $M$  disjoint sub-blocks  $\mathbf{X}_m = [\mathbf{X}_{m,0}, \dots, \mathbf{X}_{m,N-1}]^T$ ,  $m=1, \dots, M$ , such that  $\sum_{m=1}^M \mathbf{X}_m = \mathbf{X}$  and the sub-

blocks are combined to minimize the PAPR in the time domain. The time-domain signal of  $\mathbf{X}_m$  is obtained by taking a IFFT of length  $N$  on  $\mathbf{X}_m$ . These are called the partial transmit sequences. Complex phase factors,  $b_m = e^{j\phi_m}$ ,  $m = 1, \dots, M$ , are introduced to combine the partial transmit sequences. We shall write the set of the phase factors as a vector  $\mathbf{b} = [b_1, b_2, \dots, b_M]^T$ .

The time-domain signal after combining is given by

$$\mathbf{x}'(\mathbf{b}) = \sum_{m=1}^M b_m \text{IFFT}\{X_m\} = \sum_{m=1}^M b_m \mathbf{x}_m. \quad (6)$$

The objective is to find the phase factor with the aim of minimizing PAPR<sup>[7]</sup>.

The PTS algorithms are based on eq. (6).

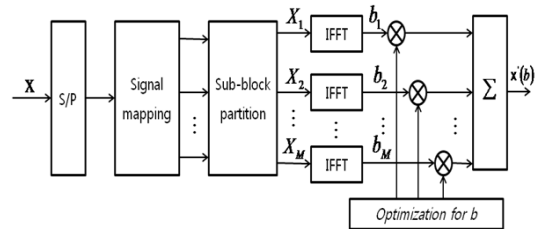


Fig. 2. Block diagram of the PTS algorithm.

Table 1. Number of computations for eq. (6).

Operation	Number of computations
Multiplication	$M \cdot N$
Addition	$(M-1) \cdot N$
IFFT	$M$

Therefore each algorithm should be calculated by using eq. (6) at least one. The table 1 shows the number of computations for eq. (6).

### 3.1 Ordinary PTS

Based on the elements of the PTSs, a peak value optimizing a combination of them is performed by suitably choosing the free optimization parameters  $b_m$ . The optimum parameters for the OFDM symbol are given by

$$\{b_1, \dots, b_M\} = \underset{\{b_1, \dots, b_M\}}{\operatorname{argmin}} \left( \max_{\rho} \left| \sum_{m=1}^M b_m \cdot x_{m,\rho} \right| \right), \quad 0 \leq \rho \leq N. \quad (7)$$

In general, the selection of the phase factors is limited to a set with finite number of elements to reduce the search complexity. The set of allowed phase factors is written as

$$F = \{e^{j2\pi l/W}, l = 0, 1, \dots, W-1\}, \quad (8)$$

where  $W$  is the number of allowed phase factors. In addition, we can set  $b_i=1$  without any loss of performance. So we perform exhaustive search for  $M$  phase factors. Hence,  $W^M$  sets of phase factors are searched to find the optimum set of phase factors. The search complexity increases exponentially with the number of sub-blocks  $M$ . As this algorithm contains the selection of the minimum peak power, it is needed to compare the peak power of each sub-carrier.

The complexity of O-PTS includes<sup>[12]</sup>: the generation of  $M$  partial transmits sequences using IFFT's; the evaluation of  $(W^M-1)$  phase patterns; for each phase pattern, the generation of a combined sequence based on eq. (7). The number of comparisons is  $(W^M-1)$ , because each comparison is performed when every time eq. (6) is calculated.

### 3.2 Non-uniform PTS<sup>[11]</sup>

For the optimized phase factor  $\mathbf{b}$ , Tsai and Huang use the characteristics that the  $M$  signal vectors, corresponding to these  $M$  sub-blocks, of the high power sample tend to have phases adjacent to each

other. The approximated distribution of the phase adjustments is shown as follow equation.

$$f(\varphi) = \eta\delta(\varphi) + \frac{1-\eta}{\sqrt{2\pi\sigma^2}} \exp(-(\varphi-\pi)^2/2\sigma^2), \quad (9)$$

where  $\eta$  is the probability of maintaining the original phase and  $\sigma$  is the standard deviation that fits in with the Gaussian-like distribution. In order to reduce the PAPR efficiently, the phase factors applied to NU-PTS, which are regarded as non-uniform, should be fitted with the distribution of eq. (9). In this paper, the approximation parameters in eq. (9) are  $\eta=0.487$  and  $\sigma=0.92$  which lead the non-uniform phase factor set with  $W=4$  being  $\{0, 0.64\pi, \pi, 1.36\pi\}$ .

As NU-PTS algorithm is based on O-PTS algorithm, the complexity of NU-PTS is the same as that of O-PTS. However, NU-PTS algorithm has to calculate phase factor set based on eq. (9). This would increase the complexity. Thus, the complexity of NU-PTS is higher than that of O-PTS algorithm.

### 3.3 Sub-optimal PTS[6]

The SO-PTS algorithm based on descent search is proposed in [6]. The objective of the algorithm is to find phase factors that achieve PAPR statistic close to that of the O-PTS algorithm with reduced complexity and little performance degradation. Here, we only consider binary (i.e. 1, -1) phase factors. After dividing the input data block into  $M$  clusters, form the  $MN$ -point PTS's. Fig. 3 shows the SO-PTS

- (1) Assume that  $b_m = 1$  for all  $m$  and compute the PAPR of the combined signal;
- (2) Invert the first phase factor ( $b_m = -1$ ) and recomputed the resulting PAPR;
- (3) If the new PAPR is lower than in the previous step, retain  $b_1$  as part of the final phase sequence, otherwise,  $b_1$  reverts to its previous;
- (4) If  $m \neq M$  then  $m = m + 1$ , and go to step (2), otherwise, go to step (5);
- (5)  $m = M$ , the algorithm is terminated.

Fig. 3. Algorithm for generation of the SO-PTS.

algorithm.

### 3.4 Proposed PTS

The PAPR of an OFDM signal to be transmitted,

$$\mathbf{x}'(\mathbf{b}) = \sum_{m=1}^M b_m \mathbf{x}_m,$$

has to be minimized for phase factor  $\mathbf{b} = [b_1, \dots, b_M]^T$  for the PTS approach. In the case of the O-PTS algorithm for minimum PAPR all phases of  $M$  sub-blocks has to be considered simultaneously. Thus, the complexity of O-PTS can be burdensome throughout implementation. On the contrary, the SO-PTS algorithm may not be expected to perform well in PAPR, because phases of  $M$  sub-blocks are considered in serial order.

If all  $M$  sub-blocks are sorted according to the PAPR of a sub-block and then SO-PTS algorithm is applied to all sub-blocks, a well-performing PAPR can be expected. The P-PTS algorithm is presented in Fig. 4.

As mentioned above, the P-PTS algorithm is almost identical to the SO-PTS algorithm except for the sort algorithm of step (3). Therefore, the complexity of the P-PTS algorithm is identical to the SO-PTS algorithm in multiplication, addition, and IFFT operations. Only the number of comparisons in the sorting algorithm of step (3) is added in the SO-PTS algorithm.

- (1) Let  $PAPR_m = PAPR$  of  $\mathbf{x}_m, m=1, \dots, M$
  - (2) Set  $i=1$  and  $\mathbf{A}=\{1, 2, \dots, M\}$
  - (3)  $Order_i = m$  for  $\max_{m \in \mathbf{A}} PAPR_m$   
 set  $\mathbf{A}=\mathbf{A}-\{m\}, i=i+1$   
 if  $\mathbf{A} \neq \phi$  then go to step (3)
  - (4) set  $i=1, m=Order_i$ , and  $b_m=1$   
 $\mathbf{x}_{\min} = \mathbf{x}_m$
  - (5) set  $i=i+1, m=Order_i$ ,  

$$PAPR_{\min} = \min_{k=1, \dots, W-1} PAPR \text{ of } \left( \mathbf{x}_{\min} + e^{\frac{2\pi k}{W}} \mathbf{x}_m \right)$$

$$b_m = e^{\frac{2\pi k}{W} |PAPR_{\min}}$$
 $\mathbf{x}_{\min} = \mathbf{x}_{\min} + \mathbf{x}_m$   
 if  $i < M$  then go to step (5)  
 else go to step (6)
  - (6)  $i=M$ , the algorithm is terminated

Fig. 4. Algorithm for generation of the P-PTS

## IV. Simulation results and discuss

We assume that the OFDM system has 128 sub-carriers, QPSK is adopted as the modulator, and the number of phase factors is four, to analyze and compare the PTS algorithms.

### 4.1 PAPR

Fig. 5 presents the complementary cumulative distribution functions (CCDFs) of PAPR for the PTS algorithms with  $M=4$ . The PAPRs are 11.8dB, 8.8dB, 7.9dB, 7dB, and 7dB for OFDM, SO-PTS, P-PTS, O-PTS, and NU-PTS, respectively for  $10^{-4}$  CCDF. The PAPR of SO-PTS shows the greatest PAPR of the PTS algorithms. PAPR of O-PTS is very close to that of NU-PTS. The PAPR of P-PTS has 0.9dB benefit from using SO-PTS. However, the

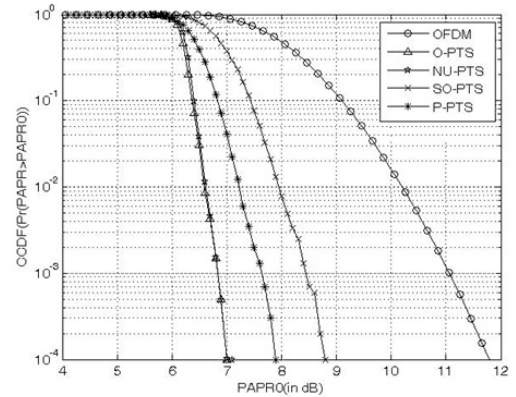


Fig. 5. CCDFs for PAPR of PTSs with  $M=4$ .

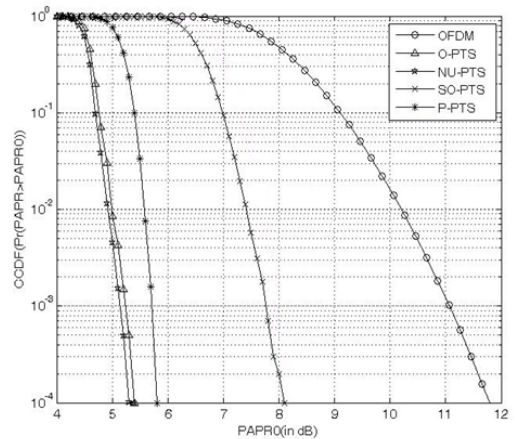


Fig. 6. CCDFs for PAPR of PTSs with  $M=8$ .

PAPR of P-PTS is 0.9dB greater than for the O-PTS algorithm.

Fig. 6 shows the PAPRs of the PTS algorithm with  $M=8$ . The curves of PAPR in Fig. 6 show a similar trend to those in Fig. 5. The PAPR of P-PTS has just 0.5dB difference to that of O-PTS for  $10^{-4}$  CCDF. Compared to the PAPR of SO-PTS, the PAPR of P-PTS is reduced by about 2.3dB. Therefore, for large values of  $M$ , we can estimate that the PAPR of P-PTS can be much closer to that of O-PTS.

#### 4.2 Complexity

In this section, all complexities of PTS algorithm introduced in this paper are analyzed with the number of sub-carriers  $N$ , number of partitioned sub-blocks  $M$ , and number of phase factors  $W$ . In the case of O-PTS, the number of computations for addition and multiplication are  $W^M$  times the number of computations for eq. (6). The number of computations for comparison is  $W^M-1$ . That is,  $W^M$  combinations of sub-blocks have to be considered for the minimum PAPR of the O-PTS algorithm.

Conversely, in the case of SO-PTS, the number of computations for addition and multiplication are  $(W-1)$  times the number of computations for eq. (6). The number of computations for the comparisons  $(W-1) \cdot M$ . That is, only  $(W-1)$  phase factors are considered for the single sub-block.

Finally, the complexity of the P-PTS algorithm is identical to that of the SO-PTS algorithm, except for the number of comparisons. Since the P-PTS algorithm is based on the SO-PTS algorithm with sorting, the number of comparisons for the P-PTS algorithm is  $M \cdot (M-1)/2 + (W-1) \cdot M$ .

The number of computations according to PTS

algorithms is compared in Table 2.

### V. Conclusions

The PTS algorithms to reduce the PAPR of the OFDM system are analyzed and compared. In order to improve both PAPR performance and complexity, the P-PTS algorithm is proposed in this paper.

Based on the analysis and comparison, the O-PTS and NU-PTS algorithm show the best PAPR performance of the introduced algorithms, but their complexity can be burdensome for implementation. Conversely, the SO-PTS algorithm exhibits very low complexity in the implementation compared to the O-PTS, NU-PTS algorithm. However, the PAPR of the SO-PTS algorithm may be intolerable in an OFDM system.

The P-PTS algorithm exhibits better PAPR performance than the SO-PTS algorithm and lower complexity than the O-PTS and NU-PTS algorithms. When  $N$  is 128,  $W$  is 4, and  $M$  is 4, for  $10^{-4}$  CCDF; the PAPR of the P-PTS algorithm is reduced by about 4dB and 1dB compared to OFDM and the SO-PTS algorithm, respectively. When  $M$  is 8, the PAPR of the P-PTS algorithm is reduced by about 6dB and 2.3dB, respectively. Compared to the O-PTS algorithm, for  $10^{-4}$  CCDF, the PAPR of the P-PTS algorithm differs by 0.9dB and 0.5dB when the values of  $M$  are 4 and 8, respectively. Therefore, for large values of  $M$ , we can estimate that the PAPR of P-PTS can be much closer to that of O-PTS.

As well as the complexity of implementation, the complexities of the SO-PTS and P-PTS algorithms are proportioned to  $N$ ,  $M$ , and  $(W-1)$ , where as the complexity of the O-PTS and NU-PTS algorithms are proportioned to  $N$ ,  $M$ , and  $W^M$ .

Therefore, when both PAPR performance and complexity of the PTS algorithms are considered simultaneously, the P-PTS algorithm can be a reasonable choice for an algorithm to reduce PAPR in OFDM system.

Table 2. Computation complexity of PTS algorithms

Operation	Number of computation			
	O-PTS	NU-PTS	SO-PTS	P-PTS
Multiplication	$M \cdot N \cdot W^M$	$>M \cdot N \cdot W^M$	$M \cdot N \cdot (W-1)$	$M \cdot N \cdot (W-1)$
Addition	$(M-1) \cdot N \cdot W^M$	$>(M-1) \cdot N \cdot W^M$	$(M-1) \cdot N \cdot (W-1)$	$(M-1) \cdot N \cdot (W-1)$
IFFT	$M$	$M$	$M$	$M$
Comparison	$W^M-1$	$W^M-1$	$M \cdot (W-1)$	$M \cdot (M-1)/2 + M \cdot (W-1)$

## References

- [1] E. Lawrey, *The suitability of OFDM as a modulation technique for wireless telecommunications, with a CDMA comparison: OFDM Model used*, Doctor thesis, degli studi di padova of university, FR. 1997.
- [2] N. Laurenti, *Implementation Issues in OFDM Systems: Overview of OFDM Systems and Synchronization of OFDM Systems*, Doctor thesis, James Cook University, AUS, 1998.
- [3] N. Chen and T. Zhou, "Peak-to-Average Power Ratio Reduction in OFDM with blind Selected Pilot Tone Modulation," *IEEE Trans. Wireless Commun.*, Vol. 3, No.7, pp.845-848, Mar. 2005.
- [4] J. H. Wen, S. H. Lee and C. C. Kung, "SLM-based data Position Permutation Method for PAPR Reduction in OFDM System," *Wireless Commun. and Mobile Comput.*, Vol.2, No.1, pp.1324-1327, Jun. 2008.
- [5] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM" *IEEE Commun. Lett.* Vol.2, No.5, pp.131-133, May 1998.
- [6] Y. Xin and I. J. Fair., "Error-control selective mapping coding for PAPR reduction in OFDM systems," *Proc. in Vehicular Technology Conference*, Vol.1, pp.583-587, Apr. 2005.
- [7] S. H. Han and J. H. Lee, "PAPR Reduction of OFDM Signals Using a Reduced Complexity PTS Technique," *IEEE Signals Processing Letters*, Vol.11, No.1, pp.887-890, Nov. 2004.
- [8] L. J. Cimini, Jr. and R. Nelson, "Peak-to-Average Power Ratio Reduction of an OFDM Signal Using Partial Transmit Sequence," *IEEE Commun. Lett.*, Vol.4, No.3, pp.86-88, Mar. 2000.
- [9] S. G. Kang, J. G. Kim, and E. K. Joo, "A novel subblock partition scheme for partial transmit sequence OFDM," *IEEE Trans. Broadcast*, Vol.45, No.3, pp.333-338, Sep. 1991.
- [10] S. H. Muller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electron. Lett.*, Vol.33, No.5, pp.368-369, Feb. 1997.
- [11] Y. R. Tsai and S. J. Huang, "PTS with Non-Uniform Phase Factor for PAPR Reduction in OFDM System," *IEEE Commun. Lett.*, Vol.12, No.1, pp.20-22, Jan. 2008.
- [12] Y. Xin and I. J. Fair, "Low Complexity PTS approaches for PAPR Reduction of OFDM Signal," *IEEE Trans. Wireless Commun.*, Vol.7, No.5, pp.1991-1995, Jan. 2005.

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