무선망에서의 상하향 링크 쌍대성 성질을 활용한 분산적 수율 최대화 기법

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Distributed Throughput-Maximization Using the Up- and Downlink Duality in Wireless Networks

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요 약

본 논문에서는 사용자들 간의 간섭이 존재하는 무선망에서 상하향 링크의 수율 최대화를 동시에 고려한다. 상향 링크에서는 라그랑지안 완화기법에 기반으로 하는 분산적이고 반복적인 알고리즘을 제안하다. 상향 링크에서의 라그랑지 곱수와 네트워크 쌍대성 성질을 이용하여 채널 이득과 최대 전력 제약이 상향 링크와 동일한 듀얼 하향 링크에서의 수율 최대화를 얻을 수 있다. 본 논문에서 증명한 네트워크 쌍대성 성질은 기존의 연구에 비해 보다 일반적인 형태를 가진다. 또한, 모의실험 결과는 채널의 상관 계수가 $\theta\in(0.5,1]$ 일 때, 상하향 링크에서 제안된 기법들이 각각 최적값에 근접하다는 것을 보여준다. 반면에 채널의 상관 계수가 낮을 때 ($\theta\in(0,0.5]$), 하향 링크에서의 성능 열화를 관찰할 수 있다. 네트워크 쌍대성 성질은 상향 링크에 비해 채널 이득과 최대 전력 제약이 다른 실제 하향 링크로 확장된다. 이러한 쌍대성 성질에 기반으로 하는 기법은 실제 하향 링크에서도 충분히 적용될 수 있음이 모의실험 결과로 보여진다. 기존에 제안된 알고리즘의 복잡도를 고려하였을 때, 본 논문의 결과는 일반화된 네트워크 쌍대성 성질의 성능과 실제 적용면에서 상당히 유용하다고 할 수 있다.

Key Words : Network Duality, Power Control, Optimization, Interference Channel, Lagrangian

ABSTRACT

We consider the throughput-maximization problem for both the up- and downlink in a wireless network with interference channels. For this purpose, we design an iterative and distributive uplink algorithm based on Lagrangian relaxation. Using the uplink power prices and network duality, we achieve throughput-maximization in the dual downlink that has a symmetric channel and an equal power budget compared to the uplink. The network duality we prove here is a generalized version of previous research [10], [11]. Computational tests show that the performance of the up- and downlink throughput for our algorithms is close to the optimal value for the channel orthogonality factor, $\theta\in(0.5,1]$. On the other hand, when the channels are slightly orthogonal ($\theta\in(0,0.5]$), we observe some throughput degradation in the downlink. We have extended our analysis to the real downlink that has a nonsymmetric channel and an unequal power budget compared to the uplink. It is

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shown that the modified duality-based approach is thoroughly applied to the real downlink. Considering the complexity of the algorithms in [6] and [18], we conclude that these results are quite encouraging in terms of both performance and practical applicability of the generalized duality theorem.

Ⅰ. 서 론

As wireless systems increasingly provide data services, intensive study has been done in rate control and packet scheduling, which are usually combined with link adaptation in the physical layer. In particular, there has been significant progress in the downlink side, of which excellent examples are CDMA-HDR [1] and HSDPA [2] [3]. On the other hand, research in the uplink is unsatisfactory, in the sense that we cannot find any practical implementation except for HSUPA [4]. One major reason for this is that it is rather difficult to coordinate multiple transmitters (mobiles) in an optimal way, even if there are theoretical results on uplink rate control [5]-[10].

Throughout this paper, our objective is to maximize the uplink throughput, iteratively and distributively. For this purpose, we have applied the Lagrangian relaxation (LR) technique [11] and have developed a heuristic algorithm. Applying LR to the uplink power/rate control was first carried out by Kim et al [5]. However, the algorithm provided in [5] considered that the rate of each mobile is a linear function of the signal-to-interference-plus-noise ratio (SINR). In [6]-[7], the authors considered the uplink throughput-maximization problem, in which the rate of each mobile is chosen to be a logarithmic function of the SINR, i.e., Shannon capacity. However, the algorithms proposed in [6]-[7] have a centralized property that causes high complexity for the base station to perform power/rate control. In [8], an uplink power control problem was formulated as a non-cooperative game where users aim selfishly at maximizing their utility-based performance. The existence of a Nash equilibrium point of each proposed game was proven, but such a nash equilibrium point may not achieve the throughput maximization for the uplink system. In [9], the problem of sum rate maximization was solved approximating the rate funcntion as log(SINR). Although this leads sum-rate maximization problem to a convex problem, the approximation can be seriously erroneous under low SINRs. The authors in [10] also consider the power optimization problem maximizing the sum rate, but the assumption that the interfering links are symmetric is critical.

When referring to the duality in the wireless network, we may naturally focus on the two communication directions: uplink and downlink. For this issue, there are two stimulating papers. The achievable capacity region of the uplink of a snapshot wireless network is mostly determined by the maximum transmittable power of each mobile. An interesting experiment is to vary such maximum transmittable power of each mobile while the total transmission power of all the mobiles is fixed to a constant, say $\overline{Q}$, and to see how the capacity region varies. In [12], the authors showed that the trace of the capacity region of the uplink (i.e., multiple access channel, MAC channel) collectively constitutes the capacity region of the downlink (i.e., broadcasting channel, BC channel), where the total power of the base station has an upper limit of $\overline{Q}$. There are two important assumptions in [12]. The first assumption is that the channel gains are symmetric between the up- and downlink. Secondly, the authors in [12] assume successive interference cancelation (SIC) in the receivers of mobiles and the base station, and the order of interference cancelation is completely reversed in the up- and downlink. A similar network duality is also presented by [13], in which spectral radius analysis was used. In [13], their analysis is rather general in the sense that the SIC was not assumed. However, what is missing in the analysis is that they did not consider the upper bound of the transmission power in the uplink.

The main purpose of this paper is threefold. First, we introduce an iterative and distributed uplink throughput-maximization algorithm which was proposed in our past work [14]. In this paper, we additionally prove the convergence of the algorithm. Second, we explain the duality properties for general wireless networks in [14]. This generalization is related to the question of how the duality theorem holds when the assumptions of SIC [12] and unlimited uplink power budget [13] are removed. We have proven that the duality between the MAC and BC channels still holds under such general conditions. Based on this, we move to the design of downlink throughput-maximization, which can be established from the uplink Lagrange multipliers (i.e., power price or power sensitivity) on each mobile and applying the network duality theorem. Third, we compensate the network duality gap in the downlink when there exist nonsymmetric channels and different power budgets between the up- and downlink. We have compared the performance of the up- and downlink throughput of our algorithms with that of [6] (uplink) and [15] (downlink), especially when the network duality gap exists or not in the downlink. The results are quite encouraging in terms of both performance and practical applicability of the generalized duality theorem.

The remainder of this paper is organized as follows. In the next section, we describe the problem definition of the uplink throughput-maximization and apply LR to this problem. In Section Ⅲ, we present a distributed uplink power control algorithm and prove the convergence of this algorithm. In Section Ⅳ, we prove the generalized network duality theorem and apply it to the downlink throughput-maximization problem. In Section Ⅴ, we introduce the network duality gap. The numerical results are given in Section Ⅵ, which is followed by the concluding remarks of Section Ⅶ.

Ⅱ. System Model and Lagrangian Relaxation

2.1 System Model

Consider an isolated single cell of a cellular radio system where N mobiles are active. In the uplink, each mobile i $(1 \le i \le N)$ can transmit with power $0 \le p_i \le \overline{p_i}$, where $\overline{p_i}$ is the maximum transmittable power of mobile i. We consider a short time interval such that the link gain between each mobile i and the base station is stationary, symmetric and given by g_i $(1 \le i \le N)$. The received signal-to-interference-plus-noise ratio (SINR) of mobile i, is defined as:

$$\gamma_i(P) = \frac{g_i p_i}{\sum_{j=1, j \ne i}^{N} \theta_{ij} g_j p_j + \nu}, \; i = 1, \cdots, N, \tag{1}$$

where the vector $P = (p_i)$ denotes the N-dimensional power vector and the positive value ν is background noise. The quantity $\theta_{ij} \in (0,1]$ is the normalized cross-correlation between p_i and p_j at the receiver of the base station; that is the effective fraction of the received signal power from transmitter j that contributes to the interference experienced by mobile i. For example in a DS-CDMA system, the spreading sequences can be chosen to be orthogonal, $\theta_{ij} = 0$ for $i \ne j$, but in reality some positive correlation will occur due to multipath propagation. Throughout the paper, we exclude the case of no intracell interference, which would make intracell coordination unnecessary.

The uplink throughput-maximization problem can be formulated as the following problem:

Problem (A):

$$z = \max \; \sum_{i=1}^{N} r(\gamma_i(P)) \tag{2}$$

$$s.t.\quad 0 \le p_i \le \overline{p_i},\ i=1,\cdots,N, \tag{3}$$

where the function $r(\gamma_i(P)) = \log(1+\gamma_i(P))$ defines the one-to-one relationship between an SINR and a data rate. In this paper, we choose the function r to be the Shannon capacity, an upper-bound on the maximum amount of error-free information that can be transmitted over a communication link using an appropriate coding. Without loss of generality, we assume that the base of the logarithm function in the channel capacity is the natural number.

2.2 Lagrangian Relaxation

Introducing nonnegative Lagrange multipliers $\lambda \in R^N$, we can form the following Lagrangian function of Problem (A):

$$L(P,\lambda) = \sum_{i=1}^{N} r(\gamma_i(P)) + \sum_{i=1}^{N} \lambda_i (\overline{p_i} - p_i). \tag{4}$$

Then a Lagrangian relaxation problem for a given λ is given by:

$$g(\lambda) = \max_{P \ge 0} L(P,\lambda), \tag{5}$$

of which the Lagrangian dual problem is:

$$w = \min_{\lambda \ge 0} g(\lambda). \tag{6}$$

The objective function in Problem (A) is neither convex nor concave with respect to P [6], [15]. Therefore, we cannot say that the strong duality theorem $(z=w)$ holds [16], and it is believed that $z < w$.

Let us now focus on the Lagrangian relaxation problem (5). The rate of mobile i depends on not only its own power allocation but also the power allocations of all the other mobiles. Power increment of mobile i increases its own rate, while decreasing the rate of all the others due to the interference. Thus the first order derivative of the Lagrangian function, with respect to p_i is the sum of the two:

$$\frac{\partial}{\partial p_i}\{r(\gamma_i(P))\} = \frac{\partial}{\partial p_i}\left\{\log\left(\frac{g_i p_i + I_i + \nu}{I_i + \nu}\right)\right\} = \frac{g_i}{g_i p_i + I_i + \nu} \tag{7}$$

$$\frac{\partial}{\partial p_i}\{r(\gamma_j(P))\} = \frac{\partial}{\partial I_j}\left\{\log\left(\frac{g_j p_j + I_j + \nu}{I_j + \nu}\right)\right\}\frac{\partial I_j}{\partial p_i} = \theta_{ij} g_i \left(\frac{1}{g_j p_j + I_j + v} - \frac{1}{I_j + v}\right), \tag{8}$$

where the term $I_i = \sum_{j=1, j\ne i}^{N} \theta_{ij} g_j p_j$ denotes the interference perceived by mobile i. Equation (7) is the derivative of mobile i's rate with respect to p_i and Equation (8) denotes that of mobile j's rate. Then the Karush-Kuhn-Tucker necessary condition of the problem (5) is reduced to finding the nonnegative power value that makes the sum of (7) and (8) equal to λ_i:

$$p_i^{opt} = \left[\frac{1}{\lambda_i + \Lambda_i} - \frac{I_i + v}{g_i}\right]^+, \tag{9}$$

where p_i^{opt} denotes the (local) optimal power of mobile i for the problem (5) and the function $[x]^+ = \max\{x,0\}$. In (9), the (local) optimal power is determined by two control variables, i.e., power price and system price. The power price of mobile i represented by the Lagrange multiplier, λ_i, is defined as the penalty that mobile i has to pay for unit power increment. The system price is defined as follows:

$$\Lambda_i = \sum_{j=1, j\ne i}^{N} \theta_{ij} g_i \left(\frac{1}{I_j + \nu} - \frac{1}{g_j p_j + I_j + \nu}\right). \tag{10}$$

The system price of mobile i, Λ_i means the effect of interference caused by the transmission of mobile i to the overall system. Therefore, the transmission power is inversely proportional to the sum of both the power price and the system price. The updates of both the power and system price are described in the next section.

Ⅲ. Distributed Uplink Power Control Algorithm

Our Distributed Uplink Power Control (DUPC) algorithm consists of iterations of three major steps as follows:

Step 0. Initialization
Set $k=0$ and arbitrary nonnegative initial values of p_i^k and λ_i^k.

Step 1. Power update

$$p_i^{k+1}=\left[\frac{1}{\lambda_i^k+\Lambda_i^k}-\frac{I_i^k+\nu}{g_i}\right]^+, \quad (11)$$

where p_i^{k+1} is a virtual transmission power of mobile i.

Step 2. Power price update

$$\lambda_i^{k+1}=\left[\lambda_i^k-\alpha^k \bullet (\overline{p_i}-p_i^{k+1})\right]^+, \quad (12)$$

where α^k denotes a sequence of positive step sizes.

Step 3. Power truncate

$$p_{i,TX}^{k+1}=\min\{p_i^{k+1},\overline{p_i}\}, \quad (13)$$

where $p_{i,TX}^{k+1}$ is an actual transmission power of mobile i.

Step 4. Repetition
Set $k=k+1$ and go to Step 1.

The base station determines the system price of each mobile by calculating the received information for a given time slot. In Equation (11), the system price of mobile i in the k-th time slot is calculated as:

$$\Lambda_i^k=\sum_{j\neq i}^{N}\theta_{ij}g_i\left(\frac{1}{I_j^k+\nu}-\frac{1}{g_jp_j^k+I_j^k+\nu}\right), \quad (14)$$

where $I_j^k=\sum_{l=1,l\neq j}^{N}\theta_{jl}g_lp_{l,TX}^k$ is the intra-cell interference at the base station caused by mobile j's actual transmission. In the next time slot, each mobile is informed of the system price by the base station.

The power price, λ_i is required to determine the transmission power of each mobile. The projected subgradient method is known to generate solutions converging to optimal λ_i^{opt} of the dual problem (6) (see [17] and references therein). In the subgradient method, a step-size sequence $\{\alpha^k\}$ is needed to update λ_i to the negative subgradient direction for the minimization problem (6), more specifically Step 2. When the norm of the subgradient is bounded, the projected subgradient update is guaranteed to converge to the optimal dual solution, $g(\lambda)$ for the problem (6) as long as $\sum_{k=1}^{\infty}\alpha^k=\infty$ and $\lim_{k\to\infty}\alpha^k=0$ [17]. Our update algorithm selects the following choice [18]:

$$\alpha^k=\frac{\beta}{\sqrt{k}}, \quad (15)$$

where β is some positive constant.

Finally, we need to modify the iterative algorithm to take into account the presence of the primal constraints, i.e., $p_i\leq\overline{p_i}$. Step 3 of DUPC introduces a projection method [19]. The projected algorithm selects the "closest" point in the primal constraint, when the virtual transmission power is out of the primal constraint. The convergence of DUPC is described as the following proposition.

Proposition 1: Assume that there is only one mobile updating its power according to (11) at each iteration. Then, for arbitrary nonnegative initial values of power, DUPC guarantees convergence to a unique equilibrium point.

Proof: The objective function (2) can be decomposed into two components according to each mobile i, i.e., the rate of mobile i and the sum rate of all the others. The rate of mobile i is a monotonically increasing and concave function with respect to the transmission power of mobile i, p_i, while the sum rate of all the others is a monotonically decreasing and convex function of p_i. This implies that the Concave-Convex Procedure (CCCP) can be applied to the problem (5) [20]. The power update procedure (11) is in fact a CCCP algorithm given by:

$$\frac{\partial}{\partial p_i}\left\{r(\gamma_i(p_i^{k+1},p_{-i}^{k+1}))+\lambda_i^k(\overline{p_i}-p_i^{k+1})\right\} \\ =-\frac{\partial}{\partial p_i}\left\{\sum_{j\neq i}^{N} r(\gamma_j(p_i^k,p_{-i}^k))+\lambda_j^k(\overline{p_j}-p_j^k)\right\}, \quad (16)$$

where p_{-i} represents the power vector except for the power of mobile i, i.e., $p_{-i}=(p_1,p_2,\cdots,p_{i-1},p_{i+1},\cdots,p_N)$. It is assumed that p_{-i} is fixed during the power update of mobile i. The left hand side of Equation (16) is the concave part of the Lagrangian function (4), while the right hand side is the convex part. In [20], Theorem 2 shows that the CCCP algorithm is guaranteed to monotonically increase the objective function that is composed of a concave part and a convex part and hence to converge to a maximum or saddle point of the objective function. In other words, the iterative CCCP algorithm (16) guarantees that the objective function monotonically increases as follows:

$$\sum_{i=1}^{N}\left\{r(\gamma_i(p_i^k,p_{-i}^k))+\lambda_i^k(\overline{p_i}-p_i^k)\right\} \\ \leq \sum_{i=1}^{N}\log(1+\gamma_i(p_i^{k+1},p_{-i}^k))+\sum_{j\neq i}^{N}\lambda_j^k(\overline{p_j}-p_j^k) \\ +\lambda_i^k(\overline{p_i}-p_i^{k+1}), \quad (17)$$

There is slight difference between (11) and (16). In (11), there is a projection procedure that sets the negative power value to zero. However, this difference will not make any change in our proof (i.e., the monotonicity of (11)). Assume now at iteration $k+1$, mobile j updates its power according to (11). Using this one-by-one update, we approach to the (local) optimal solution of the function (4), if the objective function is upper-bounded (which is our case).

So far, we have assumed that the power price of each mobile, λ_i is fixed to a constant value. However, it is varying according to (12). Since the step size α^k of the update procedure (12) converges to zero, the power price of each mobile converges to a constant value after some iterations. So, the assumption of constant power price is reasonable.

In Step 3 of DUPC, the transmission power of each mobile is constrained by the maximum transmittable power in the uplink. Since the projected algorithm selects the “closest” point in the power truncation, the actual transmission power is the most adjacent value of the (local) optimal power to the problem (5) in the feasible region. Therefore, the monotonically increasing property of the CCCP algorithm is not changed, and the above convergence proof is well defined regardless of power truncation.

The above proposition says that DUPC converges to a near optimal power vector of Problem (A), only if the power update of each mobile is done in the one-by-one fashion (i.e., one mobile per slot). One-by-one assumption is rather strong and may not be true in real situations. The question is how the convergence of DUPC when multiple mobiles update their power simultaneously. In Section Ⅵ, we numerically show the convergence of this case, answering this question.

Practical implementation of DUPC can be done for both the base station and mobiles, which can be described as follows:

BS Algorithm

Step 0. Set $k=0$.

Step 1. Using the received information, estimate the system price Λ_i^k, interference I_i^k and channel gain g_i for each mobile i.

Step 2. The above feedback is informed to each mobile.

Step 3. Set $k=k+1$ and go to Step 1.

MS Algorithm For each mobile i, $i=1,2,\cdots,N$

Step 0. Set $k=0$ and arbitrary nonnegative initial values of p_i^k and λ_i^k.

Step 1. Assume that the system price Λ_i^k, interference I_i^k and channel gain g_i are informed from the base station.

Step 2. Power update : Select the transmission power, p_i^{k+1} using (11) and (13).

Step 3. Power price update : Update λ_i^{k+1} using (12).

Step 4. Set $k=k+1$ and go to Step 1.

Ⅳ. Downlink Throughput -Maximization Using the Up- and Downlink Duality

In this section, we introduce our up- and downlink duality theorem that establishes a basis for the downlink throughput-maximization problem given by:

Problem (B):

$$\max \sum_{i=1}^{N} r(\gamma_i(Q)) \tag{18}$$

$$s.t. \quad \sum_{i=1}^{N} q_i \le \overline{Q}, \tag{19}$$

where the vector $Q=(q_i)$ denotes the N-dimensional power vector of the downlink and the function r is the same as in Problem (A). The difference between Problems (A) and (B) is in the power constraints. In the uplink case (A), the transmission power of each mobile is individually constrained, i.e., $p_i \le \overline{p_i}$, $i=1,2,\cdots,N$. On the other hand, the sum of the transmission power is constrained in the downlink problem (B), i.e., $\sum_{i=1}^{N} q_i \le \overline{Q}$.

When the channels do not change too rapidly, link gains on the up- and downlink are identical. If we assume that the orthogonality factor and thermal noise in both systems are the same, the SINR received by mobile i in the downlink is defined by:

$$\gamma_i(Q) = \frac{g_i q_i}{\theta \sum_{j=1, j\ne i}^{N} g_i q_j + \nu}, \; i=1,\cdots,N. \tag{20}$$

The power prices of DUPC give a hint about the optimal transmission power of Problem (B) based on the up- and downlink duality.

Proposition 2: If we assume that the sum of the maximum transmittable power in the uplink is the same as the power constraint of the downlink, i.e., $\sum_{i=1}^{N} \overline{p_i} = \overline{Q}$, the extreme point of the uplink rate-region traces a boundary point of the downlink rate-region by varying the maximum transmittable power of each mobile in the uplink.

Proof: Solving (1) for p_i, we have [21]:

$$p_i = \frac{\nu}{1-\sum_{j=1}^{N} \frac{\theta\gamma_j}{1+\theta\gamma_j}} \cdot \frac{\gamma_i}{g_i(1+\theta\gamma_i)}, \tag{21}$$

for $i=1,\cdots,N$. In the above, since $0 \le p_i \le \overline{p_i}$, the SINR γ_i varies according to this power range. Noting that the instantaneous rate of mobile i is a function of γ_i, we can draw the rate region of mobile i within $0 \le p_i \le \overline{p_i}$. Intersections of such rates for each mobile composes the feasible rate region of the uplink Problem (A).

When each mobile transmits with the maximum power in the uplink, i.e., $p_i = \overline{p_i}$, $i=1,2,\cdots,N$, the SINR received by mobile i is denoted as $\overline{\gamma_i^{UL}}$ and the instantaneous rate lies on an extreme point of the uplink rate-region. The maximum transmission power is denoted as follows:

$$\overline{p_i} = \frac{\overline{\alpha_i^{UL}}}{1-\sum_{j=1}^{N}\overline{\alpha_j^{UL}}} \cdot \frac{\nu}{\theta g_i}, \quad (22)$$

where $\overline{\alpha_i^{UL}} = \frac{\theta\overline{\gamma_i^{UL}}}{1+\theta\overline{\gamma_i^{UL}}}$ has a one-to-one relationship with the rate of mobile i in the uplink.

Moving to the downlink case, we have [15]:

$$\sum_{i=1}^{N} q_i = \frac{\sum_{i=1}^{N}\frac{\gamma_i}{1+\theta\gamma_i} \cdot \frac{\nu}{g_i}}{1-\sum_{j=1}^{N}\frac{\theta\gamma_j}{1+\theta\gamma_j}}. \quad (23)$$

With the constraint of $\sum_{i=1}^{N} q_i \le \overline{Q}$, we can draw the rate region of the downlink Problem (B) using the above equation. If the sum of each mobile power is the same as the power constraint, i.e., $\sum_{i=1}^{N} q_i = \overline{Q}$, the instantaneous rate lies on a boundary point of the downlink rate-region. When the SINR received by mobile i on the boundary point is defined as $\overline{\gamma_i^{DL}}$, the power constraint of the downlink is denoted as follows:

$$\frac{\sum_{i=1}^{N}\overline{\alpha_i^{DL}} \cdot \frac{v}{\theta g_i}}{1-\sum_{j=1}^{N}\overline{\alpha_j^{DL}}} = \overline{Q}, \quad (24)$$

where $\overline{\alpha_i^{DL}} = \frac{\theta\overline{\gamma_i^{DL}}}{1+\theta\overline{\gamma_i^{DL}}}$ has a one-to-one relationship with the rate of mobile i in the downlink.

If we assume that the sum of the maximum power that can be transmitted by each mobile in the uplink is the same as the power constraint of the downlink, i.e., $\sum_{i=1}^{N}\overline{p_i} = \overline{Q}$, the following condition has to be satisfied by using Equations (22) and (24):

$$\frac{\sum_{i=1}^{N}\overline{\alpha_i^{UL}} \cdot \frac{\nu}{\theta g_i}}{1-\sum_{j=1}^{N}\overline{\alpha_j^{UL}}} = \frac{\sum_{i=1}^{N}\overline{\alpha_i^{DL}} \cdot \frac{\nu}{\theta g_i}}{1-\sum_{j=1}^{N}\overline{\alpha_j^{DL}}}. \quad (25)$$

From the above condition, $\overline{\alpha_i^{UL}}$ and $\overline{\alpha_i^{DL}}$, which have one-to-one relationships with the extreme point in the uplink or a boundary point in the downlink, are equal. Therefore, the extreme point of the uplink rate-region lies on a boundary point of the downlink rate-region. If we vary the maximum transmittable power of each mobile, while the total transmission power of all the mobiles is fixed to a constant, $\overline{Q}$, the boundary of the downlink rate-region is obtained with the set of extreme points in the uplink.

As mentioned, the Lagrange multipliers, λ_i, can be interpreted as power prices of mobile i. If we have $\lambda_1 > \lambda_2$, the power constraint $\overline{p_1}$ is more restrictive than $\overline{p_2}$, that is, increasing $\overline{p_1}$ while decreasing $\overline{p_2}$ by the same amount would lead to an increase in the sum rate of the uplink. On the other hand, if all power prices are equal, each power constraint is equally hard and no tradeoff of power constraint between different mobiles would increase the sum rate.

In [12], the authors introduced channel scaling to force the power prices to be equal. Channel scaling is used to derive that the capacity region of the uplink can be characterized in terms of the capacity region of the dual downlink. Here, the dual downlink means that the channel gains are symmetric between the up- and downlink and the power constraint is the same as the sum of uplink power constraints. In channel scaling, the channel gain is scaled by a component of the positive scaling vector, $\mu=(\mu_i)$, such as $\mu_i g_i$. If λ_i is the power price of mobile i for the unscaled uplink, then $\mu_i\lambda_i$ is the power price for the uplink scaled by $\mu=(\mu_i)$. Therefore, we can scale the channel appropriately so that $\mu_i\lambda_i$ are equal for all mobile i as follows:

$$\mu_1\lambda_1=\mu_2\lambda_2=\cdots=\mu_N\lambda_N. \qquad (26)$$

The scaling of the channel and the power constraint clearly negate each other in the uplink. In other words, the power constraint of each mobile is reciprocally scaled by a component of the scaling vector. In channel scaling, the sum of the transformed power constraint has to be held to a constant as follows:

$$\frac{\overline{p_1}}{\mu_1}+\frac{\overline{p_2}}{\mu_2}+\cdots\frac{\overline{p_N}}{\mu_N}=\sum_{i=1}^{N}\overline{p_i}. \qquad (27)$$

The uplink power constraints that are reciprocally scaled in Equation (27), are the same as the downlink transmission powers that maximize the sum rate in the dual downlink. If we assume $\sum_{i=1}^{N}\overline{p_i}=\overline{Q}$, every extreme point of the uplink rate-region scaled by some scaling vector can be shown to be on a boundary of the dual downlink rate-region by Proposition 2. When we exactly select a scaling vector, $\mu=(\mu_i)$ to satisfy Equations (26) and (27), each scaled power constraint, $\overline{p_i}/\mu_i$, can maximize the sum rate of the dual downlink. Using Equations (26) and (27), the transmission powers of the dual downlink are calculated as follows:

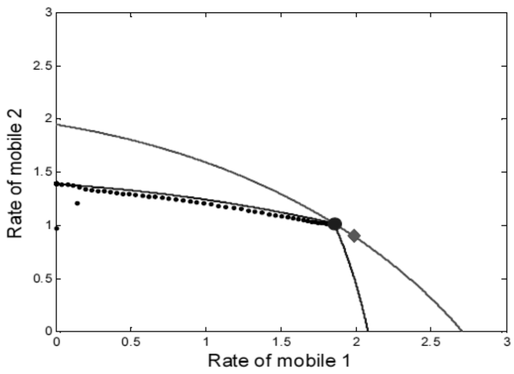

Fig. 1. Rate region of the up- and downlink system. The inner solid line represents the instantaneous rates of two mobiles in the uplink, while the outer solid line represents those in the downlink. The dots represent the iterative sequences from the DUPC algorithm

$$q_i=\frac{\overline{p_i}}{\mu_i}, \qquad where\ \mu_i=\frac{\sum_{i=1}^{M}\overline{p_i}\lambda_i}{\overline{Q}\lambda_i}. \qquad (28)$$

Example 1: For two mobiles, we can draw the up- and dual downlink rate region as in Figure 1. We apply the up- and downlink duality to the downlink power allocation problem. As shown in Figure 1, the simultaneous transmission of two mobiles is the optimal strategy in the uplink to maximize the total throughput. DUPC has iterative points that are represented by the dots. It is shown that the sequence of these points converges into the optimal point that maximizes the sum rate of two mobiles in the uplink. By Equation (28), we can find the optimal transmission power of two mobiles in the dual downlink for a given optimal power price of the uplink. The throughput-maximization point of the dual downlink is represented by the diamond point in Figure 1.

V. Network Duality Gap

So far, we assume that the channels between the up- and downlink are symmetric and the sum of the uplink power constraints is exactly the same as the downlink power constraint. Under this assumption, the downlink based on the up- and downlink duality property is defined as the dual downlink. The transmission powers of the dual downlink are obtained by using the channel scaling method (28). However, the downlink in the practical system has a nonsymmetric channel and an unequal power budget compared to the uplink. In this paper, it is defined as the real downlink, in which the channel scaling method (28) cannot maximize the throughput. We denote the throughput difference between the dual and real downlink as network duality gap. The network duality gap is determined by two factors, i.e., nonsymmetric channels and different power budgets.

When the sum of the uplink power constraints is not exactly the same as the downlink power constraint, the extreme point of the uplink rate-region does not lie on a boundary point of the downlink rate-region. Therefore, we have to compensate the network duality gap to obtain the exact transmission powers of the real downlink. If we assume that $\sum_{i=1}^{N}\overline{p_i}=\rho\overline{Q}$, ρ is a positive constant value that represents the difference of the power budgets between the up- and downlink. We simply substitute $\sum_{i=1}^{N}\overline{p_i}$ with $\rho\overline{Q}$ in Equation (27) to compensate the network duality gap as follows:

$$\frac{\overline{p_1}}{\mu_1}+\frac{\overline{p_2}}{\mu_2}+\cdots\frac{\overline{p_N}}{\mu_N}=\rho\overline{Q}. \tag{29}$$

We denote the up- and downlink channel gains of mobile i as g_i^{UL} and g_i^{DL}, respectively. When the channels between the up- and downlink are nonsymmetric, i.e., $g_i^{UL}\neq g_i^{DL}$, we have to force the power prices to be equal considering the network duality gap. Therefore, we appropriately select a scaling vector as follows:

$$\mu_1\lambda_1\frac{g_1^{DL}}{g_1^{UL}}=\mu_2\lambda_2\frac{g_2^{DL}}{g_2^{UL}}=\cdots=\mu_N\lambda_N\frac{g_N^{DL}}{g_N^{UL}}. \tag{30}$$

If we assume that the channels are symmetric between the up- and downlink, i.e. $g_i^{UL}=g_i^{DL}$, the above condition (30) is exactly equal to (26) of the dual downlink. We have multiplied the scaled power price of each mobile i by $\frac{g_i^{DL}}{g_i^{UL}}$ to compensate the difference of channel gains between the up- and downlink. If $\frac{g_i^{DL}}{g_i^{UL}}>1$, then the downlink power of q_i for the real downlink will be larger than the one for the dual downlink in proportional to $\frac{g_i^{DL}}{g_i^{UL}}$.

Due to the network duality gap, the component of channel scaling vector, $\mu=(\mu_i)$ is changed by using Equations (29) and (30) as follows:

$$\mu_i=\frac{\sum_{i=1}^{N}\overline{p_i}\epsilon_i\lambda_i}{\rho\overline{Q}\epsilon_i\lambda_i}, \tag{31}$$

where $\epsilon_i=\frac{g_i^{DL}}{g_i^{UL}}$ denotes the ratio of the channel gains between the up- and downlink. Using the modified channel scaling vector (31), we can calculate the transmission powers of the real downlink, not the dual downlink. In the next section, we apply this modified channel scaling method (31) to the downlink that has a different channel gain and power budget compared to the uplink. We will numerically compare the performance difference, when the network duality gap exists or not.

Ⅵ. Numerical Results

For simplicity, we assume that $\theta_{ij}=\theta>0$ for all i and j. We provide some simulation results to illustrate the performance of DUPC. The simulation environment is considered to be an isolated single cell of a DS-CDMA system. The cell has a radius of 1Km. For a given instance, a total of 10 mobiles are generated, the locations of which are randomly distributed over the cell. The link gain g_i is modeled as $g_i = s_i \cdot d_i^{-4}$, where s_i is the shadow fading factor and d_i is the distance between the base station and mobile i. The log-normally distributed s_i is generated according to $E[s_i]=0$ dB and $\sigma^2(s_i)=8$ dB. The power constraint of each mobile is the same as 30 dBm, and the thermal noise power is -70 dBm.

Firstly, we consider the orthogonality factor, $\theta=0.1$. With 5 mobiles deployed, Figures 2 and 3 respectively show the convergence of the power and the power price (Lagrange multiplier) for each mobile under the DUPC algorithm, starting from arbitrary nonnegative initial values. Here, we select $\beta=0.7$ for the step size in (15). For a given instance, Figure 2 describes that the opportunistic transmission in which only the best mobile transmits, is the optimal strategy in the uplink. When one mobile transmits with the maximum transmittable power, the optimal power prices of the others have zero values. By Equation (28), all mobiles except the best mobile have zero power in the downlink.

Therefore, the mobile that has the best channel gain is the only one that transmits in the downlink according to the up- and downlink duality. Unlike Figures 2 and 3, there is a scenario where the simultaneous transmission of some mobiles is the optimal strategy in the uplink. In this case, the simultaneous transmission of some weak mobiles has better performance than the opportunistic transmission due to large interference from the best gain mobile. If we determine the allocated downlink powers by Equation (28), the simultaneous transmission of some weak mobiles is also the strategy in the downlink, even if the power allocation is different from the uplink.

Next, we compare the performance of DUPC with that of power allocation proposed by Kumaran and Quian [6]. The Kumaran and Quian algorithm, the K&Q algorithm for short, has the property that each transmitting mobile transmits at the full power, i.e., $p_i=0$ for some subset S of the mobiles and $p_i=\overline{p_i}$ for the complementary set $\overline{S}$. This algorithm maximizes the sum rate of all mobiles, only if the orthogonality factor is $\theta \in (0.5,1]$. However, it is a centralized algorithm that determines the optimal transmitting mobile set from the full enumeration of all mobiles. As the number of mobiles increases, the complexity grows dramatically.

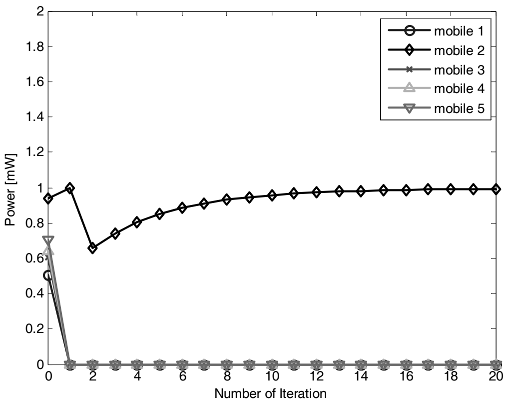

Fig. 2. Convergence of the power. Each curve corresponds to the power or the price for each mobile

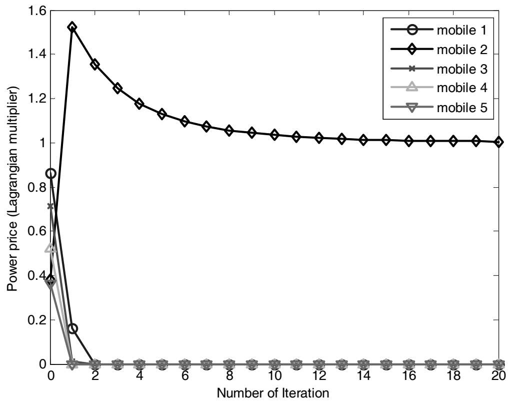

Fig. 3. Convergence of the power price (Lagragian multiplier). Each curve corresponds to the power or the price for each mobile

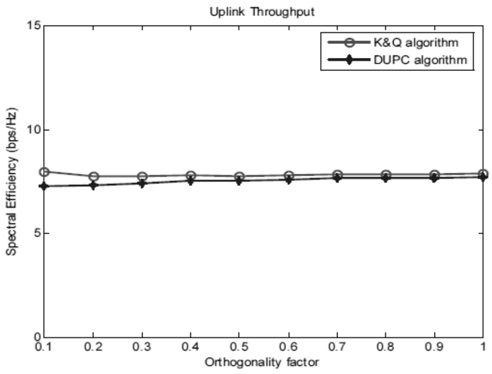

Fig. 4. Comparison of the Kumaran and Quian algorithm and DUPC algorithm in the uplink

We use the spectral efficiency as a performance measure, i.e., Shannon capacity with normalized bandwidth. We have performed 20,000 simulations to achieve the average spectral efficiency in Figures 4-6. For a given orthogonality factor, Figure 4 describes the difference between the spectral efficiency of the K&Q algorithm andDUPC. DUPC has about 97% performance compared with the K&Q algorithm in the uplink. Therefore, it has not only a "distributive" advantage, but it also has almost the same performance compared with the optimal value for the orthogonality factor $\theta \in (0.5,1]$. As we mentioned earlier in Section Ⅱ, the objective function of the uplink Problem (A) is neither convex nor concave, and finding the global optimal solution requires full enumeration of all

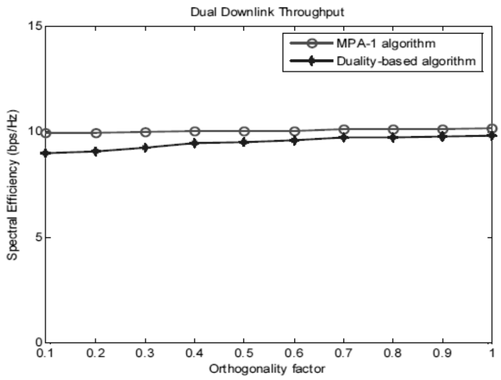

Fig. 5. Comparison of MPA-1 algorithm and duality-based algorithm in the dual downlink

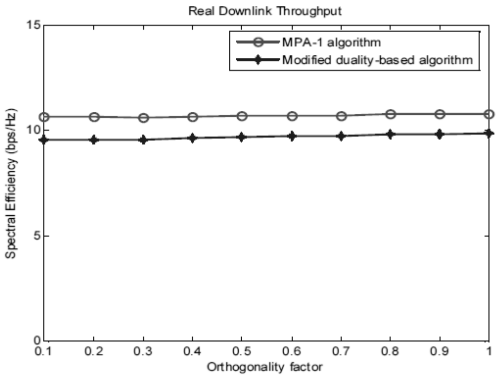

Fig. 6. Comparison of MPA-1 algorithm and modified duality-based algorithm in the real downlink

local optimal solutions, which is of combinatorial behavior. Therefore it is rather hopeless to try to find the global optimal solution in a reasonable amount of time. We numerically show that DUPC converges to a unique point. The convergence point may not exactly coincide with one of the local optimal solutions due to violation of the Karush-Kuhn-Tucker necessary condition of Problem (A). Nevertheless, DUPC has quite an encouraging result, as shown in Figure 4.

Based on the up- and downlink duality, we can determine the transmission power allocated in the dual downlink, which has a symmetric channel and an equal power budget compared to the uplink. This duality-based algorithm does not need another separate power allocation algorithm in the downlink, but directly calculates the transmission power by using the power prices from the uplink. We compare the performance of the duality-based algorithm with the MPA-1 algorithm, an efficient heuristic algorithm proposed in [15]. In Figure 5, our duality-based algorithm compared with the MPA-1 algorithm has about 96% performance for the orthogonality factor $\theta \in (0.5,1]$, and about 93% performance for $\theta \in (0,0.5]$. Therefore, the duality-based algorithm has not only a "impleness", but it also has almost the same performance for the orthogonality factor $\theta \in (0.5,1]$. Some throughput degradation under $\theta \in (0,0.5]$ shows that inaccurate power prices will decrease the system throughput more than in the case of $\theta \in (0.5, 1]$.

Finally, we consider the real downlink that has a nonsymmetric channel and an unequal power budget compared to the uplink. The power constraint of the real downlink is set to as 2 times as the sum of 10 uplink power constraints for the simulation. The shadow fading factor of channel gain in the real downlink is newly generated, while the location of each mobile is fixed to that in the uplink. In Section Ⅴ, we have established the modified channel scaling method (31) to compensate the network duality gap. Using this method, we can find the transmission power allocated in the real downlink. Figure 6 shows that the duality-based algorithm has about 90% performance compared to the MPA-1 algorithm in the real downlink. Since the modified channel scaling method (31) have been developed as a heuristic algorithm, the modified algorithm has more performance gap compared to MPA-1 algorithm in the real downlink than the dual downlink as shown in Figures 5 and 6. However, it is a slight performance gap of about 4%.

Ⅶ. Conclusions

In this paper, we considered throughput-maximization problems for both the up- and downlink by choosing a feasible power allocation of each mobile. To approach the optimal solution, we proposed a DUPC algorithm in the uplink. DUPC has about 97% performance compared with the K&Q algorithm, while each mobile respectively updates its transmission power based on the measurement feedback from the base station.

We extended the duality properties to the general wireless network. The up- and downlink duality was shown in a more realistic setting, in which the assumptions of SIC and unlimited uplink power budget are removed. Based on this up- and downlink duality, we solved the throughput-maximization problem in the dual downlink that has a symmetric channel and an equal power budget compared to the uplink. Compared with the MPA-1 algorithm, our duality-based downlink algorithm has about 96% performance for the orthogonality factor $\theta \in (0.5, 1]$ without any additional power allocation scheme in the dual downlink. Additionally, we developed the modified channel scaling method applied to the real downlink that has a nonsymmetric channel and an unequal power budget compared to the uplink. It is shown that the modified algorithm is thoroughly applied to the real downlink. From the numerical results, it is found that the proposed throughput-maximization algorithm for the orthogonality factor $\theta \in (0.5, 1]$ is an attractive method that not only has almost the same performance as that of the optimal algorithm, but also has the advantages of distributiveness and simpleness.

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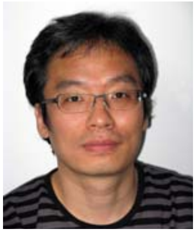

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