Blind Algorithms with Decision Feedback based on Zero-Error Probability for Constant Modulus Errors

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ABSTRACT

The constant modulus algorithm (CMA) widely used in blind equalization applications minimizes the averaged power of constant modulus error (CME) defined as the difference between an instant output power and a constant modulus. In this paper, a decision feedback version of the linear blind algorithm based on maximization of the zero-error probability for CME is proposed. The Gaussian kernel of the maximum zero-error criterion is analyzed to have the property to cut out excessive CMEs that may be induced from severely distorted channel characteristics. Decision feedback approach to the maximum zero-error criterion for CME is developed based on the characteristic that the Gaussian kernel suppresses the outliers and this prevents error propagation to some extent. Compared to the linear algorithm based on maximum zero-error probability for CME in the simulation of blind equalization environments, the proposed decision feedback version has superior performance enhancement particularly in cases of severe channel distortions.

Key Words : Constant modulus error, Decision Feedback, Blind equalizer, Zero-error probability

Ⅰ. Introduction

Blind equalizers are commonly used in many communication areas to cancel intersymbol interferences (ISI) from channel distortions because of the advantage that they do not require any training symbols[1,2]. Most blind equalizer algorithms use mean-square-error (MSE) criterion for weight-adjustment. One of the well known blind algorithms is the constant modulus algorithm (CMA) that minimizes the statistical average of the power of constant modulus error (CME), which is defined as the difference between an instant equalizer output power and a constant modulus[3]. Recently, instead of being based on MSE criterion for blind equalizer algorithms, a new constant modulus criterion that maximizes the probability that equalizer output power is equal to the constant modulus of the transmitted symbols has been proposed[4]. The probability of CME is obtained from CME samples directly by means of Parzen window estimation method[5], which is one of the bases of information-theoretic learning (ITL) introduced by Princepe[6]. The ITL methods have been developed based on a combination of Parzen probability-density-function (PDF) estimator and a procedure to compute entropy[7] and have shown superior performance as an alternative to MSE in supervised adaptive systems[8]. For unsupervised equalization, the researchers in the work [4] have also developed a new blind algorithm by applying the gradient ascent method to maximize the criterion of zero-error probability for CME. The proposed algorithm has shown a faster speed of convergence and lower steady-state MSE performance in comparison with CMA. The blind algorithm, however, is based on a linear combiner structure so that it cannot counteract ISI from worse channel environments. In this paper, in order to cope with severe channel distortions in blind equalization

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systems, we propose to employ a decision feedback structure based on the advantages from the criterion of zero-error probability for constant modulus error.

Ⅱ. Constant Modulus Error for Blind Equalization

In the linear equalizer structure of a tapped delay line (TDL), the output at symbol time k can be expressed as $y_k = \mathbf{W}_k^T \mathbf{X}_{k,N}$ where the input vector and adjustable weight vector are defined as $\mathbf{X}_{k,N} = [x_k, x_{k-1}, x_{k-2}, ..., x_{k-N+1}]^T$ and $\mathbf{W}_k^T = [w_{k,0}, w_{k,1}, w_{k,2}, ..., w_{k,N-1}]$, respectively. In the blind equalization algorithm, CMA, the power of CME $e_{CME} = |y_k|^2 - R_2$ is to be minimized as

$$P_{CMA} = E[e_{CME}{}^2] = E[(|y_k|^2 - R_2)^2], \tag{1}$$

where $R_2 = E[|d_k|^4]/E[|d_k|^2]$ and d_k is the transmitted symbol at time k.

By differentiating P_{CMA} dropping the expectation operation and using the steepest descent method, we obtain the following CMA[1] for adjusting the blind equalizer weights:

$$\mathbf{W}_{k+1} = \mathbf{W}_k - 2\mu_{CMA} \cdot \mathbf{X}_{k,N}^* \cdot y_k \cdot (|y_k|^2 - R_2) \tag{2}$$

where μ_{CMA} is the step-size parameter. We can notice in (2) that CME makes a direct impact on the weight adjustment, which means that any excessive CME from severe channel distortions can bring about a catastrophic failure to the blind equalizer.

Ⅲ. Maximum Zero-error Probability Criterion for Constant Modulus Error

To create a concentration of CME near zero, the CMA uses MSE criterion. Instead of relying on MSE criterion, we can deal with an information theoretic criterion of error probability $f_E(e)$. Recently in [4] a new blind criterion by maximizing the zero-error probability for constant modulus error e_{CME} has introduced as

$$\max_W f_E(e_{CME})\big|_{e_{CME}=0} \tag{3}$$

To obtain $f_E(\cdot)$ non-parametrically, we need the Parzen estimator[5] using Gaussian kernel as follows

$$f_X(x) \cong \frac{1}{M}\sum_{i=1}^{M} G_\sigma(x - x_i) \tag{4}$$

The zero-mean Gaussian Kernel $G_\sigma(\cdot)$ with standard deviation σ is defined as $G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp[\frac{-x^2}{2\sigma^2}]$. Inserting CME into (4) and using a block of past output samples $Y_k = \{y_k, y_{k-1}, ..., y_{k-M+1}\}$, we have

$$f_E(e_{CME}) = \frac{1}{M}\sum_{i=0}^{M-1} G_\sigma(e_{CME} - [|y_{k-i}|^2 - R_2]) \tag{5}$$

Letting e_{CME} be zero, the probability $f_e(e_{CME})$ reduces to

$$f_E(e_{CME})\big|_{e_{CME}=0} = \frac{1}{M}\sum_{i=0}^{M-1} G_\sigma(-[|y_{k-i}|^2 - R_2]) \tag{6}$$

Using a gradient ascent method for the maximization of the zero-error probability for CME based on the linear TDL structure, the maximum zero-error probability for CME (MZEP-CME) algorithm[4] is derived as

$$\mathbf{W}_{k+1} = \mathbf{W}_k + \mu_{MZEP-CME}\frac{\partial f_E(e_{CME})\big|_{e_{CME}=0}}{\partial \mathbf{W}_k} \tag{7}$$

$$\begin{aligned}\mathbf{W}_{k+1} = \mathbf{W}_k + \mu_{MZEP-CME}\frac{-2}{\sigma^2 M}\sum_{i=0}^{M-1} G_\sigma(|y_{k-i}|^2 - R_2) \\ \cdot (R_2 - |y_{k-i}|^2) \cdot y_{k-i} \cdot \mathbf{X}_{k-i,N}^*\end{aligned} \tag{8}$$

where $\mu_{MZEP-CME}$ is the step-size for convergence control.

We assume that L-ary PAM signaling systems are employed and the transmitted levels A_l takes the following discrete values

$$A_l = 2l - 1 - L, \quad l = 1, 2, ..., L \tag{9}$$

Then the constant modulus R_2 becomes

$$R_2 = E[|A_l|^4]/E[|A_l|^2] \tag{10}$$

Ⅳ. MZEP-CME Algorithm with Decision Feedback

The decision feedback equalizer comprises a feed-forward filter with weight vector W_k^F and a feedback filter with weight vector W_k^B for producing corresponding decisions $\hat{d}_k$ from input x_k. The feed-forward filter is identical to the TDL which is adopted in CMA and MZEP-CME algorithm. The feedback filter receives decisions on previously detected symbols. The residual ISI from the present estimate is to be removed by the feedback filter[9].

The feed-forward filter weights are the elements of $\mathbf{W}_k^F = [w_{k,0}^F, w_{k,1}^F, w_{k,2}^F, ..., w_{k,P-1}^F]^T$, and feedback filter weight vector is $\mathbf{W}_k^B = [w_{k,0}^B, w_{k,1}^B, w_{k,2}^B, ..., w_{k,Q-1}^B]^T$. The symbol $\hat{d}_k$ is an output of decision device for the equalizer output y_k. The input vector for the feed-forward filter section is defined as $\mathbf{X}_{k,P} = [x_k, x_{k-1}, x_{k-2}, ..., x_{k-P+1}]^T$ and the previously detected symbols for feedback section are in the decision vector $\hat{\mathbf{D}}_{k-1} = \left[\hat{d}_{k-1}, \hat{d}_{k-2}, ..., \hat{d}_{k-Q-2}\right]^T$. Then the output can be expressed as

$$y_k = [\mathbf{W}_k^F]^T \mathbf{X}_{k,P}^* + [\mathbf{W}_k^B]^T \hat{\mathbf{D}}_{k-1}^* \tag{11}$$

The filter weights are adjusted recursively in order to maximize the zero-error probability $f_E(e_{CME})|_{e_{CME}=0}$ according to the gradient ascent method.

$$\mathbf{W}_{new}^F = \mathbf{W}_{old}^F + \mu_{MZEP-CME} \frac{\partial f_E(e_{CME})|_{e_{CME}=0}}{\partial \mathbf{W}^F} \tag{12}$$

$$\mathbf{W}_{new}^B = \mathbf{W}_{old}^B + \mu_{MZEP-CME} \frac{\partial f_E(e_{CME})|_{e_{CME}=0}}{\partial \mathbf{W}^B} \tag{13}$$

The gradients are evaluated from

$$\begin{aligned}\frac{\partial f_E(e_{CME})|_{e_{CME}=0}}{\partial \mathbf{W}^F} &= \frac{1}{M}\sum_{i=0}^{M-1}\frac{\partial}{\partial \mathbf{W}^F} G_\sigma(-[|y_{k-i}|^2 - R_2]) \\ &= \frac{-2}{\sigma^2 M}\sum_{i=0}^{M-1} G_\sigma(|y_{k-i}|^2 - R_2)(R_2 - |y_{k-i}|^2)\cdot y_{k-i}\cdot \mathbf{X}_{k-i,P}^*\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial f_E(e_{CME})|_{e_{CME}=0}}{\partial \mathbf{W}^B} &= \frac{1}{M}\sum_{i=0}^{M-1}\frac{\partial}{\partial \mathbf{W}^B} G_\sigma(-[|y_{k-i}|^2 - R_2]) \\ &= \frac{-2}{\sigma^2 M}\sum_{i=0}^{M-1} G_\sigma(|y_{k-i}|^2 - R_2)(R_2 - |y_{k-i}|^2)\cdot y_{k-i}\cdot \hat{\mathbf{D}}_{k-i-1}^*\end{aligned} \tag{15}$$

where $M \geq P$ and $M \geq Q$.

Now decision feedback MZEP-CME algorithm (DF-MZEP-CME) can be summarized as

$$\begin{aligned}\mathbf{W}_{k+1}^F = \mathbf{W}_k^F + \mu_{MZEP-CME}\frac{-2}{\sigma^2 M}\sum_{i=0}^{M-1} G_\sigma(|y_{k-i}|^2 - R_2) \\ \cdot(R_2 - |y_{k-i}|^2)\cdot y_{k-i}\cdot \mathbf{X}_{k-i,P}^*\end{aligned} \tag{16}$$

$$\begin{aligned}\mathbf{W}_{k+1}^B = \mathbf{W}_k^B + \mu_{MZEP-CME}\frac{-2}{\sigma^2 M}\sum_{i=0}^{M-1} G_\sigma(|y_{k-i}|^2 - R_2) \\ \cdot(R_2 - |y_{k-i}|^2)\cdot y_{k-i}\cdot \hat{\mathbf{D}}_{k-i-1}^*\end{aligned} \tag{17}$$

Ⅴ. The Mitigation Effect on Excessive CME

In a severe channel distortion environment, most blind learning algorithms produce frequent large error signal and ensuing incorrect decisions. Incorrect decisions can cause error propagation in decision feedback equalizers. For this reason, in most blind applications, large error signal makes using decision feedback impossible. In CMA, the large CME induced by severe channel distortion can reduce weight values enough to minimize the power of CME to some acceptable extent. This can yield bursts of errors. We can notice the direct impact of CME on the weight update equation of CMA in (2).

In short, severe channel distortions can induce large error samples that hinder the application of decision feedback approach to CMA. Without any measures of reducing the direct influence of CME on the weight update equation of CMA, employment of decision feedback in CMA is considered not a

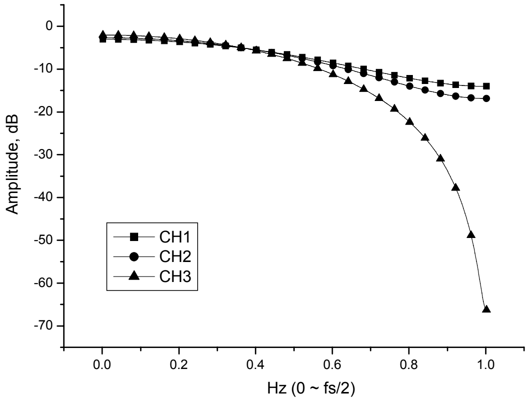

Fig. 1. Amplitude spectrum for the channel models

feasible strategy for residual ISI cancellation.

In the case of the proposed algorithm in (16) and (17), the CME goes through the Gaussian kernel. We see that the Gaussian kernel $G_{\sigma}(|y_{k-i}|^2 - R_2)$ produces an exponential decay with the distance between the instant output power and the constant modulus R_2. For proposed algorithm in severe channel conditions, therefore, the excessively large $|y_{k-i}|^2 - R_2$ induced by the channel condition becomes a very small value through the Gaussian kernels in the feedforward and feedback filter weight updates.

So we can remark that the Gaussian kernel $G_{\sigma}(|y_{k-i}|^2 - R_2)$ plays a role of reducing the impact of excessive CME on the update equations for feedforward and feedback section weights. This inherent immunity to excessive CME from severe channel distortions, that is, the immunity to error propagation has provided us with the ground for employing the decision feedback structure to MZEP-CME algorithm.

Ⅵ. Simulation results and discussion

In this section, the comparative performance of the linear MZEP-CME and the proposed DF-MZEP-CME algorithms in blind equalization is presented for three linear channels, and simulation results are discussed. The 4 level ($L = 4$) random signal is transmitted to the channel and the transfer functions $H(z)$ for each channel model[10] are

$$\text{CH1:}\ H(z) = 0.26 + 0.93z^{-1} + 0.26z^{-2} \tag{18}$$

$$\text{CH2:}\ H(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \tag{19}$$

$$\text{CH3:}\ H(z) = 0.407 + 0.815z^{-1} + 0.407z^{-2} \tag{20}$$

The number of weights is $N = 11$ in the linear TDL equalizer structure. The number of feed-forward and feedback section weights is $F = 7$ and $B = 4$, respectively. The channel noise (AWGN) variance is 0.001. As a measure of equalizer performance, we use MSE learning curves and probability densities for errors of the difference between the actual transmitted symbol and the output for linear MZEP-CME and the proposed DF-MZEP-CME. The data-block size and the kernel size are $M = 20$ and $\sigma = 6$, respectively. The step size for controlling convergence conditions is commonly set to $\mu_{\text{MZEP-CME}} = 0.02$ for both algorithms.

In the results for CH1 and CH2 of comparatively moderate channel conditions as shown in Fig.2 to 5, we observe that the performance gain in steady

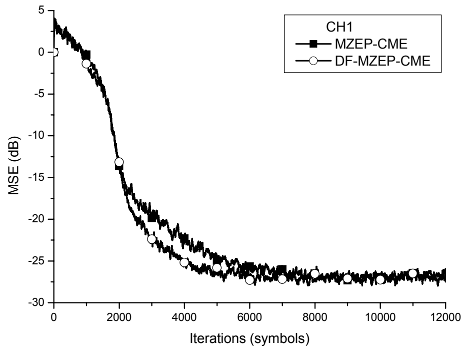

Fig. 2. MSE convergence performance in CH1

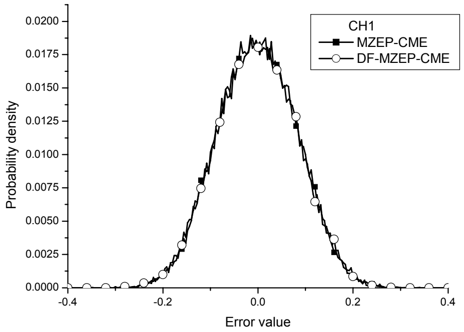

Fig. 3. Probability density for errors in CH1

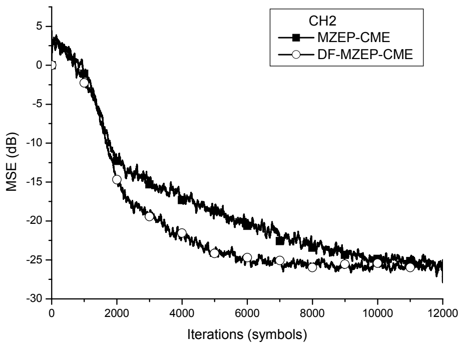

Fig. 4. MSE convergence performance in CH2

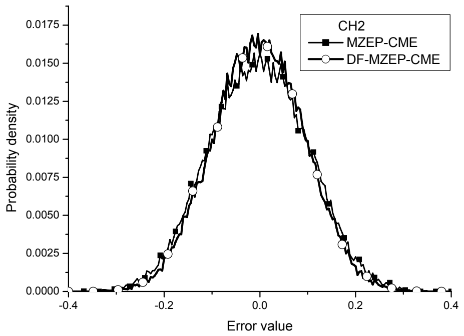

Fig. 5. Probability density for errors in CH2

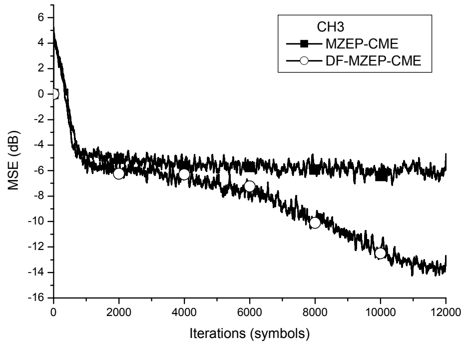

Fig. 6. MSE convergence performance in CH3

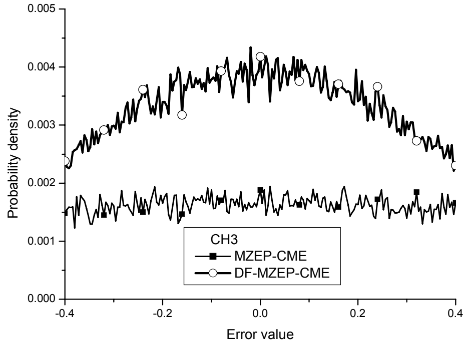

Fig. 7. Probability density for errors in CH3

state MSE is almost the same in CH1 and slight in CH2.

However, DF-MZEP-CME shows increased convergence speed in both channels. In probability density comparisons for output error shown in Fig. 3 and 5, the decision feedback approach gives error values better concentration to zero in worse channel models though not significant enhancement.

These results give us the motivation to investigate performance differences related to CME and decision feedback in much worse channel conditions than the moderate channel models of CH1 and CH2.

The MSE convergence results acquired in the worst channel model CH3 according to this motivation are shown in Fig. 6 (the step size in this channel model CH3 is 0.06 for both algorithms).

The learning curve for the linear MZEP-CME algorithm stays at almost the same MSE of -6 dB, but that of the proposed DF-MZEP-CME algorithm goes steeply down to even -14 dB as weight adjustment is proceeded. The difference of steady state MSE is over 8 dB. As analyzed in the section of V and seen in Fig. 6, we can notice the mitigation effect conspicuously on excessive CME from the severe channel model. We can conclude that the Gaussian kernel $G_\sigma(|y_{k-i}|^2 - R_2)$ plays an important role of cutting out large CMEs in severe ISI channel conditions so that the employment of decision feedback approach can yield significant performance enhancement.

Ⅶ. Conclusion

In this paper, in order to cope with severe channel distortions in blind equalization systems, a decision feedback algorithm based on zero-error probability for constant modulus error has been presented. The proposed algorithm employing decision feedback and Gaussian kernel to deal with constant modulus errors has shown superior performance particularly in severe channel models.

From the observations of the steady state MSE and error distribution and the analysis of the proposed algorithm, we have come to the conclusion that the proposed blind equalizer algorithm with decision feedback can be appropriate for the compensation of severe channel distortions.

The inherent characteristics of the proposed algorithm are that the Gaussian kernel of the proposed decision feedback algorithm plays a role of mitigating the impact of large constant modulus errors on system weight adjustment, so that the employed decision feedback structure which is vulnerable to error propagation can carry out the residual ISI cancellation effectively.

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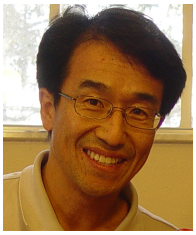

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