

# 다중 안테나 포트를 장착한 분산 안테나 시스템에서의 안테나 설계 방법

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## Antenna Placement Designs for Distributed Antenna Systems with Multiple-Antenna Ports

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### 요약

본 논문은 포트 당 일정 파워 제약을 전제한 상황에서, 다중 안테나를 장착한 분산 안테나 (distributed antenna: DA) 포트를 갖는 분산 안테나 시스템 (distributed antenna system: DAS)의 안테나 위치 설계 방법을 분석한다. 안테나 위치의 설계를 위해 복잡하게 셀 당 평균 ergodic sum rate를 최대화하는 대신, 본 논문에서는 단일 셀 상황에서는 signal-to-noise ratio (SNR) 기댓값의 lower bound에, 그리고 이중 셀 상황에서는 signal-to-leakage ratio (SLR) 기댓값의 lower bound에 각각 초점을 맞춘다. 단일 셀 상황의 경우, 기존의 반복적 알고리즘에 비해 SNR criterion의 최적화 문제는 닫힌 형태 (closed-form)의 솔루션을 제공한다. 또한, 이중 셀 상황에선 gradient ascent 방법을 이용한 알고리즘을 제안하여 SLR criterion의 최적화 솔루션을 도출한다.

**Key Words** : multiple-antenna port, distributed antenna systems (DAS), distributed antenna (DA), antenna location, antenna placement

### ABSTRACT

In this paper, we optimize antenna locations for a distributed antenna system (DAS) with distributed antenna (DA) ports equipped with multiple antennas under per-DA port power constraint. Maximum ratio transmission and scaled zero-forcing beamforming are employed for single-user and multi-user DAS, respectively. Instead of maximizing the cell average ergodic sum rate, we focus on a lower bound of the expected signal-to-noise ratio (SNR) for the single-cell scenario and the expected signal-to-leakage ratio (SLR) for the two-cell scenario to determine antenna locations. For the single-cell case, optimization of the SNR criterion generates a closed form solution in comparison to conventional iterative algorithms. Also, a gradient ascent algorithm is proposed to solve the SLR criterion for the two-cell scenario. Simulation results show that DAS with antenna locations obtained from the proposed algorithms achieve capacity gains over traditional centralized antenna systems.

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## I. Introduction

Over the past decade, a distributed antenna system (DAS) has drawn attention as a new structure for wireless communication due to its advantages over conventional centralized antenna systems (CAS). With distributed antenna (DA) ports separated geographically within a cell, DAS provides an increase in system capacity and coverage and a reduction in the access distance, which lead to decreased transmit power and co-channel interference<sup>[1-3]</sup>. Thus, designing the location of DA ports is a critical issue in enhancing the performance of DAS.

Recently, several papers proposed algorithms for determining the location of DA ports which are equipped with a single antenna. In [4] and [5], the squared distance criterion (SDC) was presented to design the antenna location which maximizes a lower bound of the cell average ergodic capacity for the single-cell DAS. The authors of [4] studied the problem of antenna positioning for the uplink DAS in which all DA ports receive the signal from a mobile station (MS). The work in [5] has applied results in [4] into downlink DAS confined to selective transmission (ST) where one DA port is selected for transmitting data to a MS. In [6] and [7], sub-optimal DA port locations which maximize lower bounds of the expected signal-to-noise ratio (SNR) and the expected signal-to-leakage ratio (SLR) were suggested for single-cell and two-cell DAS. However, aforementioned works are not suitable for DAS which is equipped with multiple antennas. The authors of [8] proposed an iterative algorithm to determine optimal deployment of antenna locations based on stochastic approximation theory for DAS with multiple antenna DA ports. However, this algorithm is feasible only for the single-cell single-user case and requires an iterative method.

In recent years, optimal transmission schemes with per antenna power constraint have been studied for traditional downlink CAS [9] and [10]. The optimal transmission was established which

achieves multiple-input single-output capacity with per-antenna power constraint in [9]. Also the authors in [10] suggested transmitter optimization using uplink-downlink duality. Furthermore, scaled down zero-forcing beamforming (ZFBF) was proposed in [11] to reduce the complexity of the optimal transmission scheme. Meanwhile, for the single-cell single-user DAS with multiple-antenna ports, maximum ratio transmission (MRT) at each DA port is shown to be the optimal beamforming scheme which can achieve the ergodic capacity under per-DA port power constraint<sup>[12]</sup>.

In this paper, we extend the works in [7] to more generalized DAS scenarios with DA ports equipped with multiple antennas under per-DA port power constraint. The MRT in [12] and the scaled ZFBF in [11] are applied for single-user and multi-user DAS, respectively. Since the extension from DA port equipped with single antenna to multiple antennas causes increase in dimensions and per-DA port power constraint, we apply some approximations and inequalities to properly formulate the optimization problem. The locations of DA ports are obtained by considering the following performance metrics. For the single-cell scenario, a lower bound of the expected SNR is maximized for identifying the locations of DA ports. This optimization problem is convex and generates a closed form solution. Simulation results show that DA port locations determined by the proposed algorithms are almost the same as the optimal locations from conventional algorithms in [8] with much lower complexity. Also, our closed form solution provides insight on the antenna placement when planning the DAS with given number of DA ports, while the conventional algorithms in [8] derives the antenna location which alters for every calculation due to different initial points and number of iterations. For the two-cell scenario, locations of DA ports are found by maximizing a lower bound of the expected SLR. The SLR criterion is suitable for multi-cell environments since other-cell interference is taken into consideration. In this case, we apply an iterative

gradient ascent algorithm as the optimization problem is non-convex.

The rest of our paper is organized as follows: Section II describes the channel model and transmission schemes. In Section III, we formulate the optimization problem and propose new algorithms for antenna location designs. Numerical results are illustrated to show performance gains of the proposed schemes in Section IV, and Section V concludes this paper.

## II. System Model

We consider a multi-cell downlink DAS. There are  $N$  DA ports equipped with  $T$  antennas, and  $K$  MSs equipped with a single antenna in each cell where all MSs are assumed to be uniformly distributed within a cell. We assume a cell with the radius of  $R$  with circular antenna layout and the distance between centers of adjacent two cells is set to  $\sqrt{3}R$ . Each cell is divided into  $N$  equal area regions and a DA port with multiple antennas is located in each region. The location of the  $i$ -th DA port is expressed as

$$P_i = R_i \exp(j\theta_i), \text{ for } i = 1, \dots, N \quad (1)$$

where  $R_i$  and  $\theta_i$  are the magnitude and the phase of the  $i$ -th DA port with  $R_{\min,i} \leq R_i \leq R_{\max,i}$  and  $\theta_{\min,i} \leq \theta_i \leq \theta_{\max,i}$ , respectively.

The channel model for DAS encompasses not only small scale fadings but also large scale fadings including shadowing and pathloss [2]. It is assumed that perfect channel state information and MS locations are known to all DA ports. We concentrate on fully cooperative networks in which the DA ports are connected via ideal backhaul in each cell but with no inter-cell cooperation. First, we define the

precoded signal vector as  $\mathbf{x}^{(l)} = \sum_{k=1}^K \mathbf{f}_k^{(l)} u_k^{(l)}$  at cell

$l$ , where  $\mathbf{f}_k^{(l)} = [\mathbf{f}_{k,1}^{(l)T} \dots \mathbf{f}_{k,N}^{(l)T}]^T$  indicates the  $NT \times 1$  beam-forming vector for the  $k$ -th MS in cell  $l$  with  $\|\mathbf{f}_{k,n}^{(i)}\|^2 \leq P$  and  $u_k^{(l)}$  represents the

complex-valued data symbol for the  $k$ -th MS in cell  $l$  with  $\mathbb{E}[|u_k^{(l)}|^2] = 1$ . Here,  $P$  denotes the power available at each DA port.

Then, the  $T \times 1$  channel vector from the  $n$ -th DA port in cell  $i$  to the  $k$ -th MS in cell  $j$  is defined as

$$\mathbf{g}_{k,n}^{(i,j)} = \sqrt{\frac{s_{k,n}^{(i,j)}}{(d_{k,n}^{(i,j)})^\alpha}} [h_{k,(n,1)}^{(i,j)} \dots h_{k,(n,T)}^{(i,j)}]^T$$

where  $h_{k,(n,t)}^{(i,j)}$  equals an independent and identically distributed zero-mean unit-variance complex Gaussian random variable representing small scale fadings from the  $t$ -th antenna of  $n$ -th DA port in cell  $i$  to the  $k$ -th MS in cell  $j$ ,  $s_{k,n}^{(i,j)}$  is a log-normal random variable corresponding to large scale fadings from the  $n$ -th DA port in cell  $i$  to the  $k$ -th MS in cell  $j$ , i.e.  $10 \log_{10} s_{k,n}^{(i,j)}$  is zero-mean Gaussian with variance  $\sigma_{sh}^2$ ,  $d_{k,n}^{(i,j)}$  stands for the distance between the  $n$ -th DA port in cell  $i$  to the  $k$ -th MS in cell  $j$ , and  $\alpha$  indicates the path loss exponent.

The received signal for the  $k$ -th MS in cell  $l$  can be expressed as

$$y_k^{(l)} = \mathbf{g}_k^{(l,l)H} \mathbf{x}^{(l)} + \sum_{l' \neq l} \mathbf{g}_k^{(l',l)H} \mathbf{x}^{(l')} + z_k^{(l)}$$

where  $z_k^{(l)}$  is the additive complex Gaussian noise at the  $k$ -th MS in cell  $l$  with zero mean and variance  $\sigma_z^2$  and  $\mathbf{g}_k^{(i,j)}$  ( $k = 1, \dots, K$ ) equals an  $NT \times 1$  channel vector from all DA ports in cell  $i$  to the  $k$ -th MS in cell  $j$ . Here,  $\mathbf{g}_k^{(i,j)}$  can be decomposed into

$$\mathbf{g}_k^{(i,j)} = [\mathbf{g}_{k,1}^{(i,j)T} \dots \mathbf{g}_{k,N}^{(i,j)T}]^T$$

For the single-user DAS, the MRT is applied for each DA port since it is the optimal precoding design in the single-cell single-user DAS under per-DA port power constraint<sup>[12]</sup>. Then, we obtain

the precoding vector of the  $n$ -th DA port in cell  $i$  as

$$\mathbf{f}_{1,n}^{(i)} = \sqrt{P} \frac{\mathbf{g}_{1,n}^{(i,i)}}{\|\mathbf{g}_{1,n}^{(i,i)}\|}. \quad (2)$$

Then, the ergodic sum rates for the single-user DAS in single-cell and two-cell cases are given, respectively,

$$R_{SC}^{SU} = \mathbb{E}_{h,s,p} \left[ \log_2 \left( 1 + \frac{P \left( \sum_{n=1}^N \|\mathbf{g}_{1,n}^{(1,1)}\| \right)^2}{\sigma_z^2} \right) \right] \quad (3)$$

$$R_{TC}^{SU} = \mathbb{E}_{h,s,p} \left[ \sum_{l=1}^2 \log_2 \left( 1 + \frac{P \left( \sum_{n=1}^N \|\mathbf{g}_{l,n}^{(1,1)}\| \right)^2}{\sigma_z^2 + |\mathbf{g}_1^{(i,l)} \mathbf{f}_1^{(i)}|^2} \right) \right]$$

where  $\mathbb{E}_{h,s,p}$  indicates the expectation with respect to small scale fading, shadowing and the position of MS and we define  $\bar{l} = 1$  if  $l = 2$  and  $\bar{l} = 2$  if  $l = 1$ .

For the case of multi-user DAS, there is no closed form solution for the optimal beamforming scheme because of the complexity in solving the optimization problem [10]. Therefore, we apply scaled ZFBF as precoding vectors for the multi-user DAS. Denoting  $\mathbf{F}^{(i)}$  as the precoding matrix for cell  $i$ ,  $\mathbf{F}^{(i)}$  can be obtained as [11]

$$\mathbf{F}^{(i)} = \frac{\sqrt{P} \mathbf{W}^{(i)}}{\max_{1 \leq n \leq N} \|\mathbf{W}_n^{(i)}\|_F}$$

where  $\mathbf{W}^{(i)} = [\mathbf{W}_1^{(i)T} \dots \mathbf{W}_N^{(i)T}]^T$  is the right Pseudo inverse  $\mathbf{G}^{(i,i)} (\mathbf{G}^{(i,i)H} \mathbf{G}^{(i,i)})^{-1}$  of  $N T \times K$  channel matrix  $\mathbf{G}^{(i,i)H}$  with  $\mathbf{G}^{(i,i)} = [\mathbf{g}_1^{(i,i)} \dots \mathbf{g}_K^{(i,i)}]$ ,  $\mathbf{W}_n^{(i)}$  equals the non-scaled beamforming matrix of the  $n$ -th DA port for all MSs in cell  $i$  and  $\|\cdot\|_F$  defines the Frobenius norm. Then, we can compute the precoding vector with the scaled ZFBF for the  $k$ -th MS in cell  $i$  as

$$\mathbf{f}_k^{(i)} = \mathbf{F}_{(k)}^{(i)} \quad (4)$$

where  $\mathbf{A}_{(k)}$  denotes the  $k$ -th column vector of a matrix  $\mathbf{A}$ .

For multi-user DAS, the cell average ergodic sum rates for single-cell and two-cell cases are expressed as

$$R_{SC}^{MU} = \mathbb{E}_{h,s,p} \left[ \sum_{k=1}^K \log_2 \left( 1 + \frac{|\mathbf{g}_k^{(1,1)} \mathbf{f}_k^{(1)}|^2}{\sigma_z^2} \right) \right]$$

$$R_{TC}^{MU} = \mathbb{E}_{h,s,p} \left[ \sum_{l=1}^2 \sum_{k=1}^K \log_2 \left( 1 + \frac{|\mathbf{g}_k^{(i,l)} \mathbf{f}_k^{(i)}|^2}{\sigma_z^2 + \sum_{k'=1}^K |\mathbf{g}_k^{(i,l)} \mathbf{f}_{k'}^{(i)}|^2} \right) \right]. \quad (5)$$

Note that precoding vectors in both scenarios satisfy per-DA port power constraint  $P$ .

### III. Antenna Placement

The optimization problem of antenna locations which maximize the cell average ergodic sum rate (3) and (5) are quite complicated to solve in general. Therefore, in this section, we formulate the performance metrics derived by lower bounds of the expected SNR and SLR for the single-cell and two-cell scenarios, respectively as suggested in [6]. Then, we introduce new algorithms to determine antenna locations by maximizing obtained performance metrics.

#### 3.1. SNR Criterion

In the single-cell DAS scenario, the cell average ergodic sum rate can be approximated by applying Jensen' inequality as

$$R_{SC} \approx \sum_{k=1}^K \log_2 \left( 1 + \mathbb{E}_{h,s,p} [SNR_k^{(1)}] \right).$$

From this relation, instead of maximizing the cell average ergodic capacity, we focus on maximizing the SNR criterion derived by the lower bound of the expected SNR.

1) Single-user Case: For the single-cell single-user case, each DA port uses a beamforming vector with

MRT in (2). Then, a lower bound of the expected SNR can be written as

$$\begin{aligned} \mathbb{E}_{h,s,p}[SNR_1^{(1)}] &= \frac{P}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \left( \sum_{n=1}^N \| \mathbf{g}_{1,n}^{(1,1)} \| \right)^2 \right] \\ &\geq \frac{P}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \sum_{n=1}^N \| \mathbf{g}_{1,n}^{(1,1)} \|^2 \right] \\ &= \frac{P}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \sum_{n=1}^N \frac{|s_{1,n}^{(1,1)}| \| \mathbf{h}_{1,n}^{(1,1)} \|^2}{(d_{1,n}^{(1,1)})^\alpha} \right] \\ &\geq \frac{PC_{\min}^{(1,1)}}{\sigma_z^2} \mathbb{E}_p \left[ \sum_{n=1}^N (d_{1,n}^{(1,1)})^{-\alpha} \right] \end{aligned}$$

where the first inequality comes from Cauchy-Schwarz inequality and the second inequality follows from the coefficient  $C_{\min}^{(1,1)} = \min [ |s_{1,1}^{(1,1)}| \| \mathbf{h}_{1,1}^{(1,1)} \|^2 \dots |s_{1,N}^{(1,1)}| \| \mathbf{h}_{1,N}^{(1,1)} \|^2 ]$ .

Assuming that MS 1 is located in the region of DA port i, we focus on  $(d_{1,n}^{(1,1)})^{-\alpha}$  for a simple derivation. Finally, applying Jensen' inequality from the fact that  $\left(\frac{1}{x}\right)^\alpha$  is convex for  $\alpha \geq 0$ , a lower bound of the expected SNR for the single-cell single-user scenario can be obtained as

$$\begin{aligned} \mathbb{E}_{h,s,p}[SNR_1^{(1)}] &\geq \frac{PC_{\min}^{(1,1)}}{\sigma_z^2} \mathbb{E}_p \left[ (d_{1,n}^{(1,1)})^{-\alpha} \right] \quad (6) \\ &\geq \frac{PC_{\min}^{(1,1)}}{\sigma_z^2} \mathbb{E}_p \left[ (d_{1,n}^{(1,1)})^2 \right]^{-\frac{\alpha}{2}}. \end{aligned}$$

2) Multi-user Case: For the single-cell multi-user case, the beamforming vector with scaled ZFBF in (4) is applied. Since every MS has the same formulation of the expected SNR, we take the expected SNR of MS 1 into consideration without loss of generality. Then, the expected SNR of MS 1 can be lower bounded as

$$\begin{aligned} \mathbb{E}_{h,s,p}[SNR_1^{(1)}] &= \frac{1}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \left\| \sum_{n=1}^N \mathbf{g}_{1,n}^{(1,1)} \mathbf{H} \mathbf{f}_{1,n}^{(1)} \right\|^2 \right] \\ &\geq \frac{1}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \sum_{n=1}^N \left| \mathbf{g}_{1,n}^{(1,1)} \mathbf{H} \mathbf{f}_{1,n}^{(1)} \right|^2 \right] \\ &= \frac{1}{\sigma_z^2} \mathbb{E}_{h,s,p} \left[ \sum_{n=1}^N \frac{|s_{1,n}^{(1,1)}| \| \mathbf{h}_{1,n}^{(1,1)} \mathbf{H} \mathbf{f}_{1,n}^{(1)} \|^2}{(d_{1,n}^{(1,1)})^\alpha} \right] \\ &\geq \frac{D_{\min}^{(1,1)}}{\sigma_z^2} \sum_{n=1}^N \mathbb{E}_p \left[ (d_{1,n}^{(1,1)})^{-\alpha} \right] \end{aligned}$$

where the first equality comes from the fact that  $\mathbb{E}_{h,s,p} [ \mathbf{g}_{1,n}^{(1,1)} \mathbf{g}_{1,m}^{(1,1)*} ] = 0$  for  $n \neq m$  and the inequality follows from  $D_{\min}^{(1,1)} = \min [ |s_{1,1}^{(1,1)}| \| \mathbf{h}_{1,1}^{(1,1)} \mathbf{H} \mathbf{f}_{1,1}^{(1)} \|^2, \dots, |s_{1,N}^{(1,1)}| \| \mathbf{h}_{1,N}^{(1,1)} \mathbf{H} \mathbf{f}_{1,N}^{(1)} \|^2 ]$ .

Similar to the single-cell single-user scenario, a lower bound of the expected SNR of the  $k$ -th MS can be derived as

$$\mathbb{E}_{h,s,p}[SNR_k^{(1)}] \geq \frac{D_{\min}^{(1,1)}}{\sigma_z^2} \mathbb{E}_{p_1} \left[ (d_{k,i}^{(1,1)})^2 \right]^{-\frac{\alpha}{2}}. \quad (7)$$

Maximizing (6) and (7) is equivalent to minimizing  $\mathbb{E}_{p_1} [ (d_{k,i}^{(1,1)})^2 ]$ . Finally, for the single-cell DAS, we determine the SNR criterion which minimizes the SNR metric  $\Gamma_{SNR}^{(i)}$  as

$$\Gamma_{SNR}^{(i)} = \mathbb{E}_{p_1} \left[ (d_{k,i}^{(1,1)})^2 \right]. \quad (8)$$

### 3.2. SLR Criterion

In the two-cell DAS scenario, the cell average ergodic sum rate can be computed by

$$R_{TC} = \mathbb{E}_{h,s,p_1,p_2} \left[ \sum_{l=1}^2 \sum_{k=1}^K \log_2 (1 + SINR_k^{(l)}) \right] \quad (9)$$

where 
$$SINR_k^{(l)} = \frac{|\mathbf{g}_k^{(l,l)} \mathbf{H} \mathbf{f}_k^{(l)}|^2}{\sigma_z^2 + \sum_{k'=1}^K |\mathbf{g}_k^{(l,l)} \mathbf{H} \mathbf{f}_k^{(l')}|^2}.$$

Similar to the single-cell scenario, we try to optimize a new metric instead of maximizing the cell average ergodic sum rate. Maximizing the expected signal-to-interference plus noise ratio (SINR) may be suitable to take other-cell interference into consideration. However, the expected SINR is still difficult to deal with because of its coupled nature in two cells causing high computational complexity. Therefore, we focus on the expected SLR as an alternative approach.

With high SINR approximation and interference limited assumptions, (9) can be expressed as a

function of SLR as

$$\begin{aligned}
 R_{TC} &\approx \mathbb{E}_{h,s,p_1,p_2} \left[ \sum_{l=1}^2 \sum_{k=1}^K \log_2(SINR_k^{(l)}) \right] \\
 &= \mathbb{E}_{h,s,p_1,p_2} \left[ \sum_{l=1}^2 \sum_{k=1}^K \log_2(SLNR_k^{(l)}) \right] \\
 &\approx \mathbb{E}_{h,s,p_1,p_2} \left[ \sum_{l=1}^2 \sum_{k=1}^K \log_2(SLR_k^{(l)}) \right]
 \end{aligned}$$

where the first equality is derived by using the property of logarithm and  $p_1$  and  $p_2$  indicate the position of MSs in cell 1 and 2, respectively. Also, signal-to-leakage plus noise ratio

$$\begin{aligned}
 SLNR_k^{(l)} &= \frac{|g_k^{(l)} H f_k^{(l)}|^2}{\sigma_z^2 + \sum_{k'=1}^K |g_k^{(l, \bar{l})} H f_k^{(l)}|^2} \quad \text{and} \\
 SLR_k^{(l)} &= \frac{|g_k^{(l)} H f_k^{(l)}|^2}{\sum_{k'=1}^K |g_k^{(l, \bar{l})} H f_k^{(l)}|^2} \quad \text{where } SLR_k^{(l)} \text{ is}
 \end{aligned}$$

defined as the ratio of the signal power of the  $k$ -th MS in cell  $l$  to the power that leaks from all MSs in cell  $l$  to the  $k$ -th MS in cell  $\bar{l}$ .

After applying Jansen's inequality, the cell-averaged sum rate can be bounded as

$$R_{TC}^{MU} \leq \sum_{l=1}^2 \sum_{k=1}^K \log_2 \left( \mathbb{E}_{h,s,p_1,p_2} [SLR_k^{(l)}] \right).$$

1) Single-user Case: For the two-cell single-user case, each DA port uses a beamforming vector with MRT in (2). Without loss of generality, we take the expected SLR of MS of cell 1 into account. Then, a lower bound of the expected SLR can be derived as

$$\begin{aligned}
 \mathbb{E}_{h,s,p} [SNR_1^{(1)}] &= \mathbb{E}_{h,s,p_1,p_2} \left[ \frac{P \left( \sum_{n=1}^N \|g_{1,n}^{(1,1)}\|^2 \right)}{|g_1^{(1,2)} H f_1^{(1)}|^2} \right] \\
 &\geq \mathbb{E}_{h,s,p_1,p_2} \left[ \frac{P \left( \sum_{n=1}^N \|g_{1,n}^{(1,1)}\|^2 \right)}{\|g_1^{(1,2)}\|^2 \|f_1^{(1)}\|^2} \right]
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{N} \mathbb{E}_{h,s,p_1} \left[ \left( \sum_{n=1}^N \|g_{1,n}^{(1,1)}\|^2 \right)^2 \right] \\
 &\quad \times \mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N \|g_{1,n}^{(1,2)}\|^2} \right] \tag{10}
 \end{aligned}$$

where the inequality comes from Cauchy Schwarz inequality and the second equality follows from the fact that  $\|f_1^{(1)}\|^2 = P$ .

With the result of (6), the first term of (10) can be lower bounded by  $C_{\min}^{(1,1)} / \mathbb{E}_{p_1} [(d_{k,i}^{(1,1)})^2]^{\frac{\alpha}{2}}$ . Also, the second term of (10) can be expressed as

$$\begin{aligned}
 &\mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N \|g_{1,n}^{(1,2)}\|^2} \right] \\
 &= \mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N (d_{1,n}^{(1,2)})^{-\alpha} |s_{1,n}^{(1,2)}| \|h_{1,n}^{(1,2)}\|^2} \right] \\
 &\geq \frac{1}{C_{\max}^{(1,2)}} \mathbb{E}_{p_2} \left[ \frac{1}{\sum_{n=1}^N (d_{1,n}^{(1,2)})^{-\alpha}} \right] \\
 &\approx \frac{1}{C_{\max}^{(1,2)} N} \mathbb{E}_{p_2} \left[ \frac{1}{(d_{1,i}^{(1,2)})^{-\alpha}} \right] \tag{11}
 \end{aligned}$$

where the inequality comes from  $C_{\max}^{(1,2)} = \max [ |s_{1,1}^{(1,2)}| \|h_{1,1}^{(1,2)}\|^2, \dots, |s_{1,N}^{(1,2)}| \|h_{1,N}^{(1,2)}\|^2 ]$  and (11) is approximated by assuming that distances from any DA port in cell 1 to MS 1 in cell 2 are equidistant. Finally, inserting the two terms into (10), a lower bound of the expected SLR of the MS in cell 1 for the two-cell single-user scenario can be obtained as

$$\mathbb{E}_{h,s,p_1,p_2} [SLR^{(1)}] \geq \frac{C_{\min}^{(1,1)}}{C_{\max}^{(1,2)} N^2} \frac{\mathbb{E}_{p_2} [(d_{1,i}^{(1,2)})^\alpha]}{\mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2]^{\frac{\alpha}{2}}}. \tag{12}$$

2) Multi-user Case: For the two-cell multi-user case, the beamforming vector with scaled ZFBF (4) is employed similar to the single-cell multi-user

case. We focus on the expected SLR of the MS in cell 1 without loss of generality. Then, a lower bound of the expected SLR of MS 1 in cell 1 can be written as

$$\begin{aligned} \mathbb{E}_{h,s,p} [SNR_1^{(1)}] &= \mathbb{E}_{h,s,p_1,p_2} \left[ \frac{|g_1^{(1,1)} H f_1^{(1)}|^2}{\sum_{k=1}^K |g_1^{(1,2)} H f_k^{(1)}|^2} \right] \\ &\geq \mathbb{E}_{h,s,p_1,p_2} \left[ \frac{|g_1^{(1,1)} H f_1^{(1)}|^2}{\sum_{k=1}^K \sum_{n=1}^N \|g_{1,n}^{(1,2)}\|^2 \sum_{n=1}^N \|f_k^{(1)}\|^2} \right] \\ &\geq \frac{1}{KNP} \mathbb{E}_{h,s,p_1} [ |g_1^{(1,1)} H f_1^{(1)}|^2 ] \\ &\quad \times \mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N \|g_{1,n}^{(1,2)}\|^2} \right] \end{aligned} \quad (13)$$

where inequalities comes from the Cauchy-Schwarz inequality and per-DA port power constraint as in the two-cell single- user scenario.

From (7), the first term of (13) can be lower bounded by  $D_{\min}^{(1,1)}/\mathbb{E}_{p_1} [(d_{k,i}^{(1,1)})^2]^{\frac{\alpha}{2}}$ . Also, the second term of (13) can be expressed as

$$\begin{aligned} &\mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N \|g_{1,n}^{(1,2)}\|^2} \right] \\ &= \mathbb{E}_{h,s,p_2} \left[ \frac{1}{\sum_{n=1}^N (d_{1,n}^{(1,2)})^{-\alpha} |s_{1,n}^{(1,2)}| \|h_{1,n}^{(1,2)}\|^2} \right] \\ &\geq \frac{1}{C_{\max}^{(1,2)}} \mathbb{E}_{p_2} \left[ \frac{1}{\sum_{n=1}^N (d_{1,n}^{(1,2)})^{-\alpha}} \right] \\ &\approx \frac{1}{C_{\max}^{(1,2)} N} \mathbb{E}_{p_2} \left[ \frac{1}{(d_{1,i}^{(1,2)})^{-\alpha}} \right] \end{aligned} \quad (14)$$

where the inequality comes from  $C_{\max}^{(1,2)} = \max [ |s_{1,1}^{(1,2)}| \|h_{1,1}^{(1,2)}\|^2, \dots, |s_{1,N}^{(1,2)}| \|h_{1,N}^{(1,2)}\|^2 ]$  and (14) follows from the equidistance assumption.

Finally, by combining the two terms of (13), a lower bound of the expected SLR of MS 1 in cell 1 for the wo-cell multi-user scenario can be given as

$$\mathbb{E}_{h,s,p_1,p_2} [SLR_1^{(1)}] \geq \frac{D_{\min}^{(1,1)}}{C_{\max}^{(1,2)} N^2} \frac{\mathbb{E}_{p_2} [(d_{1,i}^{(1,2)})^\alpha]}{\mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2]^{\frac{\alpha}{2}}}. \quad (15)$$

Maximizing lower bounds of the expected SLR (12) and (15) is equivalent to maximizing

$$J = \frac{\mathbb{E}_{p_2} [(d_{1,i}^{(1,2)})^\alpha]}{\mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2]^{\frac{\alpha}{2}}}. \text{ Since this term cannot be}$$

computed with general  $\alpha$ , we reformulate the cost function in order to make it easy to deal with. After putting the logarithm operation and Jensen' inequality, the cost function  $\gamma$  can be lower bounded as

$$\begin{aligned} \gamma &= \ln \mathbb{E}_{p_2} [(d_{1,i}^{(1,2)})^\alpha] - \ln \mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2]^{\frac{\alpha}{2}} \\ &\geq \mathbb{E}_{p_2} \left[ \ln \left\{ (d_{1,i}^{(1,2)})^2 \right\}^{\frac{\alpha}{2}} \right] - \ln \mathbb{E}_{p_1} \left[ (d_{1,i}^{(1,1)})^2 \right]^{\frac{\alpha}{2}} \\ &\geq \frac{\alpha}{2} \left\{ \mathbb{E}_{p_2} [\ln (d_{1,i}^{(1,2)})^2] - \ln \mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2] \right\}. \end{aligned}$$

Then, for the two-cell DAS, we can obtain the SLR criterion which maximizes the SLR metric  $\Gamma_{SLR}^{(i)}$

$$\Gamma_{SLR}^{(i)} = \mathbb{E}_{p_2} [\ln (d_{1,i}^{(1,2)})^2] - \ln \mathbb{E}_{p_1} [(d_{1,i}^{(1,1)})^2]. \quad (16)$$

### 3.3. Optimization of DA Port Placement

As shown in Sections III-1 and III-2 in DAS with multiple-antenna ports, the SNR and SLR criteria are expressed through deriving lower bounds of the expected SNR and SLR for the single-cell and two-cell scenarios, respectively. Therefore, an antenna placement optimization problem is determined by the SNR criterion (8) for the single-cell case and by the SLR criterion (16) for two-cell case with position of DA ports given in (1).

1) Single-Cell Case: For the single-cell scenario, the location of the i-th DA port can be obtained by the following optimization problem:

$$\begin{aligned} \{\widehat{R}_i, \widehat{\theta}_i\} &= \arg \min_{R_i, \theta_i} \Gamma_{SNR}^{(i)} \quad \text{for } i = 1, \dots, N \\ \text{s.t. } R_{\min,i} &\leq R_i \leq R_{\max,i} \quad \theta_{\min,i} \leq \theta_i \leq \theta_{\max,i} \end{aligned} \quad (17)$$

By solving (17) using the result in [7], a closed form solution for the location of the  $i$ -th DA port is given as

$$\begin{aligned} \widehat{R}_i &= \frac{2(R_{\max,i}^3 - R_{\min,i}^3)}{3(R_{\max,i}^2 - R_{\min,i}^2)} \\ &\times \frac{(\sin(\theta_{\max,i} - \widehat{\theta}_i) + \sin(\widehat{\theta}_i - \theta_{\min,i}))}{(\theta_{\max,i} - \theta_{\min,i})} \end{aligned} \quad (18)$$

$$\widehat{\theta}_i = \tan^{-1} \left( \frac{\cos \theta_{\max,i} - \cos \theta_{\min,i}}{\sin \theta_{\max,i} - \sin \theta_{\min,i}} \right). \quad (19)$$

2) Two-Cell Case: For the two-cell scenario, the location of the  $i$ -th DA port in cell 1 can be determined by solving the optimization problem:

$$\begin{aligned} \{\widehat{R}_i, \widehat{\theta}_i\} &= \arg \max_{R_i, \theta_i} \Gamma_{SLR}^{(i)} \quad \text{for } i = 1, \dots, N \\ \text{s.t. } R_{\min,i} &\leq R_i \leq R_{\max,i} \quad \theta_{\min,i} \leq \theta_i \leq \theta_{\max,i} \end{aligned} \quad (21)$$

Since (20) is not a convex problem, we employ a gradient ascent method to find a local optimal solution.

Applying the results in [7], the two gradients with respect to  $R_i$  and  $\theta_i$  are computed as

$$\begin{aligned} \nabla_{R_i} \Gamma_{SLR}^{(i)} &= \begin{cases} \frac{2}{R^2} (R_i - \sqrt{3} R \cos \theta_i) - \frac{\nabla_{R_i} \Gamma_{SNR}^{(i)}}{\Gamma_{SNR}^{(i)}} & \text{for } \rho < R \\ \frac{2}{\rho^2} (R_i - \sqrt{3} R \cos \theta_i) - \frac{\nabla_{R_i} \Gamma_{SNR}^{(i)}}{\Gamma_{SNR}^{(i)}} & \text{for } \rho \geq R \end{cases} \end{aligned} \quad (22)$$

$$\begin{aligned} \nabla_{\theta_i} \Gamma_{SLR}^{(i)} &= \begin{cases} 2\sqrt{3} \frac{R_i}{R} \sin \theta_i - \frac{\nabla_{\theta_i} \Gamma_{SNR}^{(i)}}{\Gamma_{SNR}^{(i)}} & \text{for } \rho < R \\ 2\sqrt{3} \frac{RR_i}{\rho^2} \sin \theta_i - \nabla_{\theta_i} \Gamma_{SNR}^{(i)} & \end{cases} \end{aligned} \quad (23)$$

where  $\rho = \sqrt{R_i^2 + 3R^2 - 2\sqrt{3}RR_i \cos \theta_i}$  and  $\nabla_{R_i} \Gamma_{SNR}^{(i)}$  and  $\nabla_{\theta_i} \Gamma_{SNR}^{(i)}$  are the gradients of  $\Gamma_{SNR}^{(i)}$  with respect to  $R_i$  and  $\theta_i$ , respectively.

Now, with (21) and (22), we propose an iterative algorithm which solves (20) as follows:

### Algorithm

- Step 1) Initialize the position of the  $i$ -th DA port in cell 1.
- Step 2) Update the position with  $R_i \leftarrow R_i + \delta_R \cdot \nabla_{R_i} \Gamma_{SLR}^{(i)}$  and  $\theta_i \leftarrow \theta_i + \delta_\theta \cdot \nabla_{\theta_i} \Gamma_{SLR}^{(i)}$ .
- Step 3) Compute  $\Gamma_{SLR}^{(i)}$  with the updated position.
- Step 4) Go back to Step 2) until convergence.

In our algorithm, we choose the step sizes  $\delta_R$  and  $\delta_\theta$  adopting Armijo's rule which provides provable convergence<sup>[13]</sup>. After the positions of DA ports are decided in cell 1, the positions of DA ports in cell 2 can be determined by using symmetry between cell 1 and 2.

## IV. Numerical Results

In this section, simulation results are presented to evaluate the performance of the proposed SNR and SLR criteria. We adopt the MRT and the scaled ZFBF for the single-user and multi-user scenario, respectively, and set  $R = 1$  km,  $\sigma_{sh} = 4$  dB, and  $\alpha = 3.75$  throughout the simulations. In Figure 1, we plot the locations of DA ports for  $N = 3, \dots, 8$  for the single-cell case which are computed from the SNR criterion (18) and (19). In this figure, the asterisks represent the optimal locations of DA ports from the algorithms given by [8], and the circles indicate the locations of DA ports from the proposed algorithm. Existence of a center antenna is decided by comparing the SNR criteria [6]. Note that the proposed closed form solution gives DA port



locations almost identical to the scheme in [8] with much lower computational complexity.

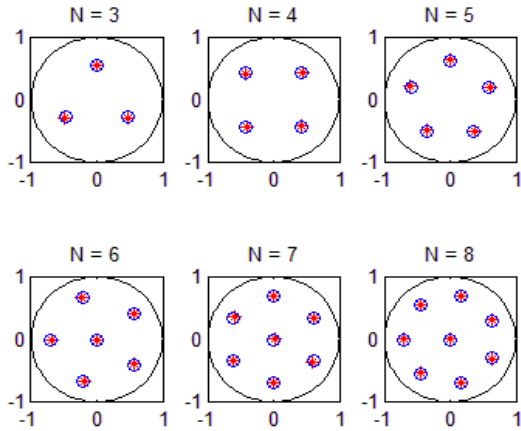


Fig. 1. Locations of DA ports in single-cell

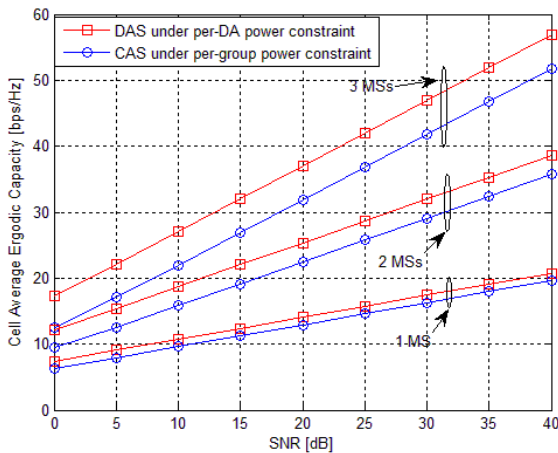


Fig. 2. The cell average ergodic sum rate for the single-cell DAS and CAS with  $N=7$

For the single-cell DAS and CAS, the cell average ergodic sum rate curves are illustrated as a function of SNR in Figure 2. We adopt antenna locations illustrated in Figure 1 for the DAS and set  $T=2$ ,  $N=7$

for  $K=1, 2$  and  $3$ . For fair comparison between DAS and CAS, CAS with per-group power constraint is considered where antennas are divided into  $N$  groups which contains  $T$  antennas in each group with power constraint 20 dB, DAS with the proposed antenna locations provides sum rate gains of about 9%, 13% and 16% over the CAS with per-group power constraint for  $K=1, 2$  and  $3$ .

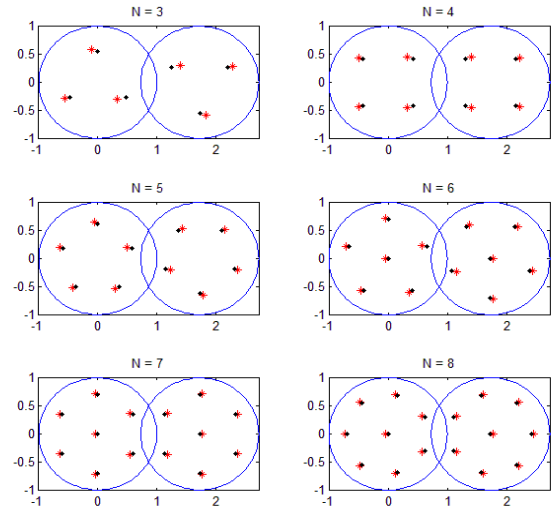


Fig. 3. Locations of DA ports for DAS in two-cell

In Figure 3, we plot the locations of DA ports for  $N=3, \dots, 8$  for the two-cell case determined by the proposed iterative algorithm in Section III-3. The dots indicate the locations of DA ports decided by the SNR criterion while the asterisks stand for the locations of DA ports obtained by the proposed SLR criterion. As shown in this figure, port locations with the SLR criterion are shifted against the other cell to reduce inter-cell interference.

For two-cell environment, we compare the performance of DAS with the port locations determined from the SNR criterion with DAS from the SLR criterion. We set  $T=2$  and

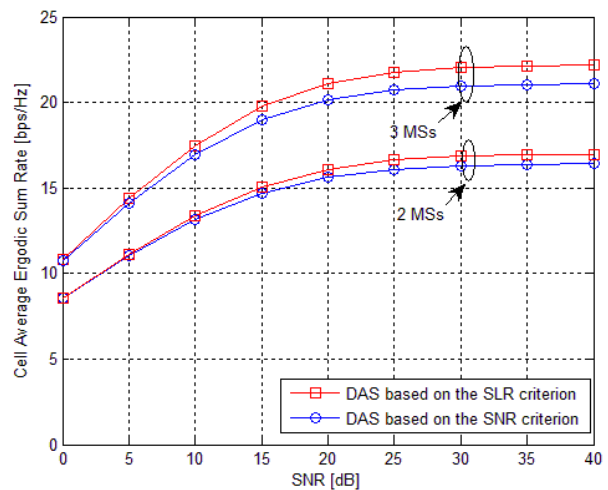


Fig. 4. The cell average ergodic sum rate for the two-cell DAS with  $N=3$

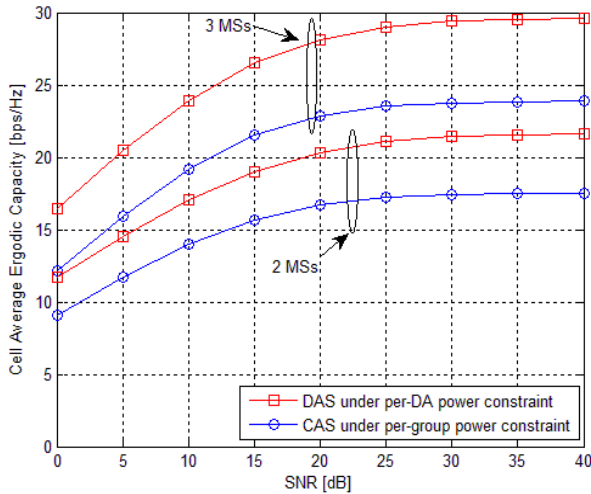


Fig. 5. The cell average ergodic sum rate for the two-cell DAS and CAS  $N=7$

$N=3$  for following simulation. In Figure 4, we illustrate the cell average ergodic sum rate curves for the two-cell case with respect to SNR. Simulation shows that DAS with the proposed SLR criterion has performance gains of about 5% compared to DAS with the SNR criterion for  $K=2, 3$  at a SNR of 30 dB. This performance gain results from the fact that the SLR criterion takes the leakage power term into consideration.

### V. Conclusion

In this paper, we have addressed the problem of antenna placements for the single-cell and two-cell DAS with DA ports equipped with multiple antennas under per-DA port power constraint. We have adopted MRT and scaled ZFBF for single-user and multi-user cases, respectively. By applying approximations and inequalities to properly manage the problem due to the extension in number of antennas of DA ports, we have formulated the optimization problems and proposed algorithms to obtain locations of DA ports. For the single-cell scenario, we have derived the SNR criterion and computed a closed form solution. The locations determined from the proposed SNR criterion are quite close to the optimal locations attained by conventional iterative algorithms. For the two-cell scenario, we have adopted the SLR criterion and

presented an iterative gradient ascent algorithm. Simulation results exhibit that both single-cell and two-cell DAS outperforms CAS in terms of the cell average ergodic sum rates.

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