논문 14-39A-09-01 The Journal of Korea Information and Communications Society '14-09 Vol.39A No.09
http://dx.doi.org/10.7840/kics.2014.39A.9.491

가우시안 동일 채널 간섭하에서 BPSK 신호의 최적 단일 사용자 검출의 정확한 BER 수식

정 규 혁•°

Analytical BER Expression of the Optimal Single User Detection of a BPSK Signal in the Presence of a Gaussian CCI

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Abstract

We derive an analytical expression for the bit-error rate (ber) of optimal single user detection (osud) of a binary phase-shift keying (bpsk) signal corrupted by a gaussian cochannel interferer (cci). the channel capacity is also calculated to investigate the ber performance.

Key Words : fading channel, cochannel interference, channel capacity, error probability, maximum likelihood detection

Ⅰ. Introduction

The problem of detecting a binary phase-shift keying (BPSK) signal corrupted by a single cochannel interferer (SCI) and additive white Gaussian noise (AWGN) has been investigated[1-10]. An optimal BPSK receiver is derived assuming Rayleigh fading and no receiver knowledge of signal parameters in [1]. A suboptimal BPSK receiver structure is proposed for a non-faded channel[2]. The optimum receiver is derived for a two-user synchronous BPSK channel[3]. The bit-error rate (BER) performance of the optimum receiver was compared with that of the conventional matched-filter receiver in [3] and the jointly optimal receiver (JOR) in [4]. The exact probability of error of an SCI-JOR was first obtained in [5] and [6]. An exact expression for the BER of an individually optimal receiver (IOR) used to detect a BPSK signal corrupted by a similar SCI and AWGN was derived[9]. When a BPSK signal corrupted by an SCI and AWGN is detected, the IOR is the optimal multiuser detector[7]. The JOR is also analyzed in [8]. On the other hand, the optimal single user detector (OSUD) in an SCI and AWGN is investigated and the BER of the OSUD is calculated in [10]. However, in [10], the cochannel interferer (CCI) is assumed to be non-fading. On the other hand, the non-fading multiple CCIs are considered in [12].

In this paper, we propose the OSUD for a BPSK signal detection in the presence of AWGN and a Gaussian fading CCI. In addition, we obtain the real roots of the equation specifically, which is formed by equating log-likelihood ratios (LLRs) with zero. The channel capacity is also calculated to investigate the BER performance.

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논문번호 : KICS2014-07-254, Received July 5, 2014; Revised July 30, 2014; Accepted August 11, 2014

Ⅱ. Signal Model and Gaussian CCI-OSUD Derivation

We consider a Gaussian CCI model. Assume that the baseband received signal is given by

$$r(t) = A_0 b_0 s_0(t) + A_1 g_1 b_1 s_1(t) + n(t) \qquad (1)$$

where b_0, A_0, and $s_0(t)$ are the information bit, amplitude, and signal waveform of the desired user, and b_1, A_1, and $s_1(t)$ are the information bit, amplitude, and signal waveform of the Gaussian cochannel interferer, respectively. $n(t)$ is an AWGN noise with zero-mean and double-sided power spectral density $\sigma_n^2 = N_0/2$, and the parameter g_1 is an independent Gaussian random variable (RV) representing the fading channel of the interfering user with mean μ_1 and unit-variance. The cross correlation is defined as $\rho_1 = \int_0^T s_0(t)s_1(t)dt$, where T is the symbol duration. Similar to [10], we assume zero timing error, zero intersymbol interference (ISI) and unit energy for signals. Then, the sampled output of the receiver filter matched to $s_0(t)$ is given by

$$r_0 = b_0 A_0 + b_1 A_1 \rho_1 g_1 + n_0 = b_0 A_0 + b_1 L g_1 + n_0 \qquad (2)$$

where $L = A_1\rho_1$, and n_0 is the component of $n(t)$ along $s_0(t)$, and n_0 is also a Gaussian RV with zero mean and variance $\sigma_{n_0}^2 = N_0/2$. We define the parameter I as I=$L\mu_1$ for a simple derivation. Then the RV Lg_1 has mean I and variance L^2, which could simplify the derivation and the final expression. Now, we consider the observation noise w, which is defined as w=b_1Lg_1+n_o. The observation noise w consists of three RVs, i.e., b_1, Lg_1, and n_o. The probability density function (PDF) $f_{b_1}(b_1)$ of the RV b_1 is $f_{b_1}(b_1) = \delta(b_1 - 1)/2 + \delta(b_1 + 1)/2$, where $\delta(\bullet)$ is the Dirac delta function. If the Gaussian distribution $\mathcal{N}_{2\sigma^2}(x)$ with zero mean and variance σ^2 is defined as $\mathcal{N}_{2\sigma^2}(x) = \exp\left(-x^2/(2\sigma^2)\right)/\sqrt{\pi(2\sigma^2)}$, the PDF $\mathcal{N}_{N_0}(n_0)$ of the RV n_0 can be expressed as $\mathcal{N}_{N_0}(n_0) = \mathcal{N}_{N_0}(n_0) = \exp(-n_0^2/N_0)/\sqrt{\pi N_0}$. The PDF of the RV Lg_1 also can be expressed as $\mathcal{N}_{2L^2}(x-I) = \mathcal{N}_{2L^2}(x-I) = \exp\left(-(x-I)^2/(2L^2)\right)/\sqrt{\pi(2L^2)}$. If binary bits are transmitted with equal probability and independent of each other and also independent of the RVs Lg_1 and n_0, then the PDF $\mathcal{N}_W(w)$ of the RV w can be calculated as follows,

$$\begin{aligned} f_W(w) &= \left[\int_{-\infty}^{\infty}\left\{\frac{1}{2}\delta(x-1)+\frac{1}{2}\delta(x+1)\right\}\mathcal{N}_{2L^2}\left(\frac{w}{x}-I\right)\left|\frac{1}{x}\right|dx\right] * \mathcal{N}_{N_0}(w) \\ &= \left[\int_{-\infty}^{\infty}\frac{1}{2}\delta(x-1)\mathcal{N}_{2L^2}\left(\frac{w}{1}-I\right)\left|\frac{1}{1}\right|dx + \int_{-\infty}^{\infty}\frac{1}{2}\delta(x+1)\mathcal{N}_{2L^2}\left(\frac{w}{-1}-I\right)\left|\frac{1}{-1}\right|dx\right] * \mathcal{N}_{N_0}(w) \\ &= \left[\mathcal{N}_{2L^2}(w-I)\int_{-\infty}^{\infty}\frac{1}{2}\delta(x-1)dx + \mathcal{N}_{2L^2}(-w-I)\int_{-\infty}^{\infty}\frac{1}{2}\delta(x+1)dx\right] * \mathcal{N}_{N_0}(w) \\ &= \frac{1}{2}\left[\mathcal{N}_{2L^2}(w-I)+\mathcal{N}_{2L^2}(w+I)\right] * \mathcal{N}_{N_0}(w) \\ &= \frac{1}{2}\left[\mathcal{N}_{N_0+2L^2}(w-I)+\mathcal{N}_{N_0+2L^2}(w+I)\right] \end{aligned} \qquad (3)$$

where * denotes the convolution operation. The expression of the angular bracket in the first line of (3) uses the fact that the PDF $f_Z(z)$ of the RV Z of the product of two independent RVs X and Y having PDFs $f_X(x)$ and $f_Y(y)$ respectively is given by $f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z/x)/|x|dx$. The right hand side of the first line of (3) uses the fact that the PDF of the sum of two or more independent RVs is the convolution of their individual PDFs, because the RV n_0 is independent of b_1 and Lg_1. The rest lines of (3) are algebraic derivations.

The optimum decision for the desired user's information bit can be obtained from the following hypothesis testing problem,

$$H_1 : r_0 = A_0 + w, \qquad H_0 : r_0 = -A_0 + w. \tag{4}$$

Using LLRs, the binary decision is given by

$$\hat{b}_0 = \operatorname{sgn}(\Lambda(r_0)) = \operatorname{sgn}\left(\log \frac{f_{R_0}(r_0 \mid H_1)}{f_{R_0}(r_0 \mid H_0)} \right) \tag{5}$$

where the LLR $\Lambda(r_0)$ is defined as $\Lambda(r_0) = \log\left(f_{R_0}(r_0 \mid H_1) \,/\, f_{R_0}(r_0 \mid H_0) \right)$ and $\operatorname{sgn}(\bullet)$ denotes signum function. Substituting (3) in (5) and after some algebraic manipulation, the OSUD in the presence of AWGN and a Gaussian SCI is given by

It is easy to show that the Gaussian SCI-OSUD of (6) simplifies to the non-fading SCI-OSUD in [10] for N_0 instead of $(N_0 + 2L^2)$, when the variance L^2 of the RV Lg_1 is zero.

Ⅲ. BER Derivation and Channel Capacity Calculation

In order to evaluate the BER, we need to find the intervals for $\Lambda(r_0) > 0$ or $\Lambda(r_0) < 0$. Based on the following five facts, we find the intervals. First, observing the last line of (6), we can know $\lim_{r_0 \to \infty} \Lambda(r_0) = \infty$. Second, similarly $\lim_{r_0 \to -\infty} \Lambda(r_0) = -\infty$. Third, $\Lambda(r_0) = -\Lambda(-r_0)$. Fourth, the equation $\Lambda(r_0) = 0$ has an exact root $r_0 = 0$. Fifth, in order to find the rest roots, we observe that the numerator and the denominator of the argument of the logarithm in $\Lambda(r_0)$ are the sums of the bell shaped curves, respectively. It is very difficult to obtain the exact real roots except a real root $r_0 = 0$. Therefore, we use the Jacobian logarithm to obtain the approximate real roots as follows,

$$\begin{aligned} \log(e^x + e^y) &= \max(x, y) + \log(1 + e^{-|y-x|}) \\ &\approx \max(x, y). \end{aligned} \tag{7}$$

Then $\Lambda(r_0) = 0$ can be written as

$$\begin{aligned}
\hat{b}_0 &= \operatorname{sgn}(\Lambda(r_0)) = \operatorname{sgn}\left(\log \frac{f_{R_0}(r_0 \mid H_1)}{f_{R_0}(r_0 \mid H_0)} \right) = \operatorname{sgn}\left(\log \frac{f_W(r_0 - A_0)}{f_W(r_0 + A_0)} \right) \\
&= \operatorname{sgn}\left(\log \frac{\frac{1}{2}\left[\mathcal{N}_{N_0+2L^2}(r_0 - A_0 - I) + \mathcal{N}_{N_0+2L^2}(r_0 - A_0 + I) \right]}{\frac{1}{2}\left[\mathcal{N}_{N_0+2L^2}(r_0 + A_0 - I) + \mathcal{N}_{N_0+2L^2}(r_0 + A_0 + I) \right]} \right) \\
&= \operatorname{sgn}\left(\log \frac{e^{\frac{-(r_0-A_0-I)^2}{N_0+2L^2}} + e^{\frac{-(r_0-A_0+I)^2}{N_0+2L^2}}}{e^{\frac{-(r_0+A_0-I)^2}{N_0+2L^2}} + e^{\frac{-(r_0+A_0+I)^2}{N_0+2L^2}}} \right) = \operatorname{sgn}\left(\log \frac{e^{\frac{-(-2r_0A_0-2r_0I+2A_0I)}{N_0+2L^2}} + e^{\frac{-(-2r_0A_0+2r_0I-2A_0I)}{N_0+2L^2}}}{e^{\frac{-(+2r_0A_0-2r_0I-2A_0I)}{N_0+2L^2}} + e^{\frac{-(+2r_0A_0+2r_0I+2A_0I)}{N_0+2L^2}}} \right) \\
&= \operatorname{sgn}\left(\frac{4A_0}{N_0 + 2L^2} r_0 + \log \frac{e^{\frac{-(-2r_0I+2A_0I)}{N_0+2L^2}} + e^{\frac{-(+2r_0I-2A_0I)}{N_0+2L^2}}}{e^{\frac{-(-2r_0I-2A_0I)}{N_0+2L^2}} + e^{\frac{-(+2r_0I+2A_0I)}{N_0+2L^2}}} \right) \\
&= \operatorname{sgn}\left(\frac{4A_0}{N_0 + 2L^2} r_0 - \log \frac{e^{\frac{2I}{N_0+2L^2}(r_0+A_0)} + e^{-\frac{2I}{N_0+2L^2}(r_0+A_0)}}{e^{\frac{2I}{N_0+2L^2}(r_0-A_0)} + e^{-\frac{2I}{N_0+2L^2}(r_0-A_0)}} \right) \\
&= \operatorname{sgn}\left[\frac{4A_0}{N_0 + 2L^2} r_0 - \log \frac{\cosh\left\{ \frac{2I}{N_0 + 2L^2}(r_0 + A_0) \right\}}{\cosh\left\{ \frac{2I}{N_0 + 2L^2}(r_0 - A_0) \right\}} \right].
\end{aligned} \tag{6}$$

Solving (10), we find the three real roots as $-I, 0,$ and I. In addition, the slope of the function $\Lambda(r_0)$ can be obtained by calculating $d\Lambda(r_0) / dr_0$ as follows,

Then $d\Lambda(0) / dr_0$ is given by

Therefore, based on the above five facts, if $d\Lambda(0) / dr_0 > 0$, it is impossible for $\Lambda(r_0) = 0$ to have the three roots and in turn $\Lambda(r_0) = 0$ has the only one root $r_0 = 0$, except that $\Lambda(r_0)$ has *x* axis as a tangent, in which case there is no effect to the intervals. Similarly, if $d\Lambda(0) / dr_0 < 0$, $\Lambda(r_0) = 0$ has the three roots $-I, 0,$ and I. Then, the decision intervals for $\Lambda(r_0) > 0$ and $d\Lambda(0) / dr_0 < 0$ are given by

$$\begin{cases} I < r_0 \\ -I < r_0 < 0 \end{cases}. \tag{11}$$

For $d\Lambda(0) / dr_0 > 0$, the BER P_b is calculated as follows,

where $Q(x) = \int_x^{\infty} (1 / \sqrt{2\pi}) \exp(-t^2 / 2) dt$. For $d\Lambda(0) / dr_0 < 0$, the BER is calculated as follows,

In addition, we calculate the channel capacity, which is the tightest upper bound on the rate of information that can be reliably transmitted over a given channel, so that it is helpful to compare a BER with its channel capacity. We consider the channel of (4) with possible inputs A_0 or $-A_0$. The capacity of this channel in bit/channel use is given by [11] where by using (3) and (4), $f_{R_0}(r_0 \mid H_1) = f_W(r_0 - A_0)$ and $f_{R_0}(r_0 \mid H_0) = f_W(r_0 + A_0)$. Then the capacity of the channel is computed by (14).

$$\begin{aligned} \frac{4A_0}{N_0 + 2L^2} r_0 &= \log \frac{e^{\frac{2I}{N_0+2L^2}(r_0+A_0)} + e^{-\frac{2I}{N_0+2L^2}(r_0+A_0)}}{e^{\frac{2I}{N_0+2L^2}(r_0-A_0)} + e^{-\frac{2I}{N_0+2L^2}(r_0-A_0)}} \\ &\approx \max\left(\frac{2I(r_0 + A_0)}{N_0 + 2L^2}, -\frac{2I(r_0 + A_0)}{N_0 + 2L^2}\right) - \max\left(\frac{2I(r_0 - A_0)}{N_0 + 2L^2}, -\frac{2I(r_0 - A_0)}{N_0 + 2L^2}\right) \end{aligned} \tag{8}$$

$$\frac{d\Lambda(r_0)}{dr_0} = \frac{4A_0}{N_0 + 2L^2} - \left(\frac{2I}{N_0 + 2L^2}\right)\left[\tanh\left\{\frac{2I}{N_0 + 2L^2}(r_0 + A_0)\right\} - \tanh\left\{\frac{2I}{N_0 + 2L^2}(r_0 - A_0)\right\}\right]. \tag{9}$$

$$\frac{d\Lambda(0)}{dr_0} = \frac{4}{N_0 + 2L^2}\left\{A_0 - I \tanh\left(\frac{2IA_0}{N_0 + 2L^2}\right)\right\}. \tag{10}$$

$$P_b = \frac{1}{2} Q\left(\frac{A_0 - I}{\sqrt{N_0 / 2 + L^2}}\right) + \frac{1}{2} Q\left(\frac{A_0 + I}{\sqrt{N_0 / 2 + L^2}}\right), \tag{12}$$

$$\begin{aligned} P_b \simeq\ & Q\left(\frac{A_0}{\sqrt{N_0 / 2 + L^2}}\right) + \frac{1}{2} Q\left(\frac{A_0 + 2I}{\sqrt{N_0 / 2 + L^2}}\right) + \frac{1}{2} Q\left(\frac{A_0 - 2I}{\sqrt{N_0 / 2 + L^2}}\right) \\ & - \frac{1}{2} Q\left(\frac{A_0 - I}{\sqrt{N_0 / 2 + L^2}}\right) - \frac{1}{2} Q\left(\frac{A_0 + I}{\sqrt{N_0 / 2 + L^2}}\right). \end{aligned} \tag{13}$$

$$C = 1 - \frac{1}{2} \int_{-\infty}^{\infty} f_{R_0}(r_0 \mid H_1) \log_2\left(1 + e^{-\Lambda(r_0)}\right) dr_0 - \frac{1}{2} \int_{-\infty}^{\infty} f_{R_0}(r_0 \mid H_0) \log_2\left(1 + e^{\Lambda(r_0)}\right) dr_0 \tag{14}$$

Ⅳ. Results

Fig. 1 shows analytical BERs, simulations, and channel capacity of the proposed Gaussian CCI-OSUD. We define the signal-to-noise ratio (SNR) as $E_b / N_0 \triangleq A_0^2 / N_0$. The upper bound, which is the hard decision for the direct outputs of the matched filter without the proposed Gaussian CCI-OSUD, is also shown. When there is no interferer, the BER is a lower bound. The analytical BER coincides with the simulation. This validates our approximation. Usually, the BER is non-increasing and the capacity is non-decreasing as the SNR increases. However, with a CCI, generally these are not true. This phenomenon is explained clearly in [12]. After fluctuations due to the Gaussian CCI, the Gaussian CCI-OSUD BER performance approaches that of the matched filter. The channel capacity is also in good agreement with the Gaussian CCI-OSUD BER, i.e., the higher the capacity is, the lower the BER is and vice versa.

In Fig. 2, we plot analytical BER curves as the variance L^2 of the RV Lg_1approaches zero with the fixed I, which means that the fading channel becomes non-fading, because generally a variance of zero for a RV indicates that all the values for the RV are identical to the mean of the RV. As it is predicted in (6), the Gaussian SCI-OSUD simplifies to the non-fading SCI-OSUD in [10] for N_0 instead of $(N_0 + 2L^2)$,

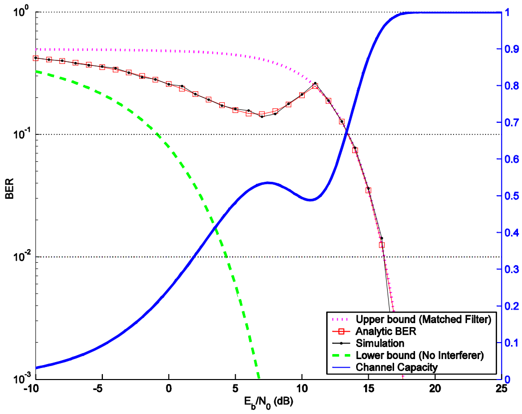

Fig. 1. The left axis is the BER and the right axis is the capacity (in bit/channel use) for I=5.0 and L^2=3.0 with N_0=2.0.

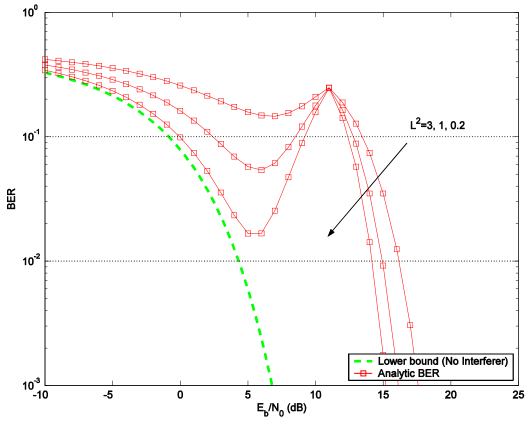

Fig. 2. Analytical BER curves for L^2=3,1,0.2 with the fixed I=5 and N_0=2.0.

Fig. 3 shows analytical BER curves as I becomes larger with the fixed L^2, which means that the dominant term of the Gaussian CCI becomes stronger relatively. The BER curves shift right, while in the low SNR region, the BER of the Gaussian CCI with the stronger dominant term becomes a little better than that with the weaker. This shows that transiently the Gaussian CCI can interfere constructively as well as destructively.

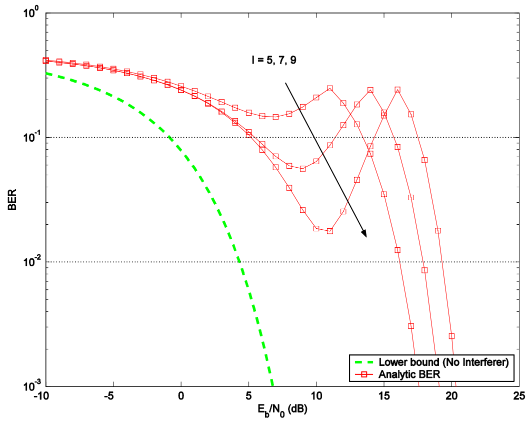

Fig. 3. Analytical BER curves for I=9,7,5 with the fixed L^2=3 and N_0=2.0

Ⅴ.Conclusion

We derived an analytical expression for the BER of the Gaussian CCI-OSUD. The effect of a

Gaussian CCI on the BER was analyzed. To investigate the BER performance, the channel capacity was also calculated. The capacity, the analytical result, and the simulation are in good agreement.

References

[1] H. Camkerten, "Optimum one-shot BPSK receiver design for multiple access mobile radio channels with rayleigh fading," in *Proc. IEEE VTC*, vol. 1, pp. 447-450, Denver, CO, May 1992.

[2] G. J. M. Janssen, "Receiver structure for simultaneous reception of two BPSK modulated cochannel signals," *Electron. Lett.*, vol. 29, no. 12, pp. 1095-1097, 1993.

[3] R. Kwan and C. Leung, "Optimal detection of a BPSK signal contaminated by interference and noise," *IEEE Commun. Lett.*, vol. 6, no. 2, pp. 225-227, 2002.

[4] T. V. Poon and N. C. Beaulieu, "Jointly and individually optimum receivers for BPSK signals in cochannel interference plus noise," in *Proc. IEEE Pacific Rim Conf. Commun., Comput., Signal Process.*, Aug. 2003.

[5] T. V. Poon and N. C. Beaulieu, "Performance analysis of a jointly optimal BPSK receiver in cochannel interference," in *Proc. IEEE Global Telecommn. Conf.*, vol. 3, pp. 1721-1725, Dec. 2003.

[6] T. V. Poon and N. C. Beaulieu, "Error performance analysis of a jointly optimal single cochannel-interferer BPSK receiver," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1051-1054, 2004.

[7] S. Verdú, *Multiuser Detection*, 1st Ed., NY: Cambridge Univ. Press, 1998.

[8] A. M. Rabiei and N. C. Beaulieu, "A simple, intuitive expression for the BER of a jointly optimal single cochannel interferer BPSK receiver," *IEEE Commun. Lett.*, vol. 9, no. 3, pp. 201-203, 2005.

[9] A. M. Rabiei and N. C. Beaulieu, "An analytical expression for the BER of an individually optimal single cochannel interferer BPSK receiver," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 60-63, 2007.

[10] S. M. Saberali, H. Amindavar, and A. R. Moghaddamjoo, "BER calculation and investigation of optimal single user detector for a BPSK signal contaminated by cochannel interferer," *IEEE Commun. Lett.*, vol. 12, no. 10, pp. 705-707, 2008.

[11] J. G. Proakis, *Digital Communications*, 4th Ed., NY: McGraw-Hill, 2001.

[12] K. Chung, "An analytical expression for the BER of optimal single user detection of a BPSK signal contaminated by multiple CCIs," *EURASIP J. Wirel. Commun. Netw.*, vol. 2012, no. 190, 2012.

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<관심분야> Channel coding, Iterative detection, LDPC codes, Turbo codes, Near-channel capacity approaching codes