

調査論文

# 이동 물체 인식을 위한 Optic Flow

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## Optic Flow for Motion Vision; Survey

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**요 약** Optic flow는 이동물체표면의 3차원 속도를 영상판(image plane)에 투영시킨 2차원 속도이다. 본 논문은 이동물체의 연속영상으로부터 optic flow를 구하는 기술들을 조사분석하고 정해진 optic flow로부터 물체의 인식 및 3차원 속도를 결정하는 기술들을 논하였다. 연속영상으로부터 구해진 optic flow는 영상압축기술인 영상간 부호화(inter-frame image coding)에 해당되며 컴퓨터비전 시스템(computer vision system)에서 이동물체 인식에 사용된다.

**ABSTRACT** Optic flow is 2D velocity projected on the image plane of 3D velocity of a moving surface element. In this paper, we survey techniques computing optic flows from an image time sequence of moving objects and techniques determining 3D velocities and surface structures of the moving objects from the optic flows determined.

### 1. Introduction

When objects in the environment are illuminated, light is reflected from surfaces of the objects. The reflected light forms a densely structured optic array at a point of observation. The optic array may be thought of as a bundle of narrow cones of light with their apices at the point of observation and their bases at distinct surface elements. Each surface element may produce a distinct light cone from a different intensity and spectral composition of the light at each surface element [LEE (1980)].

For moving objects or the observation point in motion, a light cone from a surface element moves with some velocity causing change of

the optic array continuously over time, giving rise to an optic flow field. Formally, the optic flow field is a field of 2D velocities projected on the image plane of 3D velocities of surface elements as shown in Fig 1. For convenience and realism, the optic flow field is described in terms of and extracted from the changing pattern of light incident on an image plane that intercepts the time-varying optic array.

Motion has been conceptualized classically as an inference from analyzing each static 2D brightness intensity pattern of image time sequence [HELMHOLTZ (1925)]. However, motion and structure can be perceived directly from the optic flow field as one psychological theory indicates [GIBSON (1950abc)]. In that theory, motion and change are basic for a vision system while a static intensity pattern is a rare thing.

In the following, we discuss techniques com-

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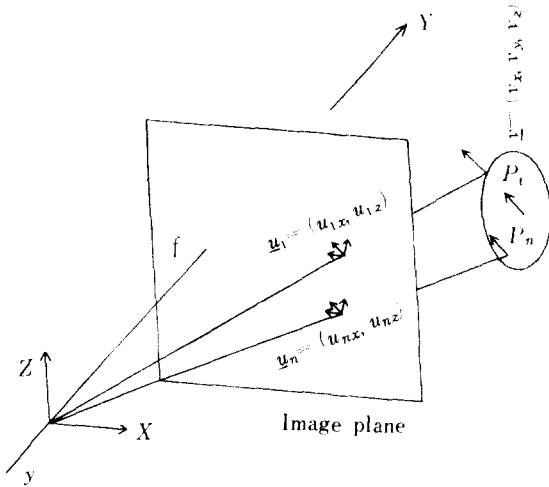


Fig 1 Geometry of optic flow vector  $\underline{u}$ .

puting optic flow directly in section 2 and techniques recovering 3D motions and depth from optic flow in section 3. In section 4 we consider techniques determining corresponding points from which optic flows can be easily computed and in section 5 we show techniques determining 3D motion and depth from corresponding points. We conclude in section 6.

## 2. Computing Optic Flow

To compute the optic flow field directly from a time sequence of image frames, numerous techniques have been proposed. A technique can be assigned to one of two categories depending upon the requirement of a matching scheme. The so called 'non-matching' approaches, which do not require the correspondence process, determine two components of a flow vector based on the temporal and spatial brightness variation of the image and motion model. When a surface models an image brightness, the brightness variation at an image point can be represented by the surface gradient at the point. Thus, the 'non-matching' approaches are sometimes called gradient based techniques [ULLMAN (1981)].

In their experimental studies of moving tele-

vision images, LIMB and MURPHY (1975) use frame brightness difference for the temporal brightness variation and consecutive row pixel brightness difference for the spatial brightness variation in estimating the horizontal component of a flow vector. These brightness differences deserve consideration in a mathematical model of image and motion for the optic flow computation [CAFFORIO and ROCCA (1976, 79), FENNEMA and THOMPSON (1979), THOMPSON and BARNARD (1981), HORN and SCHUNK (1981)].

A time-varying image can be represented mathematically by  $I(x, z, t)$  where  $x$  and  $z$  are the image plane coordinates and  $t$  is the time coordinate. Suppose that the image is displaced a distance  $\delta x$  in the  $x$ -direction and a distance  $\delta z$  in the  $z$ -direction in a time  $\delta t$ . Then the intensity at image point  $(x + \delta x, z + \delta z)$  at time  $t + \delta t$  can be represented using the Taylor series expansion as

$$\begin{aligned}
 & I(x + \delta x, z + \delta z, t + \delta t) \\
 &= I(x, z, t) + \delta x \frac{\partial I}{\partial x} + \delta z \frac{\partial I}{\partial z} + \delta t \frac{\partial I}{\partial t} \\
 &+ \frac{\delta x^2}{2} \frac{\partial^2 I}{\partial x^2} + \frac{\delta x \delta z}{2} \frac{\partial^2 I}{\partial x \partial z} \\
 &+ \frac{\delta z^2}{2} \frac{\partial^2 I}{\partial z^2} + \frac{\delta z \delta t}{2} \frac{\partial^2 I}{\partial z \partial t} \\
 &+ \frac{\delta t^2}{2} \frac{\partial^2 I}{\partial t^2} + \frac{\delta t \delta x}{2} \frac{\partial^2 I}{\partial t \partial x}
 \end{aligned} \tag{1}$$

where the partial derivatives are computed at the image point  $(x, z)$  at the time  $t$  either from a differentiable image model or using a finite difference method.

For the optic flow computation, some techniques consider only the linear and constant terms in (1) while others consider the second or higher

terms as well. In the following two subsections, we will first discuss techniques using the linear and constant terms in (1) and then techniques requiring higher terms.

## 2-1. Linear Model of Image and Motion

Neglecting the second and higher order terms as the error measurement in (1), we have

$$\begin{aligned}
 & I(x + \delta x, z + \delta z, t + \delta t) \\
 &= I(x, z, t) + \delta x \frac{\partial I}{\partial x} + \delta z \frac{\partial I}{\partial z} + \delta t \frac{\partial I}{\partial t}
 \end{aligned} \tag{2}$$

Assuming that the brightness remains the same after the motion, we can set  $I(x + \delta x, z + \delta z, t + \delta t) = I(x, z, t)$ . Thus, we have

$$\frac{\delta x}{\delta t} \frac{\partial I}{\partial x} + \frac{\delta z}{\delta t} \frac{\partial I}{\partial z} + \frac{\partial I}{\partial t} = 0 \tag{3}$$

Setting  $\delta t \rightarrow 0$ , we have a linear relationship between the two optic flow components  $u$  and  $v$  as

$$u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial z} + \frac{\partial I}{\partial t} = 0 \tag{4}$$

where  $u = \lim_{\delta t \rightarrow 0} \frac{\delta x}{\delta t}$  and  $v = \lim_{\delta t \rightarrow 0} \frac{\delta z}{\delta t}$

This relationship is called the motion constraint or the optic flow equation [FENNEMA and THOMPSON (1979), THOMPSON and BARNARD (1981), HORN and SCHUNK (1981)].

Using the optic flow equation and assuming optic flow constancy over a moving image area, a flow vector can be computed [FENNEMA and THOMPSON (1979), THOMPSON and BARNARD (1981), HORN and SCHUNK (1981)].

CAFFORIO and ROCCA (1976) take the statistical analysis on the optic flow equation and estimate motion of multiple objects using a clustering scheme. CAFFORIO and ROCCA (1979) investigate the noise effect on the optic flow equation and consider a hardware implementation of motion estimation. FENNEMA and THOMPSON (1979) represent the optic flow equation in a polar coordinate system and determine the representative flow vector in a clustering scheme. THOMPSON and BARNARD (1981) derive a number of optic flow equations from the moving image area and determine the flow vector by solving the equations using the pseudo inverse method. They compare the pseudo inverse method to the clustering method. Further discussion about these two methods is in [DUBOIS et al. (1981), NAGEL (1981a)]. DAVIS et al. (1983) use local optic flow component constancy which follows from the local 2D object motion assumption. From this constraint as well as the optic flow equation, they propagate, using geometry, the optic flow along a line segment with two given flow vectors at both end points.

Since it cannot be said generally which pixels move with the same velocity, HORN and SCHUNK (1981) use the global constraint that the estimated optic flow field should vary smoothly with the image plane coordinates. They combine the smoothness constraint and the motion constraint by a weighted sum of their error measurements and apply a minimization technique to the summed error to derive an iterative formula. The global smoothness constraint, however, causes unrealistic optic flows on stationary image areas or on the background across the occluding boundaries of moving areas.

To cope with this difficulty, YACHIDA (1983) assumes optic flows at prominent feature points like corners such as at the points A, B, ..., G in Fig 2 and propagates the flow vectors smoo-



can compute the rotational velocity  $\omega_q$  of a point Q with respect to a point P by

$$\omega_q = \frac{\Delta r \times \Delta v}{\|\Delta r\|^2} \quad (7)$$

and they define the dilation  $D_q$  by

$$D_q = \frac{\Delta r \cdot \Delta v}{\|\Delta r\|^2} \quad (8)$$

where

$$\Delta v = v' - v$$

$$\Delta r = \underline{Q} - \underline{P}$$

Thus for each pixel in a neighborhood, they can compute rotational velocity and dilation with respect to its center pixel position. For the center pixel, they average the computed values. To smooth a flow vector at a point P, they choose its neighbor P' which has the minimum variation of computed rotational velocities and dilations in its neighborhood and update the flow vector  $v_P$  at P by

$$v = v' - \Delta r \times \omega' \quad (9)$$

where  $\omega'$  is the computed rotational velocity at the point P'. In Fig 3, the flow vector at P is recomputed with respect to P' because the 3x3 neighborhood of P' has minimum variation of rotational velocities and dilations.

Similar multiple constraints are derived by TRETIK and PASTOR (1984). They derive two independent equations directly from the motion constraint by differentiating the optic flow equation with respect to the image plane coordinates. The same two equations, however, can be derived from the linear model of image and motion.

Recently SCHUNK (1984) proved that the

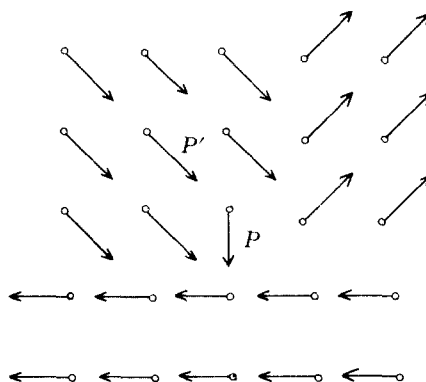


Fig. 3 Updating Optic Flow Vector at P with respect to Optic Flow Vector at P'.

motion constraint is valid at the image points of intensity discontinuities at an occluding boundary and pointed out that rotational motion or perspective projection may cause a change in the brightness pattern of an image sequence due to the foreshortening effects.

## 2-2. Nonlinear Model of Image and Motion

The linear model of intensity variation is too simple to represent edges and corners [SNYDER et al. (1980), NAGEL (1983)]. Thus, SNYDER et al. (1980) represent (1) up to the second order at the time  $t + \delta t$  as

$$\begin{aligned} & I(x + \delta x, z + \delta z, t + \delta t) \\ &= I(x, z, t + \delta t) + \delta x \frac{\partial I}{\partial x} + \delta z \frac{\partial I}{\partial z} \\ &+ \frac{\delta x^2}{2} \frac{\partial^2 I}{\partial x^2} + \frac{\delta x \delta z}{2} \frac{\partial^2 I}{\partial x \partial z} + \frac{\delta z^2}{2} \frac{\partial^2 I}{\partial z^2} \end{aligned} \quad (10)$$

where the partial derivatives are computed at the image plane coordinate  $(x, z)$  at the time Setting  $(x + \delta x, z + \delta z, t + \delta t) = I(x, z, t)$ .

(brightness constancy after displacement), they derive

$$I(x, z, t) - I(x, z, t + \delta t) = \delta x \frac{\partial I}{\partial x} + \delta z \frac{\partial I}{\partial z} + \frac{\delta x^2}{2} \frac{\partial^2 I}{\partial x^2} + \frac{\delta x \delta z}{2} \frac{\partial^2 I}{\partial x \partial z} + \frac{\delta z^2}{2} \frac{\partial^2 I}{\partial z^2} \quad (11)$$

For each pixel, they derive one quadratic equation (11) in the two unknown displacement vector components  $\delta x$  and  $\delta z$ . Thus, with the assumption of displacement vector constancy over a moving image area (i.e.  $I(x, z, t) = I(x, z, t + \delta t)$ ) they can determine the displacement vector using clustering.

NAGEL (1983) fits the image model of (10) over a local neighborhood such that the partial derivatives are substituted for the fitting coefficients minimizing the sum of weighted squared differences between the measurement and the image model. Using the determined fitting coefficients and the proper transformation of the local coordinate system, he derives two coupled nonlinear equations for the two displacement vector components from (11) by minimizing the sum of the squared differences between the measurement and the approximating equation.

For the corner points defined by DRESCHLER and NAGEL (1982) or KITCHEN and ROSENFELD (1980), which are shown to be equivalent in [NAGEL (1983)], the two nonlinear equations can be further simplified into closed forms. This method is implemented and applied to a TV frame sequence and the results, which show sparse displacement vectors computed, are reported in [NAGEL and ENKELMANN (1982)]. To overcome the sparseness, NAGEL (1983) combines the above minimization problem with a global constraint called the 'oriented smoothness' constraint to estimate displacements for image area with minor intensity variation. This const-

rain imposes smooth variation of displacements only in the direction perpendicular to the gradient direction, perpendicular to the gradient direction allowing significant displacement change in other directions. Since the intensity gradient direction usually occurs across an occluding boundary, the above constraint allows significant displacement change across such a boundary. NAGEL and ENKELMANN (1984) implement the scheme and report preliminary results.

A higher order Taylor expansion than those in (1) has been used for the optic flow computation by HARALICK and LEE (1983). They fit a polynomial function to intensities in a local 3D neighborhood consisting of two image coordinates and one coordinate for time. They determine optic flow from the fit by locating the point which matches up with the relative origin.

### 3. From Optic Flow to 3D Motion and Depth

Early systems of moving image analysis concentrated on the extraction of 2D motion information from an image sequence. LEESE et al. (1970) and ENDLICH et al. (1971) use a matching scheme to automatically determine the cloud movement on the image plane from satellite images of clouds. AGGARWAL and DUDA (1975) analyze moving polygonal images to determine motion of polygons and to segment the images. With the help of a moving object model, CHOW and AGGARWAL (1977) analyze planar curvilinear motion. POTTER (1975) uses the 2D motion information from the 'cross-shaped' template for image segmentation.

From the psychophysical and biological study of a vision system perceiving three dimensional objects, researchers began to concentrate on the 3D scene analysis containing moving objects. Some researchers identify moving elements (tokens) from changing brightness pattern

to get motion and structural information of a moving object. From the brightness difference of two image frames, LILLESTRAND (1972), ULSTAD (1973), LIMB and MURPHY (1975) obtain motion information and JAIN et al. (1977, 79), JAIN and NAGEL (1979ab), JAIN (1981), YALAMANCHILI et al. (1982) extract structural information of moving objects as well as their motion information. This differencing technique is further developed for image segmentation and motion information extraction [JAIN et al. (1979), JAIN and NAGEL (1979a, b), JAIN (1981), YALAMANCHILI et al. (1982). This differencing technique is referred to as a peripheral process in the survey by MARTIN and AGGARWAL (1978) where the motion understanding is described on three levels, peripheral, attentive, and cognitive.

Moving object boundaries can be extracted directly from an image time sequence. HAYNES and JAIN (1983) define the time-varying 'edginess' as the product of the temporal brightness change and the static 'edginess' of the Sobel edge detector. They detect the moving object boundaries by thresholding the time-varying 'edginess'. SHAH and JAIN (1984) detect the time-varying corners in a similar way.

THOMPSON (1980) segments images containing multiple moving objects based on both the motion and the brightness. He assumes brightness motion constancy at image points from the same object and uses the motion constraint described in the previous section for the Hough transform to determine dominant motions. He segments images by merging regions having the same motion and similar brightness. Motion constancy is not realistic in general, especially when the motion involves some rotation.

In a restricted scene domain consisting of only vertical and horizontal surfaces, WILLIAMS (1980) searches over distances from the camera

to the vertical surfaces and heights of horizontal surfaces which result in the best image model of one frame from the other.

Another approach which GIBSON (1950 abc) pioneered derives 3D motion and structure information from the optic flow field obtained. GORDON (1965) shows an optic flow field of the ground plane observed by a moving observer and discusses the motion and depth perception from the optic flow field.

Optic flow field from a pure translational motion contains information about moving object surface structure as well as 3D motion. NAKAYAMA and LOOMIS (1974) show that relative depth can be determined from optic flow of an observer in translational motion. CLOCKSIN (1980) derives a mathematical formula for the surface orientation and the edge discrimination from optic flows generated by a pure translational motion. RIEGER and LAWTON (1983) employ an optimization procedure on thresholded difference vectors at discontinuities of the optic flow field to determine the instantaneous axis of translational motion.

Many researchers study optic flow field from general motion. KOENDERINK and VAN DOORN (1976) discuss the surface structure from a differential geometric analysis of optic flow field. PRAZDNY (1980, 81) studies the curvilinear motion of an observer and computes the egomotion parameters from at least five optic flow points. The computation involves solving a system of nonlinear (third order polynomial) equations for the rotational components. He also computes the depth map from the motion parameters determined. LONGUET-HIGGINS and PRAZDNY (1980) assume the first and second order spatial derivatives of the optic flow field are observable and compute the surface orientation and relative motion at each retinal point. They also show that the dilation, shear, and vorti-

city components of the first derivative can be used for the above 3D information.

BRUSS and HORN (1983) measure the discrepancy between the observed optic flow and the one expected by the computed motion parameters and use a least square technique to determine the motion parameters minimizing the discrepancy. In the case of pure translational and rotational motion, they derive equations in closed form for the motion parameters from the use of a proper error norm. For the general motion, they come up with a nonlinear system for the motion parameters.

Given depth and optic flow at a point, BALDARD and KIMBALL (1983) compute 3D optic flow at the point and use it for the determination of motion parameters in decomposed parameter spaces. They assume a small external force on moving objects and compute the acceleration from 3D flows.

#### 4. Determining Corresponding Points

Optic flow can be computed without difficulty from the corresponding points in successive image frames. However, establishing the correspondence is generally recognized as a quite difficult task from the consideration of various factors [AGGARWAL et al. (1981)]. Marr and co-workers [GRIMSON and MARR (1979), MARR and HILDRETH (1980), MARR and POGGIO (1979) study the correspondence problem in human stereo vision and develop a computational theory of human vision. ESKENAZI and CUNNINGHAM (1978) and MORAVEC (1979ab) derive a set of corresponding points from two cameras in a stereo arrangement. However, stereo vision requires extremely high resolution of the cameras to get an acceptable 3D description. AGGARWAL et al. (1981) consider two general approaches to the correspondence problem in their review; one based on ico-

nic or picture-like models of an image pattern and the other based on the structure (identifiable tokens) present in the image pattern. In the next subsection, we will discuss matching techniques using a template or an iconic model followed by token matching techniques.

##### 4-1. Template Matching

The first approach usually takes a subimage (a template) containing a moving object from one frame and then matches it against the subsequent frame based on the chosen iconic representation. The iconic representation can be a portion of the image itself, a segmented binary image, or the edge image. Similarity (Dissimilarity) is measured at each possible match and the match having the maximum (minimum) cross-correlation (absolute difference sum) is considered to be the best match. An equivalent process can be carried out through the use of the Fourier Transform of the iconic representation [LEESE et al. (1970)].

This search process usually requires much computation. To speed up the search process, several schemes have been introduced. BARNEA and SILVERMAN (1972) introduce a class of fast algorithms for image registration which speed up the search by essentially doing no calculation for similarity measure on positions when errors exceed the best match so far. VANDERBURG and ROSENFELD (1977) employ a two-stage template matching scheme in which, at first, a subtemplate is selected and used to determine positions which, result in a (dis) similarity measure above (below) a specified threshold. Next, the remainder of the template is evaluated at those chosen positions for the best match. Two difficulties in the two-stage matching scheme are in selecting the subtemplate and determining a proper threshold value of (dis) similarity measure. HALL et al. (1980) discuss the selection of the



best subset for matching purposes in the sense of the minimum correlation length. GOSHTASBY et al. (1984) analytically derive the threshold value from the template size and false dismissal probability. Another scheme for the fast search is the coarse-fine matching scheme [GOSHTASBY et al. (1984)] which uses a similar strategy.

To establish point correspondence in multiple views at the specified positions, TSAI (1983) presents two methods called the joint moment method and the window variance method which result in a sharper similarity measure than the usual two frame cross-correlation. These methods, however, usually require more computation.

The techniques described up to now work only for image translation. Motion which produce a scaling or rotation of images can introduce much more computation [ALTMANN and REITBOCK (1984)].

#### 4-2. Token Matching

Instead of matching picture-like models, one can identify a set of structures in one image frame and then search the corresponding structures in the subsequent image frame. Several researchers consider the object boundary structure for the match. POTTER (1975) generates a 'cross-shaped template' at a point by calculating the distance from the point to the closest object boundary in the row and column directions, and searches the corresponding point, in the next frame, at which the same size of the template is computed. MARTIN and AGGARWAL (1978) segment boundaries detected from both image frames into primitives such as straight lines and circular arcs and match a primitive of one frame against those of the other frame based on the length and curvature of the primitives. Using the generalized Hough Transform of an object boundary in one image frame, one can find the instance of that boundary in the subsequent

frame [BALLARD (1981)].

Locating distinctive features present in images and imposing constraints on them, one can establish the correspondence of the feature points. ENDLICH et al. (1971) locate the brightness center by an ISODATA technique in successive image frames and establish the correspondence between sets of the brightness centers by assuming constant brightness motion. RENDE and ROSENFELD (1980) apply a relaxation algorithm to establish the correspondence between sets of points selected by hand from two image frames with the assumption of the global brightness displacement constancy. BARNARD and THOMPSON (1980) locate the feature points in an image pair by applying the Moravec interest operator [MORAVEC (1977)] and assign an initial match probability to each pair of feature points which lie within some specified distance of each other allowing for the possibility that the feature may not exist in the second image. They refine the probability for each pairing at a feature point iteratively by applying a relaxation algorithm such that the probability for a pairing is updated by the supporting evidence of similar pairings in the local neighborhood. DRESCHLER and NAGEL (1982) make some modifications in updating the probability by considering both evidence of the support and contradiction in the local neighborhood. At each point, the pairing assigned the highest probability is selected for the correspondence.

ULLMAN (1979) introduces a simple cost function to compare possible matches between two dot patterns and establishes the correspondence by minimizing the cost function. He shows that minimizing the cost function is optimal if the dots in the pattern move independently. PRAGER and ARBIB (1983) assume several feature types on dot patterns and they use a feature metric between feature types to define a distance between any two position-feature

pairs. Using this distance, they define an attraction function for a nearest neighbor. With the help of the attraction function and with the assumption of local consistency of displacement, they can establish the correspondence.

Some other structures present in images can be used. AYALA et al. (1982) segment image frames and establish correspondence between sets of segments based on the similarity of the segment feature values. They use features such as segment position, size, intensity, and area ratio. Since incorrect image segmentation from noise and occlusion introduces incorrect feature values, ROACH and AGGARWAL (1979) suggest using multiple methods arranged in a hierarchy. JACOBUS et al. (1980) encode boundary, region, and surface information in a graph form using a primitive called the half chunk and use a graph matching scheme to establish the correspondence.

The problem with most of these techniques is that they must employ some combinatorial computation to establish the match. This kind of computation is very expensive. Furthermore, these techniques require preprocessors to extract proper structures from image frames.

## 5. From Corresponding Points to 3D Motion and Depth

To determine 3D motion and structure of moving objects, a set of corresponding points can be used. ROACH and AGGARWAL (1980) require two known perspective views of five non-coplanar points in space to set up the problem and come up with 18 nonlinear equations in 18 unknowns. They solve the system of nonlinear equations iteratively with an initial guess for each unknown by using a modified finite difference Lavenberg-Marquardt algorithm [BROWN and DENNIS (1972), LAVENBERG (1944)]. From experiments with noisy views of 3D points,

they find that they obtain more accurate results with considerable overdetermination (two views of 12 or 15 points, three views of 7 or 8 points).

When the surface structure of a moving object is known, the motion estimation problem can be formulated differently. For a known planar surface, TSAI and HUANG (1981) define eight pure parameters which characterize the planar surface and can be uniquely determined by a linear method given two perspective views of four or more points. They derive a sixth order polynomial of one variable whose coefficients are expressed by the pure parameters and determine the motion parameters by solving the polynomial. Instead of solving the sixth order polynomial, the motion parameters can be determined by computing the singular value decomposition of a  $3 \times 3$  matrix containing the pure parameters TSAI et al. (1982). For a known curved surface, TSAI and HUANG (1984) show that two perspective views of seven points on the curved surface are sufficient to determine the 3D motion parameters. YEN and HUANG (1983) project image points centrally on the unit sphere at the origin and use the projected points in correspondence to determine the 3D motion parameters and their 3D positions. They analyze three types of motions; pure translation, pure rotation, and general motion, and interpret the uniqueness of the motion solution by a simple geometry of the points on the sphere. Unfortunately, we can not find any experimental results, especially of noisy views, in their paper.

NAGEL (1981b) uses two coordinate systems; one attached to the camera and the other to a moving object. He derives a relationship between known image point coordinates and unknown object point coordinates and sets up a minimization problem from the relationship and sets of image points in correspondence. DRESCHLER and NAGEL (1982) and NAGEL (1983) apply this technique to a sequence of feature (corner

point) images obtained from a street scene containing a moving taxicab and approximate the cab by a polyhedra.

MARTIN and AGGARWAL (1983) assume orthographic projection of images and develop a scheme describing a 3D object from multiple views of its occluding object boundary. They obtain the object boundaries from the thresholded brightness image. Their 3D description is a bounding volume approximation (to the actual object) represented by a volume segment data structure. AGGARWAL (1983) reviews techniques of deriving 3D descriptions from image sequences.

## 6. Conclusion

Two categories of techniques computing optic flows have been discussed. The one which does not require the correspondence process computes an optic flow vector based on the temporal and spatial brightness variation of the image and motion model. The other one determine the optic flow from the corresponding points in successive image frames. However, establishing the correspondence is quite a difficult task dealing with massive amounts of image data.

Techniques extracting information (3D velocity and surface structure) from either optic flows or corresponding points usually set up and solve a minimization problem which is formulated for the motion or structure information using the relationship derived.

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