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Queueing Performance Analysis of CDF-Based Scheduling over Markov Fading Channels

Yoora Kim♦°

ABSTRACT

In this paper, we analyze the queueing performance of cumulative distribution function (CDF)-based opportunistic scheduling over Nakagami-m Markov fading channels. We derive the formula for the average queueing delay and the queue length distribution by constructing a two-dimensional Markov chain. Using our formula, we investigate the queueing performance for various fading parameters.

Key Words : CDF-based scheduling, queueing analysis, Nakagami-m fading, Markov channel

Ⅰ. Introduction

In wireless networks, each user experiences time-varying channel gains due to fading. By selecting the user who has the largest channel gain at each time-slot, opportunistic scheduling can maximize the sum throughput in wireless networks.

A cumulative distribution function (CDF)-based opportunistic scheduling is designed to support precise control of fairness among users while achieving high throughput[1]. With advantages on fairness and throughput, recent studies have extended the CDF-based scheduling to various networks with practical concerns such as CDF learning[2] and limited feedback[2,3]. While there are many studies on the CDF-based scheduling, the queueing performance has been less explored. In [1], asymptotic behavior of queueing delay is analyzed for Rayleigh fading channels. In [4], the problem of controlling user service delays is addressed for Rayleigh fading channels, and the performance of the suggested solution in [4] is verified through simulation.

In this paper, we analyze the queueing performance of the CDF-based scheduling over Markov fading channels. In comparison with the previous work in [1, 4], the main contribution of this paper is summarized as follows. First, we derive the formula for the queue length distribution and the average queueing delay. Second, we present an *exact-form* mathematical analysis by considering Nakagami-m fading. Note that Nakagami-m model represents a wide range of fading: it becomes Rayleigh fading when $m=1$ and can closely approximate Rician fading when $m>1$. Hence, our analytic formula for the queueing analysis applies to various fading environments. By constructing a two-dimensional Markov chain for the joint queue and channel process, we find the queue length distribution and the average queueing delay of each user. Based on our formula, we investigate the queueing performance of the CDF-based scheduling for various fading parameters.

Ⅱ. System Model and Analysis

We consider downlink in a cellular network consisting of a base station (BS) and N mobile stations (MSs). At the BS, queue n holds packets destined for MS n ($n=1,2,...,N$).

2.1 Wireless channel model

Let $\gamma_n(t)$ be the signal-to-noise ratio (SNR) of the wireless channel between the BS and MS n at time-slot t. In the Nakagami-m fading, the CDF of $\gamma_n(t)$ is

$$F_n(x) = \frac{1}{\Gamma(m)} \int_0^x \left(\frac{m}{\mu_n}\right)^m t^{m-1} \exp\left(-\frac{m}{\mu_n} t\right) dt \quad (1)$$

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♦° First and Corresponding Author : University of Ulsan, Department of Mathematics, yrkim@ulsan.ac.kr, 정회원
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where $\mu_n := \mathrm{E}[\gamma_n(t)]$ is the average SNR, $\Gamma(\cdot)$ is the Gamma function, and $m(\geq 1/2)$ is the Nakagami fading parameter.

To describe the time-correlated fading channel, we use a finite-state Markov model[5] as follows. We partition the entire SNR range into L regions with boundary points γ_i $(i=0,1,...,L)$ where

$$0=\gamma_0<\gamma_1<\gamma_2<\cdots<\gamma_{L-1}<\gamma_L=\infty \ .$$

Then, the channel state of MS n at time-slot t is

$$C_n(t)=i \quad \text{if} \ \gamma_{i-1}\leq\gamma_n(t)<\gamma_i$$

for $i=1,2,...,L$. If $C_n(t)=i$, then the BS can transmit up to R_i packets from queue n to MS n (e.g., see Table 1). We assume that $\{C_n(t)\}$ forms a stationary Markov chain. Let $c_{i,j}^{(n)}$ be the one-step transition probability from states i to j. Then, from [5, 6], we have $c_{i,j}^{(n)}=0$ if $|i-j|>1$ and

$$c_{i,i+1}^{(n)}=L_i^{(n)}T/\rho_i^{(n)},$$
$$c_{i,i-1}^{(n)}=L_{i-1}^{(n)}T/\rho_i^{(n)},$$

where $\rho_i^{(n)}=F_n(\gamma_i)-F_n(\gamma_{i-1})$, T is the length of a time-slot, and

$$L_i^{(n)}=\frac{\sqrt{2\pi}f_d}{\Gamma(m)}\left(\frac{m\gamma_i}{\mu_n}\right)^{m-1/2}\exp\left(-\frac{m\gamma_i}{\mu_n}\right)$$

where f_d is the Doppler frequency. Fig. 1 depicts the transition diagram of $\{C_n(t)\}$.

2.2 CDF-based scheduling

Each MS measures its SNR value $\gamma_n(t)$ via e.g., a common downlink pilot channel and sends it to the BS. Then, the BS computes the CDF $F_n(\gamma_n(t))$ using Equation (1) for the measured value $\gamma_n(t)$. Note that $F_n(\gamma_n(t))$ represents the likelihood that MS n will experience the SNR lower than the measured value $\gamma_n(t)$. Hence, as the CDF $F_n(\gamma_n(t))$ increases, MS n is less likely to have the SNR higher than $\gamma_n(t)$. A CDF-based scheduling operates slot-by-slot by selecting MS $n^*(t)$ at time-slot t as follows:

$$n^*(t)=\arg\max_{1\leq n\leq N}\left[F_n(\gamma_n(t))\right]^{1/w_n}$$

where $w_n(>0)$ is the weight of MS n and satisfies $w_1+\cdots+w_N=1$. The weight w_n in fact determines the channel access probability of MS n as follows. Given that $C_n(t)=i$, the scheduling probability of MS n becomes[1]

$$s_i^{(n)} := \mathrm{P}(n^*(t)=n\,|\,C_n(t)=i)$$
$$=w_n\left\{F_n(\gamma_i)^{1/w_n}-F_n(\gamma_{i-1})^{1/w_n}\right\}/\rho_i^{(n)}.$$

Thus, the channel access probability of MS n is

$$\mathrm{P}(n^*(t)=n)=\sum_{i=1}^{L}s_i^{(n)}\rho_i^{(n)}=w_n. \qquad (2)$$

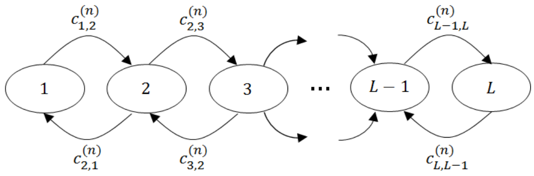

Fig. 1. Finite-state Markov chain model

Table 1. Transmission parameters[7]

Modulation	Coding Rate	Required SNR (dB)	R_i (packets/slot)
BPSK	1/2	6.4	1
QPSK	1/2	9.4	2
QPSK	3/4	11.2	3
16-QAM	1/2	16.4	4
16-QAM	3/4	18.2	6
64-QAM	2/3	22.7	8
64-QAM	3/4	24.4	9

2.3 Queueing analysis

Let $A_n(t)$ be the number of packets arriving at queue n during time-slot t. We assume that $A_n(t)$ is i.i.d. across t and follows a *general* probability distribution $a_n(i) := \mathrm{P}(A_n(t)=i)$. Let $Q_n(t)$ be the number of packets in queue n at time-slot t. Then, the queue length under the CDF-based scheduling evolves as

$$Q_n(t+1)=\left[Q_n(t)-\mathrm{I}_{\{n^*(t)=n\}}\cdot R_{C_n(t)}\right]^+ + A_n(t)$$

where $[x]^+=\max\{x,0\}$, and $\mathrm{I}_{\{\cdot\}}$ is an indicator function. In the following, we derive the limiting distribution of $Q_n(t)$ and the average delay.

First, we define $X_n(t) := (Q_n(t), C_n(t))$. Since $\{C_n(t)\}$ forms a stationary Markov chain, the joint process $\{X_n(t)\}$ becomes a two-dimensional Markov chain. Let P_n be the transition probability matrix of $\{X_n(t)\}$. Then, P_n can be written as $P_n = [H_{u,v}^{(n)}]_{u,v \geq 0}$ where, for each pair of (u,v),

$$H_{u,v}^{(n)} = [h_{u,v}^{(n)}(i,j)]_{1 \leq i,j \leq L}$$

is a $L \times L$ matrix with elements

$$h_{u,v}^{(n)}(i,j) := \mathrm{P}(X_n(t+1) = (v,j) \mid X_n(t) = (u,i)).$$

Proposition 1. $h_{u,v}^{(n)}(i,j)$ is given by

$$h_{u,v}^{(n)}(i,j) = c_{i,j}^{(n)} [s_i^{(n)} 1 - s_i^{(n)}] \begin{bmatrix} a_n(v - [u - R_i]^+) \\ a_n(v-u) \end{bmatrix}.$$

Proof. Given $X_n(t) = (u,i)$, we have

$$Q_n(t+1) = \begin{cases} [u - R_i]^+ + A_n(t) & \text{if } n^*(t) = n, \\ u + A_n(t) & \text{if } n^*(t) \neq n. \end{cases}$$

Since $\mathrm{P}(n^*(t) = n \mid C_n(t) = i) = s_i^{(n)}$, we have

$$\begin{aligned} &\mathrm{P}(Q_n(t+1) = v \mid X_n(t) = (u,i)) \\ &= s_i^{(n)} \cdot \mathrm{P}(A_n(t) = v - [u - R_i]^+) \\ &\quad + (1 - s_i^{(n)}) \cdot \mathrm{P}(A_n(t) = v - u) \\ &= [s_i^{(n)} \quad 1 - s_n^{(n)}] \begin{bmatrix} a_n(v - [u - R_i]^+) \\ a_n(v-u) \end{bmatrix}. \end{aligned}$$

In addition,

$$\mathrm{P}(C_n(t+1) = j \mid Q_n(t+1) = v, X_n(t) = (u,i)) = c_{i,j}^{(n)}.$$

Hence,

$$\begin{aligned} &h_{u,v}^{(n)}(i,j) := \mathrm{P}(X_n(t+1) = (v,j) \mid X_n(t) = (u,i)) \\ &= \mathrm{P}(Q_n(t+1) = v \mid X_n(t) = (u,i)) \\ &\quad \cdot \mathrm{P}(C_n(t+1) = j \mid Q_n(t+1) = v, X_n(t) = (u,i)) \\ &= c_{i,j}^{(n)} [s_i^{(n)} 1 - s_i^{(n)}] \begin{bmatrix} a_n(v - [u - R_i]^+) \\ a_n(v-u) \end{bmatrix}. \qquad Q.E.D. \end{aligned}$$

Next, we compute the limiting distribution of the Markov chain $\{X_n(t)\}$. Let

$$\pi_n = [(\pi_{q,1}^{(n)}, \pi_{q,2}^{(n)}, \cdots, \pi_{q,L}^{(n)})]_{q \geq 0}$$

where $\pi_{q,c}^{(n)} := \lim_{t \to \infty} \mathrm{P}(X_n(t) = (q,c))$. Then, π_n is obtained by solving $\pi_n P_n = \pi_n$, $\pi_n e = 1$, where $e = [1, 1, \cdots, 1]^T$. From π_n, we obtain the queue length distribution of MS n as

$$\lim_{t \to \infty} \mathrm{P}(Q_n(t) = q) = \sum_{c=1}^{L} \pi_{q,c}^{(n)} \quad (q = 0, 1, 2, \ldots).$$

From Little's law, the average queueing delay is

$$\overline{D}_n = \frac{\overline{Q}_n T}{\mathrm{E}[A_n]} = \frac{T}{\mathrm{E}[A_n]} \sum_{q=0}^{\infty} q \sum_{c=1}^{L} \pi_{q,c}^{(n)}$$

where $\overline{Q}_n$ is the average queue length of MS n.

Ⅲ. Numerical Study

3.1 Parameter setup

The model and parameters used in this section are as follows. We assume $N = 10$ MSs. Packet arrivals at each queue are Poisson with the rate $\lambda = 0.08$ packets/slot. The average SNRs of MSs are $\mu_1 = \cdots = \mu_{10} = 15$ dB. To these 10 MSs, we assign linear weight $(w_1, \cdots, w_{10}) = k(1, \cdots, 10)$ where $k = 1/55$. The other parameters common to all MSs are as follows: $T = 2$ msec, $f_d = 15$ Hz, and $m = 1, 3, 5$.

For wireless transmission, we adopt the IEEE 802.16 standard[7] as summarized in Table 1. Note that the required SNR in Table 1 represents the boundary points γ_i, i.e., $\gamma_1 = 6.4, \gamma_2 = 9.4, \ldots, \gamma_7 = 24.4$ dB. If $\gamma_n(t) < \gamma_1$, packets cannot be transmitted successfully since $\gamma_1 (= 6.4\,\mathrm{dB})$ is the minimum required SNR. Hence, we set $R_1 = 0$. If $\gamma_1 \leq \gamma_n(t) < \gamma_2$, one packet can be transmitted in a slot, i.e., we set $R_2 = 1$. Then, the values of $R_3, \ldots, R_8$ can be calculated in proportion to the information bits per symbol as given in Table 1.

3.2 Numerical results

Fig. 2 shows the queue length distribution of MS n for $n = 1, 5, 10$ when $m = 1$. We observe that, as the weight increases (i.e., n increases), it is more likely to have fewer packets in the queue. Fig. 3 shows the average queueing delay of MS n for $n = 1, 2, \cdots, 10$ when $m = 1, 3, 5$. As we expected, the queueing delay decreases as the weight

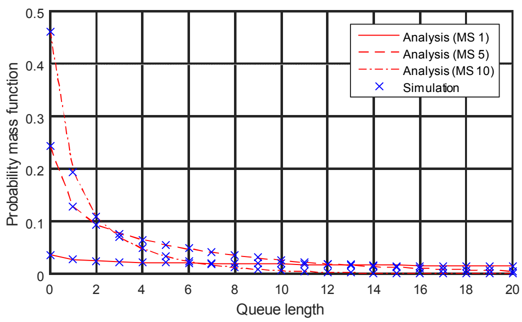

Fig. 2. The queue length distribution ($m=1$)

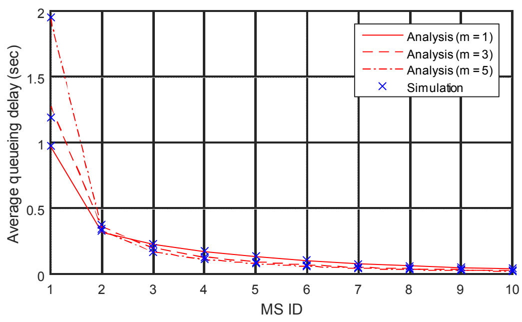

Fig. 3. The average queueing delay ($m=1$, 3, 5)

increases. In the case of linear weight, the channel access probability is also linear with n by Equation (2). Such linearity no longer holds for the queueing delay, which is convex with n. Fig. 3 also shows that Rician fading (i.e., $m>1$) induces longer/shorter delay than Rayleigh fading (i.e., $m=1$) for small/large weight.

Ⅳ. Conclusion

The weight is an important parameter for fairness control in CDF-based scheduling. In this paper, we analyze the impact of the weight on the queuing performance of the CDF-based scheduling over stationary Markov fading channels. The accuracy of our analysis is verified by simulation. Our future work is to compare the queueing performance of the CDF-based scheduling with that of other opportunistic scheduling schemes.

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