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A Survey on Approximation Algorithms for Path Planning of UAVs

JinYi Yoon◆, HyungJune Lee°

ABSTRACT

As exploiting Unmanned Aerial Vehicles (UAVs) as mobile elements is a new research trend recently, approximation algorithms to solve path planning problems for UAVs are promising approaches. In this paper, we divide problems in two main objectives, cost and reward. We introduce some representative approximation algorithms, which focus on maximizing reward or minimizing travel time. Particularly, we explore path planning for each node visit within different time deadline. In recent works, researchers extend classic algorithms to distributed path planning algorithms for multiple UAVs.

Key Words : Path Planning, Unmanned Aerial Vehicle (UAV), Traveling Salesman Problem (TSP), Orienteering, Approximation Algorithm, Time Constraints

Ⅰ. Introduction\

Path planning is a prominent problem in wide research communities: traveling salesman problem (TSP), vehicle routing problem (VRP) in operations, navigation in robotics, and scheduling data delivery in wireless networks. Since most variants of this problem proved NP-hard to determine the optimal solution, heuristic techniques such as nearest neighbor (NN) algorithms, genetic algorithms, simulated annealing, and ant colony optimization have been proposed.

One classic model of path planning is TSP, which aims to minimize the total traveling distance while covering all given locations[1,2]. TSP can be extended to VRP, where there is more than one vehicle[3,4]. Related to path planning with time constraints, Deadline-TSP and VRP with Time Windows (VRPTW) have been investigated. If all nodes have the same time deadline, the problems reduce to be the orienteering problem, which maximizes the number of visiting points within fixed travel deadline.

An usage of mobile elements can be a constructive approach to these problems. Unmanned Aerial Vehicles (UAVs) are promising as they are less constrained to movements and have re-programmable architecture. Due to these features, they can be leveraged as navigation systems for various applications such as aerial surveillance, data gathering and delivering[5-7], and communication relays for network recovery[8-10].

The remainder of this paper is organized as follows: we begin with formulating variants of path planning problems in Section 2. In Section 3, we introduce approximation path planning algorithms. We finally conclude this paper in Section 4.

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◆ First Author : Department of Computer Science & Engineering, Ewha Womans University, yjin3012@ewhain.net, 학생회원
° Corresponding Author : Department of Computer Science & Engineering, Ewha Womans University, hyungjune.lee@ewha.ac.kr, 정회원

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Ⅱ. Problem Formulation

Path planning can be specified in many different problems mainly in two aspects: 1) cost defined as travel time of the path and 2) reward, which is the number of nodes visited. Since there is a trade-off between them, it is hard to optimize both aspects. In this section, we present the problems for each objective function as in Table 1 and Fig. 1.

Table 1. Classification of problems for each objective

Objective	Constraints	Problems
Minimize cost	Lower bound reward	TSP, k-TSP
Maximize reward	Upper bound cost	Orienteering, Deadline-TSP, Mobile element scheduling
Co-optimize both	Different in cases	Prize-collecting TSP, Discounted-reward TSP

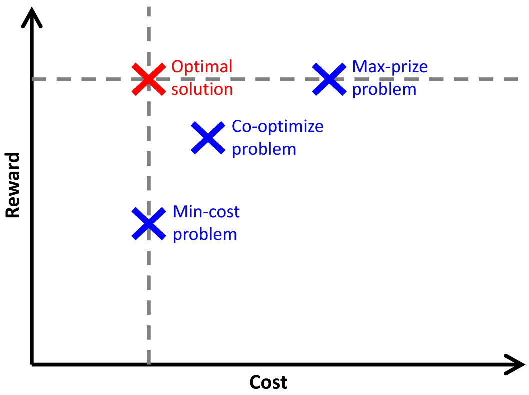

Fig. 1. A distribution of problems in path planning

2.1 Minimizing cost

In this problem, the goal is to minimize the travel time, which is related to the total distance of path. UAV has to collect at least some amount of reward. A traditional TSP finds the shortest path covering all given nodes. Given a lower bound reward k, k-TSP aims to minimize the cost[11].

2.2 Maximizing prize

In most cases, there is a maximum distance that should be traveled or the nodes have time constraints. Since it cannot be guaranteed to visit all nodes within given deadline, maximizing the reward has extensively been studied.

The orienteering problem aims to maximize the number of visiting nodes within fixed total travel time[12]. If there is a distinct time deadline for each node, we define this problem as deadline-TSP[11]. If the deadlines of all nodes are equal, it can converge to the orienteering problem. In most path planning problems, reward is collected as soon as a node is visited. However, a node need to be visited multiple times occasionally, and its deadline is updated in Mobile Element Scheduling (MES)[13].

2.3 Co-optimizing both cost and reward

If we have relaxed time deadline, it means that we are able to visit more nodes. Therefore, many researchers have attempted to co-optimize both travel cost and reward recently.

Prize-Collecting TSP (PC-TSP) aims to minimize travel distance and missed rewards[12]. In Discounted-Reward TSP, the reward for each node declines depending on serviced time[11,12].

Ⅲ. Path Planning Algorithms

In this section, we introduce several algorithms that can be applied to path planning of UAVs. While most algorithms focus on single optimized path, recent works consider multiple UAVs to collaboratively cover the given nodes. In the following, we summarize some algorithms under two main challenges[14] (Table 2).

Table 2. Summary of general approximation algorithms

Algorithm	*Problem*	*Main Idea*
Min-excess path[11-12]	k-TSP	Minimize loops
Max-prize path[12]	Orienteering	Find *min-excess* path with maximum k
Point-to-point orienteering[11]	Orienteering	Find an node pair with maximum reward
DroneNet, *DroneNet+*[8-9]	TSP	Distributed motion planning using multiple UAVs

3.1 General Approximation Algorithms

In general, there is a constraint on total travel time or a minimum bound of reward. We begin by giving some algorithms for k-TSP and orienteering problem for a single UAV. In recent years, an extension to multiple UAVs is studied in *DroneNet* and *DroneNet+*[8,9].

3.1.1 *Min-excess* path

The *min-excess* path algorithm solves minimizing cost problem, k-TSP[11,12]. Given start node s and end node t, it aims to minimize the excess of a path while collecting at least k reward. The excess of the path is denoted as $\epsilon(P)$, which is the difference between the length of the path, $d_P(s,t)$ and the distance between s and t, $d(s,t)$.

After sorting all nodes in the increasing order of distance from start point s, the path is splitted into segments as in Fig. 2. There are two types of segments: type 1 segment, *monotone*, and type 2, *wiggly*. In type 1 segment, UAV visits each node only once, and there are no intermediate nodes. It means type 1 in the computed path is an exact solution. On the other hand, in type 2 segments, the length of type 2 is at least 3 times of direct distance due to loops in the path.

By the proof in [12, 15], the excess of the computed path is less than or equal to 2.5 times of the excess of the shortest path. This approach, it guarantees at least *k* visiting points, and thus, it provides a great extension to other problems.

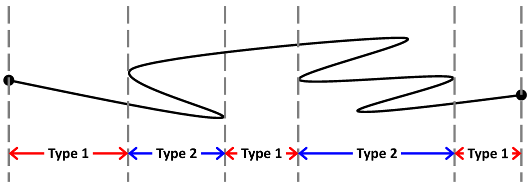

Fig. 2. Segments in *min-excess* path

3.1.2 *Max-prize* path

The *max-prize* path algorithm is one of solvable strategies for maximizing prize problem[12]. It finds the maximum-prize path within given length D, starting from s.

This approach relies on *min-excess* algorithm by guessing k. First, they compute *min-excess* path from s to each node. If there exists a node such that the length of *min-excess* path is shorter than D, it is a valid solution.

Although it is ambiguous to guess reasonable *k*, we can simply extend the *min-cost* problem to the *max-prize* problem. Also, if there is a *max-prize* path from s to e that visits k nodes, a path from s to any nodes in the path has the excess at most $\frac{D-d(s,node)}{r}$ while collecting at least $\frac{\pi}{r}$ for any integer $r \geq 1$.

3.1.3 *Point-to-point orienteering*

Maximizing reward problem can be solved by *point-to-point orienteering*[11]. Given a pair of nodes s and t, the goal is to visit as many intermediate nodes as possible in a maximum distance D.

It also borrows an idea from *min-excess* path algorithm. For each pair of points x and y, they compute a minimum excess path while visiting k nodes. From the pair of $(x,\ y,\ k)$ with the maximum k, the path is generated from the shortest path from s to x, minimum excess path from x to y, and the shortest path from y to t.

The computed path by *point-to-point* is guaranteed to visit at least $\frac{1}{3}$ of the reward of the optimal path by the proof in [11].

3.1.4 Motion Planning in *DroneNet* and *DroneNet+*

Motion planning for network traversing is one of TSP problems. In this problem, multiple UAVs find their own paths over a virtual grid topology[8,9]. The goal is to minimize travel time while covering all the vertexes and to reduce the duplicate coverage.

Each UAV moves in a *zigzag* path among 8 patterns. If all of the neighboring nodes are already visited, they select the shortest unvisited node. To leverage multiple UAVs, it is significant to avoid the duplicate coverage. Therefore, if two or more UAVs get encountered, they exchange their visited lists. UAV continues to decide the next visiting point until there remain no points to visit.

Both *DroneNet* and *DroneNet+* decide their next visiting point in a simple yet efficient manner. Also, most of previous approaches suffer from centralized computation. However, they solve the problem by traversing efficiently with multiple UAVs in a distributed way.

3.2 Path Planning under Distinct Time Constraints

In particular, there are situations such as a catastrophic disaster situation where nodes have distinct time deadline. We explore a few algorithms to solve deadline-TSP and MES (Table 3).

Table 3. Summary of distinct time-limited path planning algorithms

Algorithm	Problem	Main Idea
$O(\log n)$ *approximation*[11]	Deadline-TSP	Find the collection of disjoint rectangles
EDF and *EDF with k-lookahead*[7,13]	MES	Visit a node with close deadline
Minimum weighted sum first[7,13]	MES	Give a weight to both travel time and deadline
Distributed path planning[7]	Deadline-TSP	Collaborative divide tasks into multiple UAVs

3.2.1 $O(\log n)$ *approximation*

When each node v has its own deadline $D(v)$, it aims to visit as many nodes as possible within their designated deadlines. To tackle the respective deadline problem, they divide it into orienteering sub-problems. If the last visiting point has the shortest deadline, the deadline of all other nodes can be reduced to the smallest one. Then, it can be solved by orienteering algorithms[11].

First, they locate nodes at point $(D(v),\ C(v))$ where $C(v)$ is travel time from the start location to node v as in Fig. 3. When vertexes are visited in the order of $C(v)$, *minimal vertex* is a vertex whose deadline is shorter than any after visiting vertexes. A *rectangle* $R(h,j,k)$ consists of minimal vertexes and its bottom edge is time of h, the right edge is

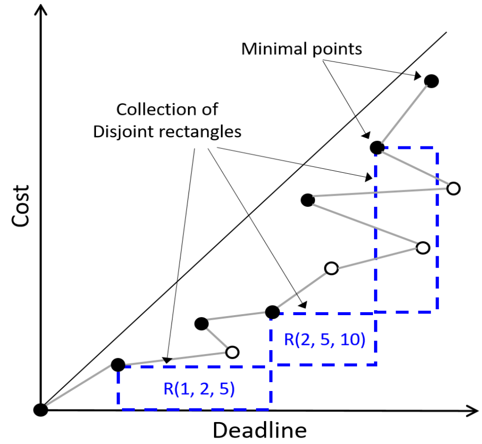

Fig. 3. An example of formulating rectangles in $O(\log n)$ *approximation* algorithm[11]

the deadline of k, and the upper left corner is j. Two rectangles are disjoint if no vertical or horizontal line intersects both of them. It makes a collection of disjoint rectangles to avoid double-counting and cover enough nodes, and runs *point-to-point orienteering* on each rectangle.

Although this approach has some difficulty in finding minimal vertexes and rectangles, it can avoid exceeding any deadlines.

3.2.2 *Earliest deadline first (EDF) and EDF with k-lookahead*

In this problem, some nodes can be visited more than once due to their designated deadlines. *Earliest Deadline First* (*EDF*) and *EDF with k-lookahead* solve the MES problem[7,13].

The main idea of *EDF* is to visit the node with the closest deadline first. First, it chooses the node whose deadline is the shortest and updates the current time, the current location to the node, and the deadline of the node to initial time deadline as in Fig. 4.

EDF can be extended to *EDF with k-lookahead*, which selects *k* nodes with the least deadline. Then, it finds an order of *k* nodes with maximum reward. UAV moves to the first node from the ordered list and updates current status.

This algorithm provides an easy way to decide the next visiting point. However, it only considers deadlines of nodes, not the cost. Due to the deadline

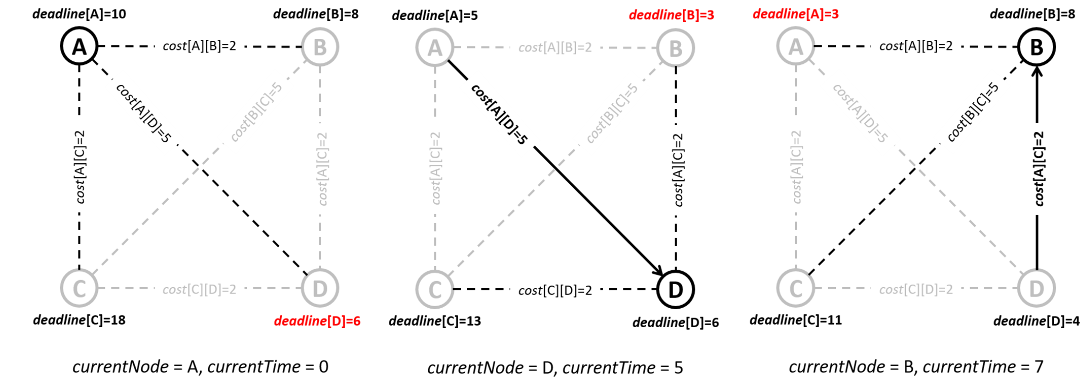

Fig. 4. An example of selecting next visiting node in *earliest deadline first* algorithm

criterion, nodes with enough deadline can be missed, while UAV serves nodes with tight deadline.

3.2.3 *Minimum weighted sum first*

This algorithm also solves the MES problem. Contrary to *EDF*, *minimum weighted sum first* considers travel time as well as deadline[7,13]. By giving weights to deadline and cost, they calculate weighted sum as follows:

$$weightedSum(v) = \alpha \times D(v) + (1-\alpha) \times C(v) \quad (1)$$

They calculate a weighted sum for all nodes and choose the node with the minimum weighted sum as a next visiting node as in Fig. 5.

This approach considers both deadline and cost. Although it is adaptive to various cases, it is hard to adjust α.

3.2.4 *Distributed path planning*

Compared to MES, each node is visited only once in this problem. It aims to find the optimal paths of multiple UAVs to maximize the number of nodes that are successfully visited within time deadline while minimizing total travel time[7].

Each UAV selects next grid point candidates to visit and finds the ordering of maximum reward as in [13]. Then, UAV moves to the first visiting point of the path. Furthermore, it collaboratively divides unvisited points when multiple UAVs encounter.

It should be pointed out that unvisited points can be distributed to the most relevant UAV. Also, it optimizes not only reward, but travel cost as well.

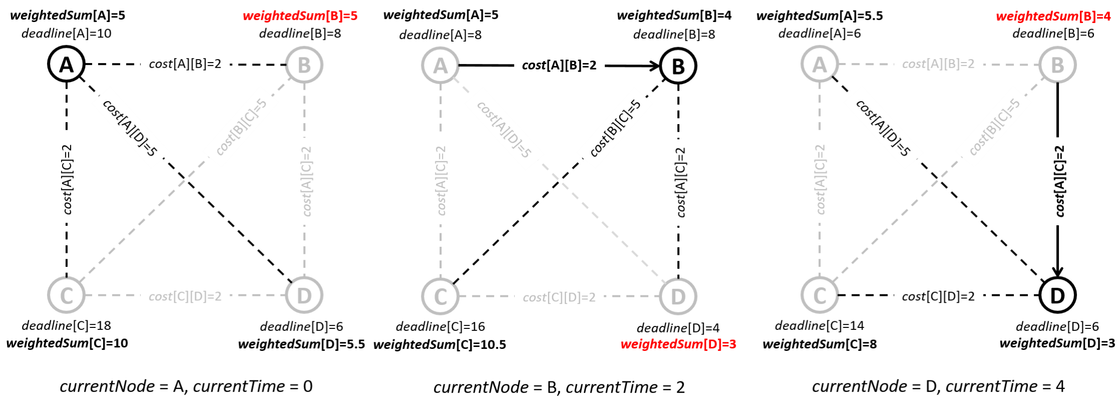

Fig. 5. An example of selecting next visiting node in *minimum weighted sum first* algorithm

Ⅳ. Conclusion

Path planning is a fundamental problem in various research areas. In this paper, we introduce some representative approximation algorithms for path planning of UAVs. They are classified into two main objectives, travel cost and the number of visited nodes.

For future work, computation complexity is still left to be discussed. The battery outage issue and real-time path planning would be studied and compared.

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윤 진 이 (JinYi Yoon)

2017년 2월 : 이화여자대학교 컴퓨터공학과 학사
2017년 3월~현재 : 이화여자대학교 컴퓨터공학과 석사과정
<관심분야> 무선 센서 네트워크, 무선 애드혹 네트워크, 네트워크 최적화, 알고리즘

이 형 준 (HyungJune Lee)

2001년 8월 : 서울대학교 전기공학부 학사
2006년 6월 : Stanford Univ. 전자공학과 석사
2010년 8월 : Stanford Univ. 전자공학과 박사
2012년 3월~현재 : 이화여자대학교 컴퓨터공학과 조교수
<관심분야> 무선 센서 네트워크, 이동 애드혹 네트워크, 네트워크 최적화, 임베디드 시스템