

# Impact of Fast Fading CCI on Performance of Optimum Combining in DF Relaying

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## ABSTRACT

We present the effect of fast fading co-channel interference (CCI) on the performance of decode-and-forward (DF) relaying. This paper derives closed-form expressions for the average symbol error rate (SER) with optimum combining (OC) in the presence of fast fading CCI. It is shown that the average SER performance of fast fading CCI is worse than that of slow fading CCI, which is the previous research assumption. In result, DF relaying systems should be designed with consideration of fast fading CCI.

**Key Words** : Decode-and-forward relaying, fast fading CCI, optimum combining, Rayleigh fading, symbol error rate

## I. Introduction

Cooperative wireless relays, which are utilized in modern and fifth generation (5G) cellular networks in LTE-A release 10, improve the throughput, reliability and coverage area. The cooperative communication has been introduced as the recent paradigm of diversity gain. In cooperative networks, there are mainly two methods, such as the amplify-and-forward (AF) relay network and the decode-and-forward (DF) relay network<sup>[1]</sup>. Then with multiple copies, the destination can achieve cooperative diversity. In order to do so, we can use maximal-ratio combining (MRC)<sup>[2]</sup> or optimum combining (OC)<sup>[3]</sup>. MRC maximizes the signal-to-noise-ratio (SNR), while OC maximizes the signal-to-interference-plus-noise ratio (SINR). When co-channel interference (CCI) is present at the destination, OC reduces CCI's power and increases diversity. Recently, there have been many research advances in the DF relay network. In [4], a novel distributed space-time coding (DSTC) transmission scheme for a two-path successive DF relay network is proposed. The average symbol error rate (SER) is

analyzed for a wireless powered three-node DF relaying system in Nakagami- $m$  fading environment<sup>[5]</sup>. In [6], successive interference cancellation in two-hop DF relay channel is investigated. An optimal source power allocation for the partial DF relay system with zero-forcing beamforming is presented<sup>[7]</sup>.

One of major assumptions for CCI in DF relaying is flat fading over time slots of a single transmission. Mobile channel states, however, can change irregularly so that CCI can be fast faded. Thus, the paper presents the effect of fast fading CCI on the performance of DF relaying. The paper is organized as follows. Section II defines the system and channel model. In Section III, the exact analytical expressions are derived for the average SER for fast or slow fading CCI, respectively, and analytical and simulation results are presented. The paper is concluded in Section IV.

## II. System and Channel Model

We assume that the destination know whether or not the relay send the symbol with the probability

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one. We suppose a time division duplex (TDD) mode. The relay system consists of a source ( $S$ ), relays ( $R$ ), a destination ( $D$ ), and an interferer ( $I$ ) at the destination. The system and channel model is depicted in Fig. 1.

In phase 1, the source transmits its data symbols. The received signal at the destination is expressed by

$$y^{(S,D,t_0)} = \sqrt{E_S} g_0^{(S,D,t_0)} b_0 + \sqrt{E_I} g_1^{(I,D,t_0)} b_1 + n^{(S,D,t_0)} \quad (1)$$

where  $E_S$ ,  $E_I$ ,  $b_0$ , and  $b_1$  are the power transmitted by the source, the power transmitted by the interferer, the source data symbol with unit average power, and the interferer data symbol with unit average power, respectively. Furthermore, the channel propagation parameters  $g_0^{(S,D,t_0)}$  and  $g_1^{(I,D,t_0)} \sim \mathcal{CN}(0,1^2)$  are Rayleigh faded and thermal noise  $n^{(S,D)} \sim \mathcal{CN}(0, N_0)$  is complex additive white Gaussian noise (AWGN), where the notation  $\mathcal{CN}(\mu, \Sigma)$  denotes the complex circularly-symmetric normal distribution with mean  $\mu$  and variance  $\Sigma$ . Assume that  $R$  relays correctly decode the symbol and forward. For the  $r$ -th time slot  $t_r$ , the received signal at the relay is expressed by

$$y^{(S,R,t_r)} = \sqrt{E_S} g_0^{(S,R,t_r)} b_0 + n^{(S,R,t_r)}, \quad r = 1, 2, \dots, R \quad (2)$$

where  $n^{(S,R,t_r)} \sim \mathcal{CN}(0, N_0)$  is complex AWGN.

In phase 2, the signal at the destination is expressed by

$$y^{(R,D,t_r)} = \sqrt{E_S} g_0^{(R,D,t_r)} b_0 + \sqrt{E_I} g_1^{(I,D,t_r)} b_1 + n^{(R,D,t_r)}, \quad r = 1, 2, \dots, R \quad (3)$$

where the channel parameters  $g_0^{(R,D,t_r)}$  and  $g_1^{(I,D,t_r)}$ ,  $r = 1, 2, \dots, R$ ,  $\sim \mathcal{CN}(0,1^2)$  are Rayleigh faded and  $n^{(R,D,t_r)} \sim \mathcal{CN}(0, N_0)$  is complex AWGN.

Thus with the total number of time slots  $N = R+1$ , the received ( $N \times 1$ ) signal vector  $\mathbf{y}$  at the destination is expressed by

$$\mathbf{y} = \begin{bmatrix} y^{(S,D,t_0)} \\ y^{(R,D,t_1)} \\ y^{(R,D,t_2)} \\ \vdots \\ y^{(R,D,t_R)} \end{bmatrix} = \sqrt{E_S} \begin{bmatrix} g_0^{(S,D,t_0)} \\ g_0^{(R,D,t_1)} \\ g_0^{(R,D,t_2)} \\ \vdots \\ g_0^{(R,D,t_R)} \end{bmatrix} b_0 + \sqrt{E_I} \begin{bmatrix} g_1^{(I,D,t_0)} \\ g_1^{(I,D,t_1)} \\ g_1^{(I,D,t_2)} \\ \vdots \\ g_1^{(I,D,t_R)} \end{bmatrix} b_1 + \begin{bmatrix} n^{(S,D,t_0)} \\ n^{(R,D,t_1)} \\ n^{(R,D,t_2)} \\ \vdots \\ n^{(R,D,t_R)} \end{bmatrix} \quad (4)$$

$$= \sqrt{E_S} g_0 b_0 + \sqrt{E_I} g_1 b_1 + \mathbf{n}.$$

Here we assume  $g_0$ ,  $g_1$ , and  $\mathbf{n}$  are ( $N \times 1$ ) zero-mean complex symmetric Gaussian random vectors.

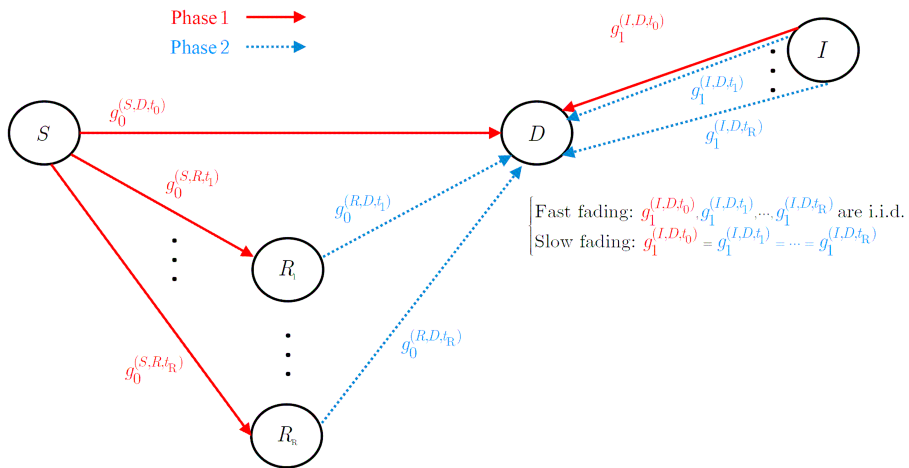


Fig. 1. System and channel model.

### III. SER Derivation and Results

The instantaneous maximum output SINR  $\gamma$  with OC at the destination is expressed as [3]

$$\gamma = E_S \mathbf{g}_0^\dagger \mathbf{R}^{-1} \mathbf{g}_0 \quad (5)$$

where the interference-plus-noise correlation matrix  $\mathbf{R} = N_0 \mathbf{I}_N + E_I \mathbf{g}_1 \mathbf{g}_1^\dagger$ , the notation  $\mathbf{I}_N$  is ( $N \times N$ ) the identity matrix and the notation  $(\cdot)^\dagger$  is the conjugation and transposition. We express the ( $N \times N$ ) Hermitian matrix  $\mathbf{R}$  as the eigenvalue decomposition

$$\mathbf{R} = \mathbf{U} \mathbf{A} \mathbf{U}^\dagger, \quad \mathbf{A} = \begin{bmatrix} \lambda_0 & 0 & 0 & \dots & 0 \\ 0 & \lambda_1 & 0 & \dots & 0 \\ 0 & 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \lambda_N \end{bmatrix} = \begin{bmatrix} N_0 + E_I \mathbf{g}_1^\dagger \mathbf{g}_1 & 0 & 0 & \dots & 0 \\ 0 & N_0 & 0 & \dots & 0 \\ 0 & 0 & N_0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & N_0 \end{bmatrix} \quad (6)$$

where  $\mathbf{U}$  is the ( $N \times N$ ) unitary matrix. Note that when CCI is fast faded, the elements of  $\mathbf{g}_1$  are independent and identically distributed (i.i.d.). Specifically, the coherence time  $T_c$  of the channel is defined as the period of time over which the fading process is correlated. Then  $T_c^{(fast)}$  becomes a single time slot duration  $t_c$  for fast fading CCI, and  $T_c^{(slow)}$  becomes the transmission duration  $t_c N$  for slow fading CCI, respectively. The instantaneous maximum output SINR  $\gamma$  at the destination is given as

$$\begin{aligned} \gamma &= E_S \mathbf{g}_0^\dagger (\mathbf{U} \mathbf{A} \mathbf{U}^\dagger)^{-1} \mathbf{g}_0 \\ &= E_S (\mathbf{U}^\dagger \mathbf{g}_0)^\dagger \mathbf{A}^{-1} (\mathbf{U}^\dagger \mathbf{g}_0) \\ &= E_S \mathbf{h}_0^\dagger \mathbf{A}^{-1} \mathbf{h}_0 \end{aligned} \quad (7)$$

where  $\mathbf{h}_0 = \mathbf{U}^\dagger \mathbf{g}_0$  and  $\mathbf{h}_0$  has the same statistics as  $\mathbf{g}_0$  because  $\mathbf{U}$  is a unitary matrix. The conditional moment generating function (MGF)  $M_{\gamma|\lambda_0}(s)$  is given by

$$\begin{aligned} M_{\gamma|\lambda_0}(s) &= \mathbb{E}_{\gamma|\lambda_0} \left[ e^{\gamma s} \right] \\ &= \mathbb{E}_{\mathbf{h}_0|\lambda_0} \left[ e^{\gamma s} \right] \\ &= \frac{1}{\left(1 - \frac{E_S}{\lambda_0} s\right) \left(1 - \frac{E_S}{N_0} s\right)^R}. \end{aligned} \quad (8)$$

The average SER  $P_e$  with the coherent binary phase shift keying (BPSK) is calculated as

$$\begin{aligned} P_e &= \mathbb{E}_{\gamma} \left[ \frac{P_{e|\gamma}}{2} \right] \\ &= \mathbb{E}_{\gamma} \left[ Q(\sqrt{2\gamma}) \right] \end{aligned} \quad (10)$$

where  $Q(x) \triangleq 1 / \sqrt{2\pi} \int_x^\infty e^{-y^2/2} dy$ . In particular,  $Q(x)$  can also be represented as  $Q(x) = 1 / \pi \int_0^{\pi/2} e^{-x^2/(2\sin^2\theta)} d\theta$ . Then the average SER  $P_e$  is directly expressed in terms of the conditional MGF  $M_{\gamma|\lambda_0}(s)$

$$\begin{aligned} P_e &= \mathbb{E}_{\mathbf{h}_0, \lambda_0} \left[ Q(\sqrt{2\gamma}) \right] \\ &= \mathbb{E}_{\lambda_0} \left[ \mathbb{E}_{\mathbf{h}_0|\lambda_0} \left[ Q(\sqrt{2\gamma}) \right] \right] \\ &= \mathbb{E}_{\lambda_0} \left[ \mathbb{E}_{\mathbf{h}_0|\lambda_0} \left[ \frac{1}{\pi} \int_0^{\pi/2} e^{-\gamma/\sin^2\theta} d\theta \right] \right] \\ &= \mathbb{E}_{\lambda_0} \left[ \frac{1}{\pi} \int_0^{\pi/2} \mathbb{E}_{\mathbf{h}_0|\lambda_0} \left[ e^{-\gamma/\sin^2\theta} \right] d\theta \right] \\ &= \mathbb{E}_{\lambda_0} \left[ \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma|\lambda_0} \left( -\frac{1}{\sin^2\theta} \right) d\theta \right] \\ &= \int_{N_0}^\infty \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma|\lambda_0} \left( -\frac{1}{\sin^2\theta} \right) d\theta f_{\lambda_0}(\lambda_0) d\lambda_0 \end{aligned} \quad (10)$$

where using the fact that  $\lambda_0$  is chi-squared distributed, the probability density function (PDF)  $f_{\lambda_0}(\lambda_0)$  is given by

$$f_{\lambda_0}(\lambda_0) = \frac{(\lambda_0 - N_0)^{N-1} e^{-\frac{\lambda_0 - N_0}{E_I}}}{(N-1)! E_I^N}, \quad \lambda_0 \geq N_0. \quad (11)$$

If CCI is slow faded, then the CCI channel gains do not change, i.e.,  $g_1^{(I,D,t_0)} = g_1^{(I,D,t_1)} = \dots = g_1^{(I,D,t_N)}$ . In this case, the random eigenvalue of  $\mathbf{R}$  becomes  $\lambda_0 = N_0 + N E_I \left| g_1^{(I,D,t_0)} \right|^2$ . Therefore, for the average SER  $P_e$  at the destination, the only change is the PDF  $f_{\lambda_0}(\lambda_0)$ , which is given by

$$f_{\lambda_0}(\lambda_0) = \frac{1}{NE_I} e^{-\frac{\lambda_0 - N_0}{NE_I}}, \quad \lambda_0 \geq N_0. \quad (12)$$

With this PDF  $f_{\lambda_0}(\lambda_0)$ , the average SER  $P_e$  at the destination is computed similarly. Note that slow fading CCI is the one of the main assumptions of the previous research. We present the effects of fast fading CCI on the performance of DF relaying, compared with slow fading CCI.

We define the SNR as  $\Gamma_0 \triangleq E_S / N_0$  and the interference-to-noise-ratio INR as  $\Gamma_1 \triangleq E_I / N_0$ . It is assumed that two relays correctly decode the symbol and forward, with  $\Gamma_0 = \Gamma_1$ . In Fig. 2, the average SER performance is shown versus the SNR  $\Gamma_0$ . We observe in Fig. 2 that the average SER performance of fast fading CCI is worse than that of slow fading CCI. The gap is about 4 dB. This amount loss of the SNR  $\Gamma_0$  is not negligible. We also show simulation results, which are in good agreement with analytical results. For simulations, enough errors are collected, over 100 errors at each SNR  $\Gamma_0$  and the coherent BPSK modulation is used. The channel parameters of fast fading CCI are generated independently using different random number generators, based on the coherence time  $T_c^{(fast)} = t_c$ , while the channel parameter of slow fading CCI is generated one time and used

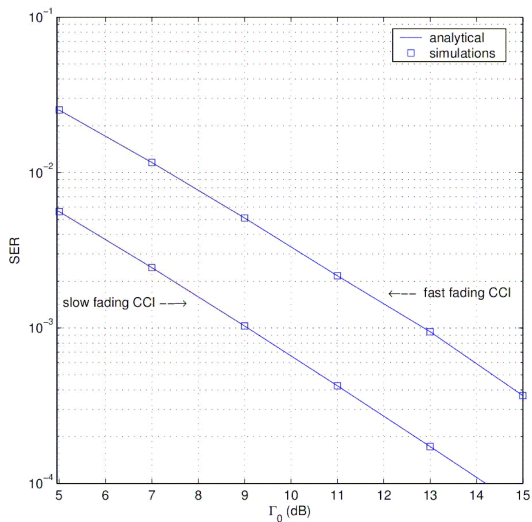


Fig. 2. SER in the presence of fast/slow fading CCI with  $\Gamma_0 = \Gamma_1$  and two relays.

repeatedly over the transmission duration, with the coherence time  $T_c^{(slow)} = t_c N$ .

#### IV. Conclusion

In this paper, we investigated the effects of fast/slow fading CCI on the performance of the DF relaying OC system. We first developed the MGF of the instantaneous maximum output SINR and derived closed-form expressions for the average SER at the destination. With these analytical expressions, it was shown that the average SER performance of fast fading CCI is worse than that of slow fading CCI. In result, DF relaying systems should be designed with consideration of fast fading CCI.

#### References

- [1] J. N. Laneman, D. N. C. Tse, and G.W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [2] W. C. Jakes Jr. et al., *Microwave Mobile Communications*. New York: Wiley, 1974.
- [3] J. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE Trans. Veh. Technol.*, vol. 33, pp. 144-155, Aug. 1984.
- [4] M. S. Gilan and A. Olfat, "New beamforming and space-time coding for two-path successive decode and forward relaying," *IET commun.* vol. 12, no. 13, pp. 1573-1588, 2018.
- [5] P. Kumar and K. Dhaka, "Performance analysis of wireless powered DF relay system under Nakagami-m Fading," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7073-7085, 2018.
- [6] X. Jin, "Successive interference cancellation on zero-forcing receiver in two-hop relay channel," *J. KICS*, vol. 42, no. 10, pp. 1947-1950, Oct. 2017.
- [7] X. Jin, "Source power allocation on end-node zero-forcing beamforming in two-hop relay channel," *J. KICS*, vol. 42, no. 8, pp. 1521-

1527, Aug. 2017.

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