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Channel Capacity for NOMA under BPSK Modulation

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ABSTRACT

We present the channel capacity for non-orthogonal multiple access (NOMA). This paper considers the binary phase shift keying (BPSK) modulation, compared to the ideal signal modulation. It is shown that the channel capacity of the BPSK modulation is different from that of the ideal signal modulation. In result, the power should be allocated effectively, based on the channel capacity of the BPSK modulation.

Key Words : Non-orthogonal multiple access, channel capacity, binary phase shift keying, multiple-input multiple-output, power allocation.

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) has received tremendous attention for the design of radio access techniques for fifth generation (5G) wireless networks[1,5]. In NOMA, multiple users utilizes resources non-orthogonally to improve a high spectral efficiency while allowing some degree of multiple access interference at receivers. Among different NOMA technologies, power-domain NOMA is the most frequently used, in which the user with better channel condition employs successive interference cancelation (SIC) to detect and remove the signals of users with worse channel conditions, and in turn improves its own signal reception[2]. By appropriately allocating the transmit power at the base station to multiple users with diverse channel conditions, NOMA can achieve a balance between network throughput and user fairness. To further improve spectral efficiency and reduce latency, NOMA combined with multiple-input multiple-output (MIMO), namely MIMO-NOMA, is considered as a key technology in the 5G communication system[3]. In [6], a new dynamic bandwidth adjusting algorithm is suggested for NOMA to provide the backward compatibility for orthogonal multiple access (OMA). The performance of a NOMA adopted two-cell downlink communication scenario is studied in [7].

Most of previous researches analyze NOMA systems, based on the ideal signal modulation, in which the number of signal levels are infinite and the signal is Gaussian distributed. This assumption could result in misleading understandings. In this paper, we consider the binary phase shift keying (BPSK) modulation, with binary signal levels. It is shown that the channel capacity of the practical signal modulation is quite different from that of the ideal signal modulation. Therefore, the power allocation should be changed, according to the channel capacity of the practical signal modulation. The paper is organized as follows. Section II defines the system and channel model. In Section III, the channel capacities are derived for the practical signal modulation. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Given the total transmit power P, αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with

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$\mathbb{E}\left[\left|s_1\right|^2\right]=\mathbb{E}\left[\left|s_2\right|^2\right]=1$ and $0 \le \alpha \le 1$. The superimposed signal is expressed by

$$x=\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2. \tag{1}$$

The received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= \left|h_1\right|x+n_1 \\ r_2 &= \left|h_2\right|x+n_2 \end{aligned} \tag{2}$$

where h_1 and h_2 are the channel gains, n_1 and $n_2 \sim \mathcal{N}\left(0,N\right)$ are additive white Gaussian noise (AWGN), where the notation $\mathcal{N}\left(\mu,\Sigma\right)$ denotes the normal distribution with mean μ and variance Σ. Assuming $\left|h_1\right|>\left|h_2\right|$, after the SIC is performed on the user-1 wi

th the better channel condition, the received signals of the user-1 and the user-2 are given as

where $n_3=\left|h_2\right|\sqrt{\alpha P}s_1+n_2$.

Ⅲ. Channel Capacity Calculations

The channel capacity is defined as [4]

$$C=\max_{p_X(x)} H(y)-H(n), \tag{4}$$

where $p_X(x)$ is the probability density function (PDF), the entropy of *x* is $H(x)=-\mathbb{E}_x\left[\log_2 p_X(x)\right]$, $y=x+n$ and *n* is AWGN. The previous researches for NOMA calculate the channel capacity based on the ideal signal modulation, i.e., $x \sim \mathcal{N}\left(0,\mathbb{E}\left[\left|x\right|^2\right]\right)$. The channel capacities in bits/s/Hz for the user-1 and the user-2 are calculated as

$$\begin{aligned} C_1 &= \frac{1}{2}\log_2\left(1+\frac{\left|h_1\right|^2\alpha P}{N}\right) \\ C_2 &= \frac{1}{2}\log_2\left(1+\frac{\left|h_2\right|^2(1-\alpha)P}{\left|h_2\right|^2\alpha P+N}\right). \end{aligned} \tag{5}$$

Note that for the ideal signal modulation, the number of signal levels is infinite, i.e., $-\infty<x<\infty$.

In this paper, we consider the practical signal modulation. Specifically, the BPSK is assumed. In this case, $s_1,s_2 \in \{+1,-1\}$ and the channel capacity for the user-1 is calculated with $p_{S_1}(s_1)=\frac{1}{2}\delta(s_1-1)+\frac{1}{2}\delta(s_1+1)$ by

where $p_{Y_1}(y_1)$ is calculated as

where $\delta(x)$ is the Dirac delta function. The channel capacity for the user-2 is calculated with $p_{S_2}(s_2)=\frac{1}{2}\delta(s_2-1)+\frac{1}{2}\delta(s_2+1)$ by

where $p_{Y_2}(y_2)$ is calculated as

and $p_{N_3}(n_3)$ is calculated as

$$\begin{aligned} y_1 &= \left|h_1\right|\sqrt{\alpha P}s_1+n_1 \\ y_2 &= r_2=\left|h_2\right|\left(\sqrt{\alpha P}s_1+\sqrt{(1-\alpha)P}s_2\right)+n_2=\left|h_2\right|\sqrt{(1-\alpha)P}s_2+\left(\left|h_2\right|\sqrt{\alpha P}s_1+n_2\right)=\left|h_2\right|\sqrt{(1-\alpha)P}s_2 \end{aligned} \tag{3}$$

$$\begin{aligned} C_1^{(b)} &= \max_{p_{S_1}(s_1)} H(y_1)-H(n_1) \\ &= -\int_{-\infty}^{\infty} p_{Y_1}(y_1)\log_2\left(p_{Y_1}(y_1)\right)dy_1+\int_{-\infty}^{\infty} p_{N_1}(n_1)\log_2\left(p_{N_1}(n_1)\right)dn_1 \\ &= -\int_{-\infty}^{\infty} p_{Y_1}(y_1)\log_2\left(p_{Y_1}(y_1)\right)dy_1-\frac{1}{2}\log_2\left(2\pi eN\right), \end{aligned} \tag{6}$$

$$\begin{aligned} p_{Y_1}(y_1) &= \int_{-\infty}^{\infty} p_{Y_1|S_1}(y_1)p_{S_1}(s_1)\,ds_1 \\ &= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi N}}e^{-\frac{\left(y_1-\left|h_1\right|\sqrt{\alpha P}s_1\right)^2}{2N}}\left(\frac{1}{2}\delta(s_1-1)+\frac{1}{2}\delta(s_1+1)\right)ds_1 \\ &= \frac{1}{2}\frac{1}{\sqrt{2\pi N}}e^{-\frac{\left(y_1-\left|h_1\right|\sqrt{\alpha P}\right)^2}{2N}}+\frac{1}{2}\frac{1}{\sqrt{2\pi N}}e^{-\frac{\left(y_1+\left|h_1\right|\sqrt{\alpha P}\right)^2}{2N}}. \end{aligned} \tag{7}$$

Ⅳ. Results and Discussions

Assume that the channel gain of the user-1 is ten times as strong as that of the user-2, with $|h_1| = 2$ and $|h_2| = 0.2$. The total transmit signal power to noise power ratio (SNR) is given as $P / N = 30$. The channel capacity pairs $\left(C_1, C_2\right)$ and $\left(C_1^{(b)}, C_2^{(b)}\right)$ are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. The power allocated to the strong user is αP and the power allocated to the weak user is $(1-\alpha)P$. As shown in Fig. 1, the channel capacity pair $\left(C_1^{(b)}, C_2^{(b)}\right)$ is quite different from the channel capacity pair $\left(C_1, C_2\right)$. The channel capacity $C_1^{(b)}$ reaches the maximum 1 bits/s/Hz of the BPSK. Note that this maximum can be obtained with only 5 % power allocation ($\alpha = 0.05$) to the strong user. Increasing the power of the strong user from 5 % ($\alpha = 0.05$) to 100 % ($\alpha = 1$) does not increase $C_1^{(b)}$, which results in the severe decrease of $C_2^{(b)}$. Based on the channel capacity pair $\left(C_1, C_2\right)$, however, such predictions are not considered.

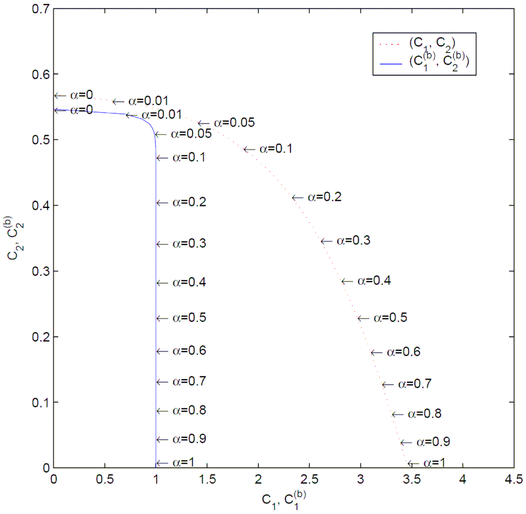

Fig. 1. Channel capacities of the ideal signal modulation and the practical signal modulation with various power allocations.

Ⅴ. Conclusion

In this paper, we calculated the channel capacity for NOMA under the practical signal modulation.

$$
\begin{aligned}
C_2^{(b)} &= \max_{p_{S_2}(s_2)} H(y_2) - H(n_3) \\
&= -\int_{-\infty}^{\infty} p_{Y_2}(y_2)\log_2\left(p_{Y_2}(y_2)\right) dy_2 + \int_{-\infty}^{\infty} p_{N_3}(n_3)\log_2\left(p_{N_3}(n_3)\right) dn_3
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
p_{Y_2}(y_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{Y_2|S_1,S_2}(y_2) p_{S_1,S_2}(s_1,s_2)\, ds_1\, ds_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{Y_2|S_1,S_2}(y_2) p_{S_1}(s_1) p_{S_2}(s_2)\, ds_1\, ds_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(y_2 - |h_2|\sqrt{\alpha P}s_1 - |h_2|\sqrt{(1-\alpha)P}s_2\right)^2}{2N}} \left(\frac{1}{2}\delta(s_1-1) + \frac{1}{2}\delta(s_1+1)\right)\left(\frac{1}{2}\delta(s_2-1) + \frac{1}{2}\delta(s_2+1)\right) ds_1\, ds_2 \\
&= \frac{1}{4}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(y_2 - |h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P}\right)^2}{2N}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(y_2 + |h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P}\right)^2}{2N}} \\
&\quad + \frac{1}{4}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(y_2 - |h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P}\right)^2}{2N}} + \frac{1}{4}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(y_2 + |h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P}\right)^2}{2N}}.
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
p_{N_3}(n_3) &= \int_{-\infty}^{\infty} p_{N_3|S_1}(n_3) p_{S_1}(s_1)\, ds_1 \\
&= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(n_3 - |h_2|\sqrt{\alpha P}s_1\right)^2}{2N}} \left(\frac{1}{2}\delta(s_1-1) + \frac{1}{2}\delta(s_1+1)\right) ds_1 \\
&= \frac{1}{2}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(n_3 - |h_2|\sqrt{\alpha P}\right)^2}{2N}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N}} e^{-\frac{\left(n_3 + |h_2|\sqrt{\alpha P}\right)^2}{2N}}.
\end{aligned} \tag{10}
$$

This capacity was compared to the capacity with the ideal signal modulation. It was shown that the channel capacity of the practical signal modulation is different from that of the ideal signal modulation. In result, the power should be allocated effectively, based on the channel capacity of the practical signal modulation.

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