Optimal Detection for NOMA Weak Channel User

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ABSTRACT

We present the optimal detection for the weak channel user of non-orthogonal multiple access (NOMA) with two users. This paper compares the optimal detection to the standard detection. It is shown that the performance of the optimal detection is better than that of the standard detection. In result, the optimal detection could be a promising scheme for the receiver of the NOMA weak channel user.

Key Words : Non-orthogonal multiple access, optimal detection, maximum likelihood, binary phase shift keying, power allocation.

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) has been recognized as one of promising multiple access techniques for fifth generation (5G) networks due to its superior spectral efficiency [1-5]. In NOMA, the user with the better channel condition employs successive interference cancelation (SIC) to remove the signals of users with the worse channel conditions, whereas the user with the weaker channel condition treats the other user's signal as noise and decodes its own signal. In this paper, the optimal detection for the weak channel user of NOMA with two users is proposed, considering the statistical structure of the other user's signal. The paper is organized as follows. Section II defines the system and channel model. In Section III, the optimal detection is derived for the user with the weaker channel condition. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \quad (1)$$

After the SIC is performed on the user-1 with the better channel condition, the received signal of the strong channel user is given by

$$r_1 = |h_1| \sqrt{\alpha P} s_1 + n_1 \quad (2)$$

where $n_1 \sim \mathcal{N}(0, N_0/2)$ is additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ and N_0 is one-sided power spectral density. The SIC is not performed on the user-2 with the worse channel condition. Then the received signal of the weak channel user is given by

$$\begin{aligned} r_2 &= |h_2| \left(\sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 \right) + n_2 \\ &= |h_2| \sqrt{(1-\alpha)P} s_2 + |h_2| \sqrt{\alpha P} s_1 + n_2 \end{aligned} \quad (3)$$

where $n_2 \sim \mathcal{N}(0, N_0/2)$ is AWGN.

Ⅲ. Optimal Detection

We consider the binary phase shift keying (BPSK), with $s_1, s_2 \in \{+1, -1\}$. In the standard

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논문번호 : 201812-399-A-LU, Received December 31, 2018 ; Revised January 10, 2019; Accepted January 10, 2019

receiver, the user with the weak channel condition treats the other user's signal as noise and decodes its own signal. In this case, the decision region for $s_2 = +1$ is simply given by

$$r_2 > 0, \qquad \text{for all } \alpha. \tag{4}$$

The probability of error $P_e^{(standard)}$ for all α is calculated as

$$\begin{aligned} P_e^{(standard)} &= \frac{1}{2} Q\left(\frac{|h_2|\sqrt{P}\left(\sqrt{(1-\alpha)} - \sqrt{\alpha}\right)}{\sqrt{N_0/2}} \right) \\ &+ \frac{1}{2} Q\left(\frac{|h_2|\sqrt{P}\left(\sqrt{(1-\alpha)} + \sqrt{\alpha}\right)}{\sqrt{N_0/2}} \right). \end{aligned} \tag{5}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$.

Now, we derive the optimal receiver. The likelihoods $p_{R_2|S_2}(r_2 \mid s_2 = +1)$ and $p_{R_2|S_2}(r_2 \mid s_2 = -1)$ are expressed as

$$\begin{aligned} p_{R_2|S_2}(r_2 \mid s_2) &= \int_{-\infty}^{\infty} p_{R_2|S_2,S_1}(r_2 \mid s_2, s_1) p_{S_1}(s_1)\, ds_1 \\ &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 - |h_2|\sqrt{\alpha P}s_1\right)^2}{N_0}} \\ &\quad \times \left(\frac{1}{2}\delta(s_1 - 1) + \frac{1}{2}\delta(s_1 + 1) \right) ds_1 \\ &= \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 - |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P}s_2 + |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \end{aligned} \tag{6}$$

where $p_X(x)$ is the probability density function (PDF) and $\delta(x)$ is the Dirac delta function. The optimum detection is made, based on the maximum likelihood, as

$$s_2 = \underset{s_2 \in \{+1,-1\}}{\arg\max}\, p_{R_2|S_2}(r_2 \mid s_2). \tag{7}$$

If $\alpha < 0.5$, the one exact decision boundary, $r_2 = 0$, is obtained from

$$p_{R_2|S_2}(r_2 \mid s_2 = +1) = p_{R_2|S_2}(r_2 \mid s_2 = -1), \tag{8}$$

which is

$$\begin{aligned} &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P} - |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P} + |h_2|\sqrt{\alpha P}\right)^2}{N_0}} = \\ &\frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 + |h_2|\sqrt{(1-\alpha)P} - |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{\pi N_0}} e^{-\frac{\left(r_2 + |h_2|\sqrt{(1-\alpha)P} + |h_2|\sqrt{\alpha P}\right)^2}{N_0}}. \end{aligned} \tag{9}$$

Then the decision region for $s_2 = +1$ is given by

$$r_2 > 0, \qquad \text{if } \alpha < 0.5. \tag{10}$$

If $\alpha > 0.5$, however, the optimum detection in (7) has the three decision boundaries, as follows; the first exact decision boundary, $r_2 = 0$, is the same as that in case of $\alpha < 0.5$. In order to obtain the second approximate decision boundary, first taking the logarithm to the both sides of the equation (9), we have

$$\begin{aligned} &\log\left(e^{-\frac{\left(r_2 + |h_2|\sqrt{(1-\alpha)P} + |h_2|\sqrt{\alpha P}\right)^2}{N_0}} + e^{-\frac{\left(r_2 + |h_2|\sqrt{(1-\alpha)P} - |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \right) \\ &= \log\left(e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P} + |h_2|\sqrt{\alpha P}\right)^2}{N_0}} + e^{-\frac{\left(r_2 - |h_2|\sqrt{(1-\alpha)P} - |h_2|\sqrt{\alpha P}\right)^2}{N_0}} \right) \end{aligned} \tag{11}$$

Then we can have the following equation,

$$-\frac{\left(r_2+\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}$$
$$+\log\left(1+e^{-\frac{\left(r_2+\left|h_2\right|\sqrt{(1-\alpha)P}+\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}+\frac{\left(r_2+\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}}\right)$$
$$=$$
$$-\frac{\left(r_2-\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}$$
$$+\log\left(1+e^{-\frac{\left(r_2-\left|h_2\right|\sqrt{(1-\alpha)P}+\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}+\frac{\left(r_2-\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\right)^2}{N_0}}\right). \tag{12)}$$

After some algebraic manipulations, the second approximate decision boundary, $r_2 \simeq \left|h_2\right|\sqrt{\alpha P}$, is obtained from

$$4\left(r_2-\left|h_2\right|\sqrt{\alpha P}\right)\left|h_2\right|\sqrt{(1-\alpha)P}\,/\,N_0 = \log\frac{1+e^{-\frac{4\left(r_2+\left|h_2\right|\sqrt{(1-\alpha)P}\right)\left|h_2\right|\sqrt{\alpha P}}{N_0}}}{1+e^{-\frac{4\left(r_2-\left|h_2\right|\sqrt{(1-\alpha)P}\right)\left|h_2\right|\sqrt{\alpha P}}{N_0}}} \tag{13}$$

where we use the fact that the negative exponentials are almost zeros. Similarly, the third approximate decision boundary is $r_2 \simeq -\left|h_2\right|\sqrt{\alpha P}$. Then, the decision region for $s_2=+1$ is given by

$$\begin{cases}-\left|h_2\right|\sqrt{\alpha P} < r_2 < 0 \\ \left|h_2\right|\sqrt{\alpha P} < r_2\end{cases},\quad \text{if } \alpha > 0.5. \tag{14}$$

The probability of error $P_e^{(optimal)}$ for $\alpha < 0.5$ is calculated as

$$P_e^{(optimal)} = \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) \tag{15)}$$

and for $\alpha > 0.5$,

$$P_e^{(optimal)} \simeq Q\left(\frac{\left|h_2\right|\sqrt{P}\sqrt{(1-\alpha)}}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}+2\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}-2\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) - \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right) - \frac{1}{2}Q\left(\frac{\left|h_2\right|\sqrt{P}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right). \tag{16}$$

Note that for $\alpha < 0.5$, $P_e^{(optimal)}$ is exactly the same as $P_e^{(standard)}$.

Ⅳ. Results and Discussions

Assume that the channel gain of the user-2 is $\left|h_2\right| = 0.9$. The total transmit signal power to one-sided power spectral density ratio is $P/N_0 = 50$. Then we define the signal-to-noise ratio (SNR) as $\gamma \triangleq \left|h_2\right|^2(1-\alpha)P/N_0$. The probabilities of error $P_e^{(standard)}$ and $P_e^{(optimal)}$ are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. As shown in Fig. 1, The performance of the optimal detection is quite better than that of the standard receiver for $\gamma < 13.06\,\text{dB}$, $(\alpha > 0.5)$. The local maximum of $P_e^{(optimal)}$ is at $\alpha = 0.5$, which is obtained from

$$-\left|h_2\right|\sqrt{(1-\alpha)P}+\left|h_2\right|\sqrt{\alpha P} = \left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}. \tag{17)}$$

On the other hand, the local minimum of $P_e^{(optimal)}$ is at $\alpha = 4/5$, $(\gamma = 9.08\,\text{dB})$, which is obtained from

$$\begin{aligned}&\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\\&=(-\left|h_2\right|\sqrt{(1-\alpha)P}-\left|h_2\right|\sqrt{\alpha P}\\&\quad-\left|h_2\right|\sqrt{(1-\alpha)P}+\left|h_2\right|\sqrt{\alpha P})\ /\ 2.\end{aligned} \tag{18}$$

We also show simulation results in Fig. 1, which are in good agreement with analytical results.

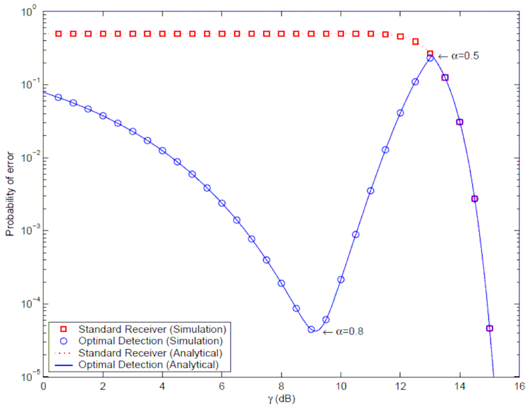

Fig. 1. Probabilities of error for the standard receiver and the optimum detection.

V. Conclusion

We presented the optimal detection for the weak channel user of NOMA with two users. This paper compared the optimal detection to the standard detection. It was shown both in computer simulations and by analytical expressions that the performance of the optimal detection is better than that of the standard detection. In result, the optimal detection could be a promising scheme for the receiver of the NOMA weak channel user.

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