

Performance Analysis on Non-Perfect SIC in NOMA

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ABSTRACT

We present non-orthogonal multiple access (NOMA) with non-perfect successive interference cancellation (SIC). It is shown that the performance of non-perfect SIC NOMA is almost the same as that of perfect SIC NOMA for the power allocation factor less than 20 %. However, the performance of non-perfect SIC NOMA becomes worse than that of perfect SIC NOMA for the power allocation factor greater than 20 %. Consequently, NOMA should be designed to take into account non-perfect SIC.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying

I. Introduction

As a promising candidate for the fifth generation (5G) mobile networks, non-orthogonal multiple access (NOMA) has been considered for high system capacity and low latency^[1,2,5-7]. In NOMA, the performance is greatly dependent on the successive interference cancellation (SIC). Recently, the NOMA SIC receiver for a downlink channel is implemented in [8]. However, in [8], only the average computation time for the SIC implementation is presented by the simulation without the derivation of analytical expressions. In this paper, the performance of non-perfect SIC NOMA is compared to that of perfect SIC NOMA with the analytical expression derivation and we show how non-perfect SIC affects the performance of NOMA. The paper is organized as follows. Section II defines the system and channel model. In

Section III, the performance of non-perfect SIC NOMA is derived. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

II. System and Channel Model

Assume that the total transmit power is P , the power allocation factor is α with $0 \leq \alpha \leq 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1 - \alpha)P$ is allocated to the user-2 signal s_2 , with $\mathbb{E}[|s_1|^2] = \mathbb{E}[|s_2|^2] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1 - \alpha)P} s_2. \tag{1}$$

Before SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1| \sqrt{\alpha P} s_1 + (|h_1| \sqrt{(1 - \alpha)P} s_2 + n_1) \\ r_2 &= |h_2| \sqrt{(1 - \alpha)P} s_2 + (|h_2| \sqrt{\alpha P} s_1 + n_2) \end{aligned} \tag{2}$$

where n_1 and $n_2 \sim \mathcal{CN}(0, N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The notation $\mathcal{CN}(\mu, \Sigma)$ denotes the complex circularly-symmetric normal distribution with mean μ and variance Σ . In standard NOMA, SIC is performed only on the user-1. Then the received signals are given by, with perfect SIC,

$$\begin{aligned} y_1 &= |h_1| \sqrt{\alpha P} s_1 + n_1 \\ y_2 &= r_2. \end{aligned} \tag{3}$$

We consider the binary phase shift keying (BPSK)

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modulation, with $s_1, s_2 \in \{+1, -1\}$.

III. Non Perfect SIC Receiver

Standard NOMA assumes perfect SIC. However, the SIC errors are inevitable due to the decoding errors of the inter user-1 interference. The best we can do for SIC is the maximum likelihood (ML) decoding of the inter user-1 interference. Then we consider the probability of errors for the user-1 with non-perfect SIC as

$$\begin{aligned}
 & P_e^{(1; \text{non-perfect SIC})} \\
 &= \left(1 - P_e^{(2; ML)}\right) P_e^{(1; \text{perfect SIC})} \\
 &+ \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=+1)} P_e^{(1; \text{non-perfect SIC; } s_2=+1, s_1=+1)} \\
 &+ \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=-1)} P_e^{(1; \text{non-perfect SIC; } s_2=+1, s_1=-1)}
 \end{aligned} \tag{4}$$

where $P_e^{(1; \text{perfect SIC})}$ is the probability of errors for the user-1 with the perfect SIC, and $P_e^{(2; ML)}$ is the probability of errors for the inter user-1 interference with the ML decoding, which is expressed as

$$P_e^{(2; ML)} = \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=+1)} + \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=-1)}. \tag{5}$$

The probabilities of errors $P_e^{(1; \text{non-perfect SIC; } s_2=+1, s_1=+1)}$ and $P_e^{(1; \text{non-perfect SIC; } s_2=+1, s_1=-1)}$ are for the user-1 with non-perfect SIC, conditioned on $(s_2 = +1, s_1 = +1)$ and $(s_2 = +1, s_1 = -1)$, respectively. The reason why these conditions are required is that $P_e^{(2; ML)}$ and $P_e^{(1; \text{non-perfect SIC})}$ are not independent, while $P_e^{(2; ML)}$ and $P_e^{(1; \text{perfect SIC})}$ are independent. Then $P_e^{(1; \text{perfect SIC})}$ is simply the probability of errors for the BPSK modulation, for all α ,

$$P_e^{(1; \text{perfect SIC})} = Q\left(\frac{|h_1| \sqrt{\alpha P}}{\sqrt{N_0 / 2}}\right) \tag{6}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. The probability of errors for the user-2 with the ML decoding over the weak channel h_2 is presented in [3]. We present the results over the strong channel h_1 as the probability of error $P_e^{(2; ML)}$ for the inter user-1 interference, with the conditioning, for $\alpha < 0.5$,

$$\begin{aligned}
 P_e^{(2; ML)} &= \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=+1)} \\
 &+ \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=-1)} \\
 &= \frac{1}{2} Q\left(\frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} + \sqrt{\alpha})}{\sqrt{N_0 / 2}}\right) \\
 &+ \frac{1}{2} Q\left(\frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} - \sqrt{\alpha})}{\sqrt{N_0 / 2}}\right)
 \end{aligned} \tag{7}$$

and for $\alpha > 0.5$,

$$\begin{aligned}
 P_e^{(2; ML)} &= \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=+1)} \\
 &+ \frac{1}{2} P_e^{(2; ML; s_2=+1, s_1=-1)} \\
 &\approx \frac{1}{2} \left(Q\left(\frac{|h_1| \sqrt{(1-\alpha)P}}{\sqrt{N_0 / 2}}\right) \right. \\
 &- Q\left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0 / 2}}\right) \\
 &+ Q\left(\frac{|h_1| \sqrt{P} (2\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0 / 2}}\right) \Big) \\
 &+ \frac{1}{2} \left(Q\left(\frac{|h_1| \sqrt{(1-\alpha)P}}{\sqrt{N_0 / 2}}\right) \right. \\
 &+ Q\left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0 / 2}}\right) \\
 &- Q\left(\frac{|h_1| \sqrt{P} (2\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0 / 2}}\right) \Big).
 \end{aligned} \tag{8}$$

We note that if the correct SIC is performed, the decision region is represented by, for all α , for

$$(s_2 = +1, s_1 = +1),$$

$$r_1 - |h_1| \sqrt{(1-\alpha)P} = y_1 > 0. \quad (9)$$

However, if the wrong SIC is performed, the decision region is severely distorted as, for all α ,

$$\text{for } (s_2 = +1, s_1 = +1),$$

$$r_1 + |h_1| \sqrt{(1-\alpha)P} = y_1 + 2|h_1| \sqrt{(1-\alpha)P} > 0. \quad (10)$$

The probabilities of errors

$$P_e^{(1; \text{non-perfect SIC}; s_2=+1, s_1=+1)} \quad \text{and}$$

$$P_e^{(1; \text{non-perfect SIC}; s_2=+1, s_1=-1)} \quad \text{are given by}$$

$$P_e^{(1; \text{non-perfect SIC}; s_2=+1, s_1=+1)} = Q \left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} + 2\sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \quad (11)$$

and

$$P_e^{(1; \text{non-perfect SIC}; s_2=+1, s_1=-1)} = Q \left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} - 2\sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right). \quad (12)$$

These probabilities are calculated based on the wrong SIC, for all α , for $(s_2 = +1, s_1 = +1)$,

$$r_1 + |h_1| \sqrt{(1-\alpha)P} < 0 \quad (13)$$

and for $(s_2 = +1, s_1 = -1)$,

$$r_1 + |h_1| \sqrt{(1-\alpha)P} > 0 \quad (14)$$

with the conditional probability density functions (PDFs),

$$p_{R_1|s_2,s_1}(r_1 | s_2 = +1, s_1 = +1) = \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_1 - |h_1| \sqrt{\alpha P} - |h_1| \sqrt{(1-\alpha)P})^2}{2N_0/2}} \quad (15)$$

and

$$p_{R_1|s_2,s_1}(r_1 | s_2 = +1, s_1 = -1) = \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{(r_1 + |h_1| \sqrt{\alpha P} - |h_1| \sqrt{(1-\alpha)P})^2}{2N_0/2}}. \quad (16)$$

For comparison, we present the probability of errors for the user-1 with the ML decoding over the strong channel h_1 in [4] as follows, for $\alpha > 0.5$,

$$P_e^{(1; \text{non-SIC ML})} = \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \quad (17)$$

and for $\alpha < 0.5$,

$$P_e^{(1; \text{non-SIC ML})} \simeq Q \left(\frac{|h_1| \sqrt{\alpha P}}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} - \sqrt{\alpha})}{\sqrt{N_0/2}} \right) - \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} + \sqrt{\alpha})}{\sqrt{N_0/2}} \right) - \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (2\sqrt{(1-\alpha)} - \sqrt{\alpha})}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left(\frac{|h_1| \sqrt{P} (2\sqrt{(1-\alpha)} + \sqrt{\alpha})}{\sqrt{N_0/2}} \right). \quad (18)$$

IV. Results and Discussions

Assume that the channel gain is $|h_1| = 2.5$. The

total transmit signal power to one-sided power spectral density ratio is $P / N_0 = 20$. The probabilities of errors with perfect SIC and non-perfect SIC for the user-1 are shown in Fig. 1, with different power allocations, $0 \leq \alpha \leq 1$. As shown in Fig. 1, the performance of non-perfect SIC NOMA is almost the same as that of perfect SIC NOMA for the power allocation factor less than 20%. However, the performance of non-perfect SIC NOMA becomes worse than that of perfect SIC NOMA for the power allocation factor greater than 20%. We also show in Fig. 1 the performance of non-SIC ML for the user-1 for comparison. It is reasonable that the performance of non-perfect SIC does not outperform that of non-SIC ML, because ML is optimal in that it minimizes the probability of errors.

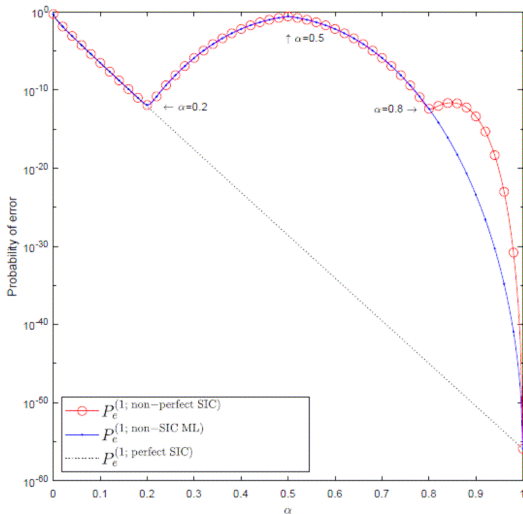


Fig. 1. Probabilities of errors with perfect SIC, non-perfect SIC, and non-SIC ML for the user-1.

V. Conclusion

We presented NOMA with non-perfect SIC. It was shown that the performance of non-perfect SIC NOMA is almost the same as that of perfect SIC NOMA for the power allocation factor less than 20%. However, the performance of non-perfect SIC NOMA became worse than that of perfect SIC

NOMA for the power allocation factor greater than 20%. In result, NOMA should be designed with consideration of non-perfect SIC.

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