

# Performance Comparison of NOMA to Cooperative NOMA

Kyuhuk Chung\*

## ABSTRACT

Recently, a newly deployed communications paradigm, cooperative communications, such as decode-and-forward (DF) relaying, have been one of promising candidates for the fifth generation (5G) mobile network. Another strong candidate for 5G radio access is non-orthogonal multiple access (NOMA). In NOMA, successive interference cancellation (SIC) is performed on the strong channel user to compensate for small power allocation. Then as the by-product of SIC, DF relaying can be easily implemented in NOMA. This paper compares NOMA to DF NOMA. We show how much DF NOMA is better than NOMA. This DF NOMA gain is due to the use of the additional time slot resource. In order to compare the two schemes fairly, we assume that NOMA retransmits the information one more time with the additional time slot resource. The contributions of this paper, opposed to the existing researches based on the channel capacity, is that the analytical expressions for the bit error rate (BER) performance are derived and with the use of such analytical expressions, the performance comparisons can be made intensively and flexibly for various system scenarios and channel conditions under the fair conditions.

**Key Words** : Cooperative communications, non-orthogonal multiple access, decode-and-forward relaying, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying.

## I. Introduction

As a promising application to fifth generation (5G) mobile networks, such as non-orthogonal multiple access (NOMA)<sup>[1-5]</sup>, the cooperative NOMA<sup>[6]</sup> has been proposed recently. In NOMA, successive interference cancellation (SIC) should be performed on the user of the better channel condition, which could naturally be used in the decode-and-forward (DF) relaying cooperative NOMA. A historical development is presented as follows; In 5G mobile communications, the number of users served in a single cell coverage increases dramatically, and in turn a new paradigm is required to accommodate the increased number of users. However, channel resources, such as time, frequency, code, and space, are already fully utilized in orthogonal multiple access (OMA), i.e., time division multiple access (TDMA), frequency division multiple access (FDMA), orthogonal frequency division multiplexing (OFDM), code division multiple access (CDMA), and multiple input multiple output (MIMO). Therefore the standard body for 5G mobile networks has considered new techniques. A candidate for this requirement is NOMA, which is the superposition based multi-user access technique, to provide high system capacity and low latency. It is also called as Multi-User Superposition Transmission (MUST). In NOMA, the users in the stronger channel conditions perform SIC to remove the inter user interference. In this case, in order to improve the system performance, the decoded inter user interference can be forwarded to the corresponding users. This motivates DF NOMA. This paper compares the cooperative NOMA to NOMA and we show how much the gain of the DF NOMA is over NOMA. The paper is organized as follows. Section II defines the system and channel model. In Section III, the comparison of the DF NOMA to NOMA is made. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

\* First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 종신회원  
논문번호 : 201903-032-A-LU, Received March 27, 2019; Revised April 4, 2019; Accepted April 8, 2019

## II. System and Channel Model

Suppose that a time division duplex (TDD) mode. DF NOMA system is composed of a source (S; base station), a relay (R; user-1), and a destination (D; user-2).

In phase 1, the base station (S) broadcasts the superimposed signal, which is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 \quad (1)$$

where  $P$  is the total transmit power, the power allocation factor is  $\alpha$  with  $0 \leq \alpha \leq 1$ , and  $\alpha P$  and  $(1-\alpha)P$  are allocated to the user-1 (R) signal  $s_1$  and the user-2 (D) signal  $s_2$ , respectively, with  $\mathbb{E}[|s_1|^2] = \mathbb{E}[|s_2|^2] = 1$ . Assume that the channel gains are  $h_1$ ,  $h_3$ , and  $h_2$  with  $|h_1| > |h_3| > |h_2|$ . Before SIC is performed on the user-1 (R) with the better channel condition, the received signals of the user-1 (R) and the user-2 (D) are represented as

$$\begin{aligned} r_1 &= |h_1| \sqrt{\alpha P} s_1 + (|h_1| \sqrt{(1-\alpha)P} s_2 + n_1) \\ r_2 &= |h_2| \sqrt{(1-\alpha)P} s_2 + (|h_2| \sqrt{\alpha P} s_1 + n_2) \end{aligned} \quad (2)$$

where  $n_1$  and  $n_2 \sim \mathcal{CN}(0, N_0)$  are complex additive white Gaussian noise (AWGN) and  $N_0$  is one-sided power spectral density. The notation  $\mathcal{CN}(\mu, \Sigma)$  denotes the complex circularly-symmetric normal distribution with mean  $\mu$  and variance  $\Sigma$ . The decision  $\hat{s}_2$  for DF and SIC is defined as

$$\hat{s}_2 = \arg \max_{s_2 \in \{+1, -1\}} p_{R_1|S_2}(r_1 | s_2) \quad (3)$$

where  $p_{R_1|S_2}(r_1 | s_2)$  is the probability density function (PDF) conditioned on  $s_2$ .

In phase 2, the relay (R) forwards the decoded symbol  $\hat{s}_2$ . Then the received signal of the user-2 (D) is expressed as

$$r_3 = |h_3| \sqrt{(1-\alpha)P} \hat{s}_2 + n_3. \quad (4)$$

where  $n_3 \sim \mathcal{CN}(0, N_0)$ . We consider the binary phase shift keying (BPSK) modulation, with  $s_1, s_2 \in \{+1, -1\}$ .

## III. Cooperative NOMA Performance

The best we can do for DF is the maximum likelihood (ML) decoding of the inter user-1 interference  $s_2(h_1)$  at the relay (R). The probability of errors  $P_e^{(2; h_1; ML)}$  for the inter user-1 interference  $s_2(h_1)$  with the ML decoding over the strong channel  $h_1$  is presented in [7], as follows; for  $\alpha < 0.5$ ,

$$\begin{aligned} P_e^{(2; h_1; ML)} &= \frac{1}{2} Q \left( \frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} + \sqrt{\alpha})}{\sqrt{N_0/2}} \right) \\ &+ \frac{1}{2} Q \left( \frac{|h_1| \sqrt{P} (\sqrt{(1-\alpha)} - \sqrt{\alpha})}{\sqrt{N_0/2}} \right) \end{aligned} \quad (5)$$

and for  $\alpha > 0.5$ ,

$$\begin{aligned} P_e^{(2; h_1; ML)} &\simeq Q \left( \frac{|h_1| \sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right) \\ &- \frac{1}{2} Q \left( \frac{|h_1| \sqrt{P} (\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \\ &+ \frac{1}{2} Q \left( \frac{|h_1| \sqrt{P} (2\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \\ &+ Q \left( \frac{|h_1| \sqrt{P} (\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \\ &- Q \left( \frac{|h_1| \sqrt{P} (2\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) \end{aligned} \quad (6)$$

where  $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$ . Now the

decision  $\hat{s}_2$  is forwarded to the user-2 (D). Then  $P_e^{(2; h_3; correct DF)}$  is simply the probability of errors for the BPSK modulation, for all  $\alpha$ ,

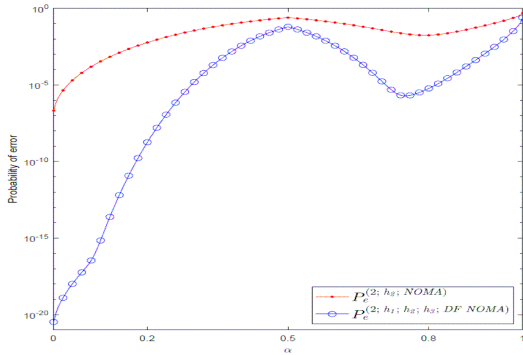


Fig. 1. Probabilities of errors of DF NOMA and NOMA for user-2 (D) with channel gains  $|h_1| = 1.4$ ,  $|h_3| = 1.2$ , and  $|h_2| = 0.8$ .

$$P_e^{(2; h_3; \text{correct DF})} = Q \left( \frac{|h_3| \sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right). \quad (7)$$

Then the user-2 (D) collects  $r_2$  and  $r_3$ , and decodes its own signal. The probability of errors  $P_e^{(2; h_2; NOMA)}$  for the user-2 (D) with the ML decoding over the weak channel  $h_2$  is presented in [8], for  $\alpha < 0.5$ ,

$$P_e^{(2; h_2; NOMA)} = \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (\sqrt{(1-\alpha)} + \sqrt{\alpha})}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (\sqrt{(1-\alpha)} - \sqrt{\alpha})}{\sqrt{N_0/2}} \right) \quad (8)$$

and for  $\alpha > 0.5$ ,

$$P_e^{(2; h_2; NOMA)} \simeq Q \left( \frac{|h_2| \sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right) - \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (2\sqrt{\alpha} + \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) + \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right) - \frac{1}{2} Q \left( \frac{|h_2| \sqrt{P} (2\sqrt{\alpha} - \sqrt{(1-\alpha)})}{\sqrt{N_0/2}} \right). \quad (9)$$

Then we consider the probability of errors  $P_e^{(2; h_1; h_2; h_3; DF NOMA)}$  for the user-2 (D) with the

non-perfect DF as

$$\begin{aligned} & P_e^{(2; h_1; h_2; h_3; DF NOMA)} \\ &= \left( 1 - P_e^{(2; h_1; ML)} \right) P_e^{(2; h_3; \text{correct DF})} P_e^{(2; h_2; NOMA)} \\ &+ P_e^{(2; h_1; ML)} P_e^{(2; h_3; \text{wrong DF})} P_e^{(2; h_2; NOMA)} \\ &= \left( 1 - P_e^{(2; h_1; ML)} \right) P_e^{(2; h_3; \text{correct DF})} P_e^{(2; h_2; NOMA)} \\ &+ P_e^{(2; h_1; ML)} \left( 1 - P_e^{(2; h_3; \text{correct DF})} \right) P_e^{(2; h_2; NOMA)} \\ &= P_e^{(2; h_3; \text{correct DF})} P_e^{(2; h_2; NOMA)} + P_e^{(2; h_1; ML)} P_e^{(2; h_2; NOMA)} \\ &- 2P_e^{(2; h_3; \text{correct DF})} P_e^{(2; h_1; ML)} P_e^{(2; h_2; NOMA)} \\ &= P_e^{(2; h_2; NOMA)} \\ &\times \underbrace{\left( P_e^{(2; h_3; \text{correct DF})} + P_e^{(2; h_1; ML)} - 2P_e^{(2; h_3; \text{correct DF})} P_e^{(2; h_1; ML)} \right)}_{DF \text{ NOMA gain over NOMA}} \end{aligned} \quad (10)$$

For the above derivation of the equation (10), we use the fact that  $r_1$ ,  $r_2$ , and  $r_3$  are independent on conditioned on  $s_2$ .

#### IV. Results and Discussions

Assume that the channel gains are  $|h_1| = 1.4$ ,  $|h_3| = 1.2$ , and  $|h_2| = 0.8$ . The total transmit signal power to one-sided power spectral density ratio is  $P/N_0 = 30$ . The probabilities of errors of DF NOMA and NOMA for the user-2 (D) are shown in Fig. 1, with different power allocations,  $0 \leq \alpha \leq 1$ . As shown in Fig. 1, the performance of DF NOMA is much better than that of NOMA for the entire operating range of the power allocation factor,  $0\% \leq \alpha \leq 100\%$ . Major difference between DF NOMA and NOMA is that there exists the additional DF channel of the user-1 (R) to the user-2 (D). The DF NOMA gain is due to the existence of this (R) to (D) channel. By using the additional channel, DF NOMA can outperform NOMA. Now, for fair comparison, we include the cost of adding this additional channel. If we add this additional channel, the user-2 (D) needs the additional time slot for DF relaying. In order to compare the two systems fairly, we assume the user-2 (D) in NOMA can use the additional time slot. In this case, NOMA system is assumed to retransmit the information one more time. In this retransmission scenario, the probability of errors

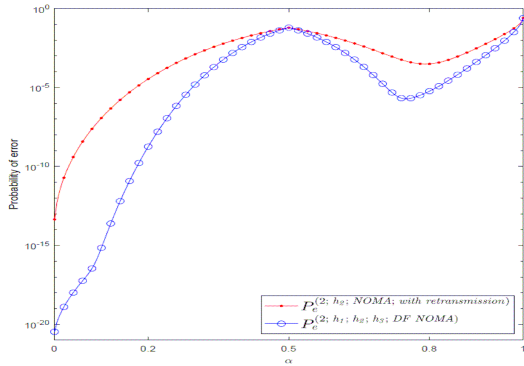


Fig. 2. Probabilities of errors of DF NOMA and NOMA for user-2 (D) with channel gains  $|h_1| = 1.4$ ,  $|h_3| = 1.2$ , and  $|h_2| = 0.8$  (NOMA uses retransmission for fair comparison).

$P_e^{(2; h_2; NOMA; with retransmission)}$  for the user-2 (D) is represented as

$$P_e^{(2; h_2; NOMA; with retransmission)} = P_e^{(2; h_2; NOMA)} \times P_e^{(2; h_2; NOMA)} \quad (11)$$

where we assume that the original transmission and the retransmission are statically independent. As shown in Fig. 2, now the DF NOMA gain over NOMA with retransmission decreases. In addition, we consider the comparison more fairly, with the DF relaying channel gain  $h_3$  being worse up to NOMA user-2 (D) channel gain  $h_2$ . As shown in Fig. 3, the DF NOMA gain over NOMA with retransmission dramatically decreases. Then we conjecture that the DF NOMA gain over NOMA mainly comes from the channel gain  $h_1$ .

### V. Conclusion

We compared the performance of DF NOMA to that of NOMA. It was shown that DF NOMA performs better than NOMA for the entire operating range of the power allocation factor. Consequently, DF NOMA could be considered for 5G and beyond mobile networks, with the by-product of SIC in NOMA.

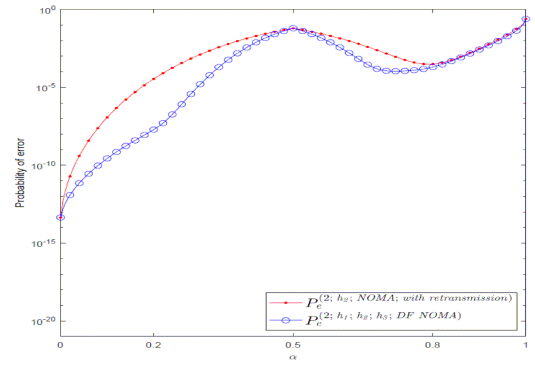


Fig. 3. Probabilities of errors of DF NOMA and NOMA for user-2 (D) with channel gains  $|h_1| = 1.4$ ,  $|h_3| = 0.8$ , and  $|h_2| = 0.8$  (NOMA uses retransmission for fair comparison).

### References

- [1] Y. Saito, et al., "Non- orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1-5, 2013.
- [2] Z. Ding, et al., "Impact of user pairing on 5G nonorthogonal multiple- access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010-6023, Aug. 2016.
- [3] S. R. Islam, et al., "On non-orthogonal multiple access (NOMA) in 5G systems," *J. KICS*, vol. 40, no. 12, pp. 2549-2558, Dec. 2015.
- [4] M. H. Lee, et al., "Dynamic bandwidth allocation of NOMA and OMA for 5G," *J. KICS*, vol. 42, no. 12, pp. 2383-2390, Dec. 2017.
- [5] M. B. Uddin, et al., "Power optimization of NOMA for multi-cell networks," *J. KICS*, vol. 43, no. 7, pp. 1182-1190, Jul. 2018.
- [6] Z. Ding, et al., "Cooperative non-orthogonal multiple access in 5G systems," *IEEE Commun. Lett.*, vol. 19, no. 8, pp. 1462-1465, Aug. 2015.
- [7] K. Chung (May 2019), "Performance analysis on Non-Perfect SIC in NOMA," *J. KICS*, vol. 44, no. 5, pp. 855-858, May 2019.
- [8] K. Chung, "Optimal detection for NOMA weak channel user," *J. KICS*, vol. 44, no. 2, pp. 270-273, Fec. 2019.