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SSC NOMA Capacity:Zero Capacity Bit to Symbol Mappingover Gaussian Mixture Channel

Kyuhyuk Chung•

ABSTRACT

Recently, the symmetric superposition coding (SSC)[6] is proposed for a solution for the error propagation (EP) due to the non-perfect successive interference cancellation (SIC) in non-orthogonal multiple access (NOMA). This paper calculates the channel capacity of NOMA with the SSC. First, we prove that the channel capacity of the strong channel user in the SSC NOMA is zero, if the perfect SIC is assumed. However, it is shown that if this zero capacity bit to symbol mapping is sent over Gaussian mixture channel, the SSC NOMA capacity is greater than that of NOMA with the normal superposition coding (NSC) for the power allocation factor greater than about 20 % or less than about 80 %. We also show that the capacityof the SSC NOMA is the same as that of the NSC NOMA for the power allocation factor less than about 20 %, and the SSC NOMA capacity is worse than the NSC NOMA capacity for the power allocation factor greater than about 80 %. As a result, the SSC should be used to take into account the power allocation.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, symmetric superposition coding, binary phase shift keying.

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) is the superposition based multi-user access technique for the fifth generation (5G) mobile networks, to provide high system capacity and low latency[1-5]. It is also called as Multi-User Superposition Transmission (MUST). In NOMA, the performance is greatly dependent on the successive interference cancellation (SIC). Recently, the symmetricsuperposition coding (SSC)[6] is proposed for a solution for the error propagation (EP) due to the non-perfect SIC. In this paper, the SSC NOMA capacity is compared to the capacity of NOMA with the normal superposition coding (NSC) and we show how the SSC affects the capacity of NOMA. The paper is organized as follows. Section II defines the system and channel model. In Section III, we prove that the channel capacity of the strong channel user in the SSC NOMA is zero, if the perfect SIC is assumed, and the capacity of the SSC NOMA is calculated. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \leq \alpha \leq 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{1}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P} s_1 + \left(|h_1|\sqrt{(1-\alpha)P} s_2 + n_1\right) = |h_1|\sqrt{\alpha P} s_1 + n_4 \\ r_2 &= |h_2|\sqrt{(1-\alpha)P} s_2 + \left(|h_2|\sqrt{\alpha P} s_1 + n_2\right) \\ &= |h_2|\sqrt{(1-\alpha)P} s_2 + n_3 \end{aligned} \tag{2}$$

where n_1 and $n_2 \sim \mathcal{N}\left(0, N_0/2\right)$ areadditive white Gaussian noise (AWGN) and N_0 is one-sided

• First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 종신회원
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power spectral density. The notation $\mathcal{N}(\mu,\Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the perfect SIC is performed only on the user-1. Then the received signals are given by

$$\begin{aligned} y_1 &= |h_1|\sqrt{\alpha P}s_1 + n_1 \\ y_2 &= r_2. \end{aligned} \tag{3}$$

We consider the binary phase shift keying (BPSK) modulation, with $s_1, s_2 \in \{+1,-1\}$. Let the information input bits for the user-1 and the user-2 be $b_1, b_2 \in \{0,1\}$. Then in normal superposition coding (NSC), the modulation signal mapping is normal as

$$\begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases}$$
$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}. \tag{4}$$

However, in symmetric superposition coding (SSC), the modulation signal mapping is changed as

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = +1 \\ s_1(b_1 = 1, b_2 = 0) = -1 \end{cases} \quad \begin{cases} s_1(b_1 = 0, b_2 = 1) = -1 \\ s_1(b_1 = 1, b_2 = 1) = +1 \end{cases}$$
$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}. \tag{5}$$

In this paper, the channel capacity is defined as the maximum mutual information, maximized with an input probability density function (PDF), without the shaping, (often called as the mutual information with equiprobable *M*-ary constellations[7]). The capacity of equiprobable *M*-ary constellations asymptotically approaches a straight line parallel to the capacity of the ideal Gaussian modulation[8], shifted right by $\pi e / 6$ (1.53 dB), which is the shaping loss. The capacity of equiprobable *M*-ary constellations saturates because information cannot be sent at a rate higher than $\log_2 M$.

Ⅲ. Zero Capacity Bit to Symbol Mapping and SSC NOMA Capacity

In this section, first, we prove that the channel capacity of the strong channel user in the SSC NOMA is zero, if the perfect SIC is assumed. Later, the capacity of the SSC NOMA is calculated.

In the SSC NOMA, it is interesting that if no power is allocated to the user-2 ($\alpha = 1$), then the user-1 cannot receive any information from the base station, even if all power is allocated to the user-1[9].

Theorem 1: The channel capacity of the strong channel user in the SSC NOMA is zero, if the perfect SIC is assumed.

Proof) If the perfect SIC is assumed, then the received signal for the strong channel user is given by

$$y_1 = |h_1|\sqrt{\alpha P}s_1 + n_1. \tag{6}$$

In this case, we consider the reasonable capacity in bit/s/Hz, for the channel

$$y_1 = |h_1|\sqrt{\alpha P}s_1(b_1) + n_1 \tag{7}$$

as

$$\begin{aligned} &C_1^{(b;\ perfect-SIC;\ SSC)}(b_1 \to s_1 \to y_1) = \max_{p_{B_1}(b_1)} H(y_1) - H(y_1 \mid b_1) \\ &= -\int_{-\infty}^{\infty} p_{Y_1}(y_1)\log_2 p_{Y_1}(y_1)\,dy_1 \\ &+\int_{-\infty}^{\infty} p_{Y_1|B_1}(y_1 \mid b_1 = 0)p_{B_1}(b_1 = 0)\log_2 p_{Y_1|B_1}(y_1 \mid b_1 = 0)\,dy_1 \\ &+\int_{-\infty}^{\infty} p_{Y_1|B_1}(y_1 \mid b_1 = 1)p_{B_1}(b_1 = 1)\log_2 p_{Y_1|B_1}(y_1 \mid b_1 = 1)\,dy_1 \end{aligned} \tag{8}$$

instead of

$$\begin{aligned} &C_1^{(b;\ perfect-SIC;\ SSC)}(s_1 \to y_1) = \max_{p_{S_1}(s_1)} H(y_1) - H(y_1 \mid s_1) \\ &= -\int_{-\infty}^{\infty} p_{Y_1}(y_1)\log_2 p_{Y_1}(y_1)\,dy_1 \\ &+\int_{-\infty}^{\infty} p_{Y_1|S_1}(y_1 \mid s_1 = +1)p_{S_1}(s_1 = +1)\log_2 p_{Y_1|S_1}(y_1 \mid s_1 = +1)\,dy_1 \\ &+\int_{-\infty}^{\infty} p_{Y_1|S_1}(y_1 \mid s_1 = -1)p_{S_1}(s_1 = -1)\log_2 p_{Y_1|S_1}(y_1 \mid s_1 = -1)\,dy_1 \end{aligned} \tag{9}$$

where the entropy $H(x) = -\mathbb{E}_x\left[\log_2 p_X(x)\right]$ and $p_X(x)$ is the PDF. Then the PDF of the received signal is represented as

$$p_{Y_1}(y_1) = p_{Y_1|B_1}(y_1 \mid b_1 = 0) = p_{Y_1|B_1}(y_1 \mid b_1 = 1)$$
$$= \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(y_1 - |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(y_1 + |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}}. \tag{10}$$

Note that y_1 and b_1 are independent. Therefore, $C_1^{(b;\, perfect-SIC;\, SSC)}(b_1 \rightarrow s_1 \rightarrow y_1) = 0.$ ***Q.E.D.***

However, if there exists the inter user-1 interference, the user-1 can receive the information as if there is no interference for $\alpha < 0.5$ [9]. In effect, the perfect SIC is achieved for $\alpha < 0.5$. The channel capacity for the SSC NOMA strong channel user is calculated by

$$C_1^{(b;\, non-SIC;\, SSC)} = \max_{p_{B_1}(b_1)} H(r_1) - H(r_1 \mid b_1)$$
$$= -\int_{-\infty}^{\infty} p_{R_1}(r_1)\log_2 p_{R_1}(r_1)dr_1$$
$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1 \mid b_1 = 0)p_{B_1}(b_1 = 0)\log_2 p_{R_1|B_1}(r_1 \mid b_1 = 0)\,dr_1$$
$$+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1 \mid b_1 = 1)p_{B_1}(b_1 = 1)\log_2 p_{R_1|B_1}(r_1 \mid b_1 = 1)\,dr_1 \tag{11}$$

where

$$p_{R_1|B_1}(r_1 \mid b_1 = 0) = \frac{1}{2}p_{R_1|B_2,B_1}(r_1 \mid b_2 = 0, b_1 = 0) + \frac{1}{2}p_{R_1|B_2,B_1}(r_1 \mid b_2 = 1, b_1 = 0)$$
$$= \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} \tag{12}$$

and

$$p_{R_1|B_1}(r_1 \mid b_1 = 1) = \frac{1}{2}p_{R_1|B_2,B_1}(r_1 \mid b_2 = 0, b_1 = 1) + \frac{1}{2}p_{R_1|B_2,B_1}(r_1 \mid b_2 = 1, b_1 = 1)$$
$$= \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} - |h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} + |h_1|\sqrt{(1-\alpha)P}\right)^2}{2N_0/2}}. \tag{13}$$

Note that we cannot use $H(n_4)$ instead of $H(r_1 \mid b_1)$, as in [8], *Theorem 16*, p. 43], because b_1 and n_4 are not independent,

$$p_{N_4}(n_4) \neq p_{N_4|B_1}(n_4 \mid b_1 = 0) \neq p_{N_4|B_1}(n_4 \mid b_1 = 1). \tag{14}$$

Also note that this channel is one of examples of the input signal and the noise signal being dependent.

Ⅳ. Results and Discussions

The channel capacity $C_1^{(b;\, non-SIC;\, NSC)}$ for the NSC NOMA strong channel user is calculated in [10] and the channel capacity $C_2^{(b)}$ for the NOMA weak channel user is calculated in [11]. Assume that the channel gains are $|h_1| = 1.5$ and $|h_2| = 0.5$. The total transmit signal power to one-sided power spectral densityratio is $P / N_0 = 15$, ($11.76\text{ dB} = 10\log_{10}(15)$). The channel capacity regions of the non-SIC NSC NOMA [10] and the non-SIC SSC NOMA under the BPSK modulation are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. As shown in Fig. 1, the SSC NOMA capacity is greater than that of NOMA with the NSC for the power allocation factor greater than about 20 % or less than about 80 %. It is also shown that the capacity of the SSC NOMA is the same as that of the NSC NOMA for the power allocation factor lessthan about 20 %. However, the SSC NOMA capacity is worse than the NSC NOMA capacity for the power allocation factor greater than about 80 %. It is reasonable that the SSC NOMA is compared to the NSC NOMA without SIC, because in the SSC NOMA, even if the inter user-1 interference signal is decoded and is subtracted from the received signal, we cannot decode the user-1 signal due to SSC[9], which is proven in this paper with zero capacity bit to symbol mapping

Ⅴ. Conclusion

This paper calculated the channel capacity of

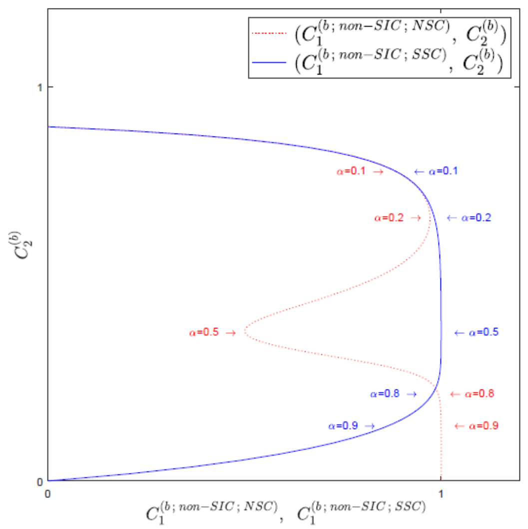

Fig. 1. Channel capacity regions of the non-SIC NSC NOMA and the non-SIC SSC NOMA under the BPSK

NOMA with SSC. We proved that the channel capacity of the strong channel user in the SSC NOMA is zero, if the perfect SIC is assumed. However, it was shown than if this zero capacity bit to symbol mapping is sent over Gaussian mixture channel, the SSC NOMA capacity is greater than that of NOMA with the NSC for the power allocation factor greater than about 20 % or less than about80 %. We also showed that the capacity of the SSC NOMA is the same as that of the NSC NOMA for the power allocation factor less than about 20 %, and the SSC NOMA capacity is worse than the NSC NOMA capacity for the power allocation factor greater than about 80 %. As a result, the SSC should be used to take into account the power allocation.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1-5, 2013.

[2] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010-6023, Aug. 2016.

[3] S. R. Islam, J. M. Kim, and K. S. Kwak, "On non-orthogonal multiple access (NOMA) in 5G systems," *J. KICS*, vol. 40, no. 12, pp. 2549-2558, Dec. 2015.

[4] M. H. Lee, V. C. M. Leung, and S. Y. Shin, "Dynamic bandwidth allocation of NOMA and OMA for 5G," *J. KICS*, vol. 42, no. 12, pp. 2383-2390, Dec. 2017.

[5] M. B. Uddin, M. F. Kader, A. Islam, and S. Y. Shin, "Power optimization of NOMA for multi-cell networks," *J. KICS*, vol. 43, no. 7, pp. 1182-1190, Jul. 2018.

[6] H. Li, Z. Huang, Y. Xiao, S. Zhan, and Y. Ji, "Solution for error propagation in a NOMA-based VLC network: symmetric superposition coding," *Opt. Exp.*, vol. 25, no. 24, pp. 29856-29863, Nov. 2017.

[7] G. D. Forney Jr. and G. Ungerboeck, "Modulation and coding for linear Gaussian channels," *IEEE Trans. Inform. Theory*, vol. 44, pp. 2384-2415, Oct. 1998.

[8] C. E. Shannon, "A mathematical theory of communication," *The Bell Syst. Tech. J.*, vol. 27, no. 3, pp. 379-423, Oct. 1948.

[9] K. Chung, "Performance analysis of NOMA with symmetric superposition coding," *J. IKEEE*, vol. 23, no. 1, pp. 314-317, Mar. 2019.

[10] K. Chung (in press Apr. 2019), "Non-SIC NOMA: Gaussian mixture channel capacity under BPSK and QPSK modulations," *J. KICS*, vol. 44, no. 4, pp. 666-669, Apr. 2019.

[11] K. Chung, "On channel capacity for NOMA under BPSK modulation," *J. KICS*, vol. 44, no. 2, pp. 208-211, Feb. 2019.