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Orthogonal NOMA

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ABSTRACT

We invent polar on-off keying (Polar OOK; POOK). This modulation is the inter user interference dependent OOK. If there exists interference, polar OOK gets away from interference in the direction from the origin to interference. Then one of the users in non-orthogonal multiple access (NOMA) can be served orthogonally in power domain. In this case NOMA comes back partially to orthogonal multiple access (OMA), with help of polar OOK, namely power-domain orthogonal NOMA (O NOMA).

Key Words : On-off keying, non-orthogonal multiple access, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) has been a promising communication technique for fifth generation (5G) mobile networks with high system capacity and low latency[1-5]. Such gains over orthogonal multiple access (OMA) stand on the perfect successive interference cancellation (SIC). However, the existing practical modulation techniques can hardly achieve the perfect SIC performance on the stronger channel user. In this paper, we invent polar on-off keying (Polar OOK; POOK) to achieve the perfect SIC performance with maximum likelihood (ML) decoding. If it is possible to do so, NOMA can be again categorized as OMA partially, namely power-domain orthogonal NOMA (O NOMA). We should mention the motivation of this paper; is it meaningful to transmit the superimposed binary phase shift keying (BPSK) signals of two users, compared to the quadrature phase shift keying (QPSK) signal of one user? The positive answer for this question motivates this paper as follows; If two BPSK signals can be transmitted over the same channel resource (here over power domain), we increase the system capacity double. In addition, for QPSK modulations, we can consider the superimposed QPSK signals. The paper is organized as follows. Section II defines the system and channel model. In Section III, polar OOK is presented and the performance of O NOMA is calculated. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is given by

$$x = \sqrt{\alpha P}s_1 + \sqrt{(1-\alpha)P}s_2. \tag{1}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals are represented as

$$\begin{aligned} z_1 &= h_1\sqrt{\alpha P}s_1 + \left(h_1\sqrt{(1-\alpha)P}s_2 + w_1\right) \\ z_2 &= h_2\sqrt{(1-\alpha)P}s_2 + \left(h_2\sqrt{\alpha P}s_1 + w_2\right) \end{aligned} \tag{2}$$

where w_1 and $w_2 \sim \mathcal{CN}(0, N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The notation $\mathcal{CN}(\mu, \Sigma)$ denotes the complex circularly symmetric normal distribution with mean μ and variance Σ. In addition, if the channel gains are assumed to be Rayleigh faded, then h_1 and $h_2 \sim \mathcal{CN}(0, 1^2)$. The coherent receivers of Rayleigh fading channels constructs the following metrics from the received signals;

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$$\begin{aligned} h_1^* z_1 &= |h_1|^2 \sqrt{\alpha P} s_1 + \left(|h_1|^2 \sqrt{(1-\alpha)P} s_2 + h_1^* w_1 \right) \\ h_2^* z_2 &= |h_2|^2 \sqrt{(1-\alpha)P} s_2 + \left(|h_2|^2 \sqrt{\alpha P} s_1 + h_2^* w_2 \right). \end{aligned} \tag{3}$$

Furthermore, the receivers process the above metrics one step more;

$$\begin{aligned} \frac{h_1^*}{|h_1|} z_1 &= |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + \frac{h_1^*}{|h_1|} w_1 \right) \\ \frac{h_2^*}{|h_2|} z_2 &= |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + \frac{h_2^*}{|h_2|} w_2 \right). \end{aligned} \tag{4}$$

Note that the noise $\frac{h_1^*}{|h_1|} w_1$ and $\frac{h_2^*}{|h_2|} w_2$ have the same statistics as w_1 and w_2, because $\frac{h_1^*}{|h_1|} = e^{j\theta}$ with θ uniformly distributed in $[0, \pi)$. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 &= \mathrm{Re}\left\{ \frac{h_1^*}{|h_1|} z_1 \right\} = |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + \mathrm{Re}\left\{ \frac{h_1^*}{|h_1|} w_1 \right\} \right) \\ &= |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + n_1 \right) \\ r_2 &= \mathrm{Re}\left\{ \frac{h_2^*}{|h_2|} z_2 \right\} = |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + \mathrm{Re}\left\{ \frac{h_2^*}{|h_2|} w_2 \right\} \right) \\ &= |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + n_2 \right) \end{aligned} \tag{5}$$

where n_1 and $n_2 \sim \mathcal{N}\left(0, N_0 / 2\right)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}\left(\mu, \Sigma\right)$ denotes the normal distribution with mean μ and variance Σ.

Ⅲ. O NOMA Performance

On-off keying (OOK) is the simplest modulation technique. The carrier is sent or not. Assume the BPSK modulation for the user-1, with $s_1 \in \{+1, -1\}$. Then Polar OOK, with $s_2 \in \{+\sqrt{2}, 0, -\sqrt{2}\}$, is the inter user interference s_1 dependent OOK. (The normalized power is $\mathbb{E}\left[|s_2|^2\right] = \frac{1}{4}\left(+\sqrt{2}\right)^2 + \frac{1}{2}\left(\sqrt{0}\right)^2 + \frac{1}{4}\left(-\sqrt{2}\right)^2 = 1$.) (Compare the standard OOK, $s_{OOK} \in \{+\sqrt{2}, 0\}$with $\mathbb{E}\left[|s_{OOK}|^2\right] = \frac{1}{2}\left(\sqrt{2}\right)^2 + \frac{1}{2}\left(0\right)^2 = 1$.) If there exists interference, polar OOK gets away from interference in the direction from the origin to interference. Therefore we give polarity to OOK, with the information input bits for the user-1 and the user-2 being $b_1, b_2 \in \{0, 1\}$, as

$$\begin{gathered} \begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases} \\ \begin{cases} s_2(b_2 = 0 \mid b_1 = 0) = +\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 0) = 0 \end{cases} \quad \begin{cases} s_2(b_2 = 0 \mid b_1 = 1) = -\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 1) = 0. \end{cases} \end{gathered} \tag{6}$$

Let us start the performance analysis of the user-1, which suffers the non-perfect SIC. In [6], it is shown that the performance of the non-perfect SIC user-1 receiver does not outperform that of the non-SIC ML user-1 receiver, because ML is optimal in that it minimizes the probability of errors. Therefore, in the practical environments, the best we can do for the user-1 is the non-SIC ML receiver, of which the performance $P_e^{(1;\, NOMA;\, non-SIC\ ML;\, practical)}$ is presented in [7]. If the perfect SIC is assumed, then the performance is simply the probability of errors of the BPSK modulation, for all α,

$$P_e^{(1;\, NOMA;\, perfect\ SIC;\, ideal)} = Q\left(\frac{|h_1| \sqrt{\alpha P}}{\sqrt{N_0 / 2}} \right) \tag{7}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Now we derive the performance, the probability of errors for the O NOMA user-1; The likelihood $p_{R_1|B_1}(r_1 \mid b_1 = 0)$ is expressed as

$$\begin{aligned} p_{R_1|B_1}(r_1 \mid b_1 = 0) &= \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} - |h_1|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} \end{aligned} \tag{8}$$

where $p_X(x)$ is the probability density function (PDF). The likelihood $p_{R_1|B_1}(r_1 \mid b_1 = 1)$ is expressed as

$$\begin{aligned} p_{R_1|B_1}(r_1 \mid b_1 = 1) &= \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}}. \end{aligned} \tag{9}$$

The ML detection is made as

$$\hat{b}_1 = \underset{b_1 \in \{0,1\}}{\arg\max}\, p_{R_1|B_1}(r_1 \mid b_1). \tag{10}$$

The equal likelihood equation is given by

$$p_{R_1|B_1}(r_1 \mid b_1 = 0) = p_{R_1|B_1}(r_1 \mid b_1 = 1) \tag{11}$$

which is

$$\begin{aligned} &\frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P} - |h_1|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} = \\ &\frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P}\right)^2}{2N_0/2}} + \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} e^{-\frac{\left(r_1 + |h_1|\sqrt{\alpha P} + |h_1|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}}. \end{aligned} \tag{12}$$

The equal likelihood equation (12) has the one exact decision boundary, $r_1 = 0$, which is obtained directly from the equation (12). Then, the decision region for $b_1 = 0$ is given by

$$r_1 > 0, \qquad \text{for all } \alpha. \tag{13}$$

The probability of error $P_e^{(1;\, O\ NOMA)}$ is calculated as, for all α,

$$P_e^{(1;\,O\ NOMA)} = \frac{1}{2}Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{2(1-\alpha)}+\sqrt{\alpha}\right)}{\sqrt{N_0/2}}\right). \quad (14)$$

Note that

$$P_e^{(1;\,O\ NOMA)} \le P_e^{(1;\,NOMA;\,perfect\ SIC;\,ideal)} \quad (15)$$

where the equality holds for $\alpha = 1$. Such the performance improvement is due to the non-interfering superposition coding of polar OOK. Actually, with the equation (15), O NOMA performs even better than the ideal perfect SIC NOMA for the user-1. This gain is the by-product of searching for the perfect SIC NOMA. One more comment on the performance of the user-1 in O NOMA is that as shown in the decision region in the equation (13), effectively there is no need for SIC, i.e., orthogonal in power domain.

Now we derive the probability of errors for the user-2 in O NOMA; The likelihood $p_{R_2|B_2}(r_2 \mid b_2 = 0)$ is expressed as

$$p_{R_2|B_2}(r_2 \mid b_2 = 0) = \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}-|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}+|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}}. \quad (16)$$

The likelihood $p_{R_2|B_2}(r_2 \mid b_2 = 1)$ is expressed as

$$p_{R_2|B_2}(r_2 \mid b_2 = 1) = \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}}. \quad (17)$$

The ML detection is made as

$$\hat{b}_2 = \underset{b_2\in\{0,1\}}{\arg\max}\, p_{R_2|B_2}(r_2 \mid b_2). \quad (18)$$

The equal likelihood equation is given by

$$p_{R_2|B_2}(r_2 \mid b_2 = 0) = p_{R_2|B_2}(r_2 \mid b_2 = 1). \quad (19)$$

For all α, the equal likelihood equation (19) has the two decision boundaries as follows; The first approximate decision boundary, $r_2 \simeq |h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P/2}$, is obtained from

$$\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}-|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} = \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \quad (20)$$

where we use the following observation, at $r_2 = |h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P/2}$, for all α,

$$e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}+|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \simeq 0,\ e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \simeq 0. \quad (21)$$

Similarly, the second approximate decision boundary, $r_2 \simeq -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P/2}$, is obtained from

$$\frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}+|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} = \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2+|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \quad (22))$$

where we use the following observation, at $r_1 = -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P/2}$, for all α,

$$e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}-|h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \simeq 0,\ e^{-\frac{\left(r_2-|h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \simeq 0. \quad (23)$$

Then, the decision region for $b_2 = 0$ is given by, for all α,

$$\begin{cases} r_2 > +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P/2} \\ r_2 < -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P/2} \end{cases} \quad (24)$$

and the decision region for $b_2 = 1$ is given by, for all α,

$$\begin{cases} r_2 < +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P/2} \\ r_2 > -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P/2} \end{cases}. \quad (25)$$

The probability of errors for the user-2 in O NOMA, for all α,

$$P_e^{(2;\,O\ NOMA)} \simeq Q\left(\frac{|h_2|\sqrt{(1-\alpha)P/2}}{\sqrt{N_0/2}}\right) - \frac{1}{2}Q\left(\frac{|h_2|\sqrt{P}\left(2\sqrt{\alpha}+3\sqrt{(1-\alpha)/2}\right)}{\sqrt{N_0/2}}\right) + \frac{1}{2}Q\left(\frac{|h_2|\sqrt{P}\left(2\sqrt{\alpha}+\sqrt{(1-\alpha)/2}\right)}{\sqrt{N_0/2}}\right). \quad (26)$$

Ⅳ. Results and Discussions

Assume that the channel gains are $|h_1| = 1.1$ and $|h_2| = 0.9$. The total transmit signal power to one-sided power spectral density ratio is $P/N_0 = 15$. The probabilities of errors with the non-perfect SIC NOMA in [6], non-SIC ML NOMA in [7] and the perfect SIC NOMA in the equation (7) are compared to that of O NOMA in the equation (14), for the user-1, in Fig. 1, with different power

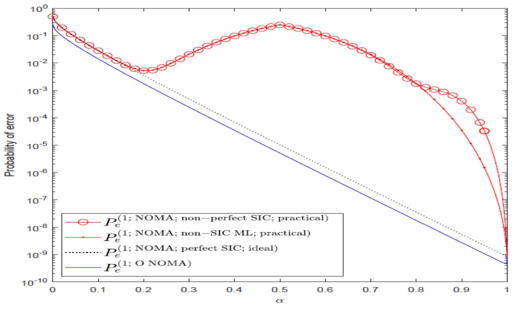

Fig. 1. Probabilities of errors with non-perfect SIC/ non-SIC ML/perfect SIC NOMA and O NOMA for the user-1.

allocations, $0 \le \alpha \le 1$. As shown in Fig. 1, for all α, the performance of O NOMA is better than that of the perfect SIC NOMA for the user-1. We also compare the probability of errors with ML NOMA for the user-2 in [8] to that of O NOMA for the user-2 in the equation (26), in Fig. 2. As shown in Fig. 2, for the user-2, the probability of errors of O NOMA is better than that of ML NOMA for the power allocation factor greater than about 10% and less than about 70%. Note that NOMA operates usually on $10\% \le \alpha \le 70\%$. An additional comment on $10\% \le \alpha \le 70\%$ is that even if we increase the power enormously, the NOMA performance never improve in the vicinity of $\alpha = 50\%$. However, in O NOMA, the performance can improve linearly. Lastly, we should mention the weakness of OOK, thus, weakness of polar OOK. In nature, OOK is not a constant-amplitude modulation, which is not a good property for modulations. Such non-constant amplitude property of polar OOK could be understood as the price for improved performance. In future works, improving NOMA performance with

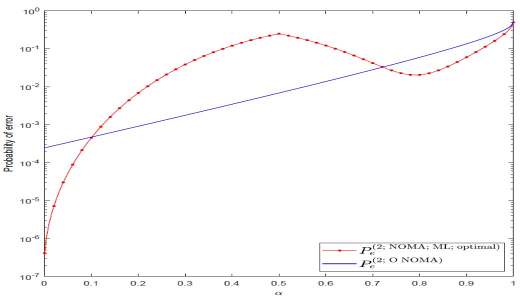

Fig. 2. Probabilities of errors with ML NOMA and O NOMA for the user-2.

constant-amplitude modulation schemes will be interesting and precious.

Ⅴ. Conclusion

We invented Polar OOK. With this modulation, power-domain orthogonal NOMA (O NOMA) was proposed. It was shown that, for all α, the performance of O NOMA user-1 is better than that of the ideal perfect SIC NOMA user-1. We also showed that the performance of O NOMA user-2 is better than that of the optimal ML NOMA user-2, for about $0.1 \le \alpha \le 0.7$. Therefore we achieved the perfect SIC performance, and in turn one of the users could be served orthogonally in power domain. Consequently, NOMA could be categorized as OMA partially, with help of polar OOK. In future work, as mentioned in introduction section, for the QPSK scenario, it is meaningful to research the superimposed QPSK modulations.

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