논문 19-44-08-04 The Journal of Korean Institute of Communications and Information Sciences '19-08 Vol.44 No.08
https://doi.org/10.7840/kics.2019.44.8.1467

Polar Multilevel On-Off Keying in NOMA

Kyuhyuk Chung◆

ABSTRACT

To increase the rate of transmission, it is natural to consider the multilevel modulations. However, in non-orthogonal multiple access (NOMA), the straight application of multilevel modulations can be hardly made due to the severe effect of the superposition. Recently, to mitigate the destructive impact of the superposition, polar on-off keying (POOK) and quadrature POOK (QPOOK) have been proposed in [1-2]. This paper expands such concept to multilevel modulations, e.g., polar 4-ary on-off keying (P4OOK), which is compared to the 4-ary pulse amplitude modulation (4PAM). It is shown that first, for the strong channel user with binary phase shift keying (BPSK), the BPSK/P4OOK NOMA performance is better than the perfect successive interference cancellation (SIC) performance of BPSK/4PAM NOMA, and second, for the weak channel user, the BPSK/P4OOK NOMA performance is comparable to the perfect SIC BPSK/4PAM NOMA. In result, P4OOK could be considered for NOMA.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying.

Ⅰ. Introduction

ALL channel resources for multiple accesses, such as time, frequency, space, and code, have been fully exploited for the current communication systems. Thus the standard bodies have been searching for other possible channel resources for fifth generation (5G) and beyond mobile networks. One candidate is power, i.e., non-orthogonal multiple access (NOMA) [3-7]. In NOMA, the superposition principle is applied for the power-domain. At the same time, such principle is a limitation to the NOMA performance. The severe effect of the superposition is recently mitigated with the concept of polarity modulation, such as polar on-off keying (POOK) [1] and quadrature POOK (QPOOK) [2]. We propose the polar multilevel on-off keying (OOK), e.g., polar 4-ary OOK (P4OOK). The contributions of this paper are summarized as follows; first, we invent the 4-ary OOK (4OOK), which is the multilevel version of OOK and corresponds to the existing modulation scheme of the 4-ary pulse amplitude modulation (4PAM). Second, we also invent P4OOK, which is the multilevel extension of POOK. The paper is organized as follows. Section II defines the system and channel model. In Section III, the BPSK/P4OOK NOMA performance is derived. Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is given by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{1}$$

Before the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signals are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P} s_1 + \left(|h_1|\sqrt{(1-\alpha)P} s_2 + n_1\right) \\ r_2 &= |h_2|\sqrt{(1-\alpha)P} s_2 + \left(|h_2|\sqrt{\alpha P} s_1 + n_2\right) \end{aligned} \tag{2}$$

◆ First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 종신회원
논문번호 : 201905-055-A-LU, Received May 1, 2019; Revised May 22, 2019; Accepted May 22, 2019

where n_1 and $n_2 \sim \mathcal{N}(0, N_0/2)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signals are given by, if the perfect SIC is assumed,

$$y_1 = r_1 - |h_1|\sqrt{(1-\alpha)P}s_2 = |h_1|\sqrt{\alpha P}s_1 + n_1 \quad (3)$$

Ⅲ. BPSK/P4OOK NOMA Performance Derivations

Assume the binary phase shift keying (BPSK) modulation for the user-1, with $s_1 \in \{+1, -1\}$. Now, we design P4OOK. We consider 4PAM, with

$$s_{\text{4PAM}} \in \left\{+\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\}. \quad (4)$$

The power is normalized as

$$\mathbb{E}\left[|s_{\text{4PAM}}|^2\right] = \frac{1}{4}\left(\frac{3}{\sqrt{5}}\right)^2 + \frac{1}{4}\left(\frac{1}{\sqrt{5}}\right)^2 + \frac{1}{4}\left(-\frac{1}{\sqrt{5}}\right)^2 + \frac{1}{4}\left(-\frac{3}{\sqrt{5}}\right)^2 = 1. \quad (5)$$

Then the corresponding 4OOK is represented as

$$s_{\text{4OOK}} \in \left\{0, +\frac{2}{\sqrt{14}}, +\frac{4}{\sqrt{14}}, +\frac{6}{\sqrt{14}}\right\}. \quad (6)$$

The power is normalized as

$$\mathbb{E}\left[|s_{\text{4OOK}}|^2\right] = \frac{1}{4}(0)^2 + \frac{1}{4}\left(\frac{2}{\sqrt{14}}\right)^2 + \frac{1}{4}\left(\frac{4}{\sqrt{14}}\right)^2 + \frac{1}{4}\left(\frac{6}{\sqrt{14}}\right)^2 = 1. \quad (7)$$

Then P4OOK with

$$s_{\text{P4OOK}} \in \left\{-\frac{6}{\sqrt{14}}, -\frac{4}{\sqrt{14}}, -\frac{2}{\sqrt{14}}, 0, +\frac{2}{\sqrt{14}}, +\frac{4}{\sqrt{14}}, +\frac{6}{\sqrt{14}}\right\} \quad (8)$$

is the inter user interference s_1 dependent 4OOK. The power is normalized as

$$\mathbb{E}\left[|s_{\text{P4OOK}}|^2\right] = \frac{2}{8}(0)^2 + \frac{1}{8}\left(-\frac{2}{\sqrt{14}}\right)^2 + \frac{1}{8}\left(-\frac{4}{\sqrt{14}}\right)^2 + \frac{1}{8}\left(-\frac{6}{\sqrt{14}}\right)^2 + \frac{1}{8}\left(\frac{2}{\sqrt{14}}\right)^2 + \frac{1}{8}\left(\frac{4}{\sqrt{14}}\right)^2 + \frac{1}{8}\left(\frac{6}{\sqrt{14}}\right)^2 = 1. \quad (9)$$

If there exists the inter user interference s_1, P4OOK gets away from the interference in the direction from the origin to interference. Therefore we give polarity to 4OOK, as

$$\begin{cases} s_2(s_1 = +1) = 0 \\ s_2(s_1 = +1) = +2/\sqrt{14} \\ s_2(s_1 = +1) = +4/\sqrt{14} \\ s_2(s_1 = +1) = +6/\sqrt{14} \end{cases} \quad \begin{cases} s_2(s_1 = -1) = 0 \\ s_2(s_1 = -1) = -2/\sqrt{14} \\ s_2(s_1 = -1) = -4/\sqrt{14} \\ s_2(s_1 = -1) = -6/\sqrt{14} \end{cases} \quad (10)$$

First, let us start the performance analysis of the user-1. If the perfect SIC is assumed, then the performance is simply the probability of errors of the BPSK modulation, for all α,

$$P_e^{(1;\,BPSK;\,BPSK/4PAM\ NOMA;\,perfect\ SIC;\,ideal)} = Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0/2}}\right) \quad (11)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Now we derive the probability of errors for the BPSK/P4OOK NOMA user-1. The likelihood $p_{R_1|S_1}(r_1 \mid s_1)$ is expressed as

$$\begin{aligned} p_{R_1|S_1}(r_1 \mid s_1) = &\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}s_1 - |h_1|\sqrt{(1-\alpha)P}\frac{6}{\sqrt{14}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}s_1 - |h_1|\sqrt{(1-\alpha)P}\frac{4}{\sqrt{14}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}s_1 - |h_1|\sqrt{(1-\alpha)P}\frac{2}{\sqrt{14}}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_1 - |h_1|\sqrt{\alpha P}s_1\right)^2}{2N_0/2}} \end{aligned} \quad (12)$$

where $p_X(x)$ is the probability density function (PDF). The maximum likelihood (ML) detection is made as

$$\hat{s}_1 = \underset{s_1 \in \{+1,-1\}}{\arg\max}\, p_{R_1|S_1}(r_1 \mid s_1). \quad (13)$$

For all α, there is the one exact decision boundary, $r_1 = 0$, which is obtained directly from the equation

(13). Then, the decision region for $s_1 = +1$ is given by

$$r_1 > 0, \qquad \text{for all } \alpha. \tag{14}$$

The probability of error $P_e^{(1;\, BPSK;\, BPSK/P4OOK\ NOMA;\, ML;\, practical)}$ is calculated as, for all α,

$$P_e^{(1;\, BPSK;\, BPSK/P4OOK\ NOMA;\, ML;\, practical)} = q_{h_1}^{(1;0)} + q_{h_1}^{(1;2)} + q_{h_1}^{(1;4)} + q_{h_1}^{(1;6)} \tag{15}$$

where for the simplification, we define the notation as

$$q_{h_1}^{(I;A)} = \frac{1}{4} Q\left(\frac{|h_1| \sqrt{P} \left(\sqrt{\alpha} \cdot I + \sqrt{(1-\alpha)} \cdot A / \sqrt{14} \right)}{\sqrt{N_0 / 2}} \right). \tag{16}$$

Note that

$$P_e^{(1;\, BPSK;\, BPSK/P4OOK\ NOMA;\, ML;\, practical)} \le P_e^{(1;\, BPSK;\, BPSK/4PAM\ NOMA;\, perfect\ SIC;\, ideal)} \tag{17}$$

where the equality holds for $\alpha = 1$. Such the performance improvement is due to the non-interfering superposition coding of P4OOK. Actually, with the equation (17), BPSK/P4OOK NOMA performs even better than the ideal perfect SIC BPSK/4PAM NOMA for the user-1. One more comment on the performance of the user-1 in BPSK/P4OOK NOMA is that as shown in the decision region in the equation (14), effectively there is no need for SIC, i.e., orthogonal in power domain.

Now, we derive the probability of errors for the user-2 in BPSK/P4OOK NOMA; the likelihood $p_{R_2|S_2}(r_2 \mid s_2)$ is expressed as

$$p_{R_2|S_2}(r_2 \mid s_2) = \frac{1}{2} \frac{1}{\sqrt{2\pi N_0 / 2}} \left(e^{-\frac{\left(r_2 - |h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} s_2\right)^2}{2N_0/2}} + e^{-\frac{\left(r_2 + |h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} s_2\right)^2}{2N_0/2}} \right). \tag{18}$$

The ML detection is made as

$$\hat{s}_2 = \underset{s_2 \in \left\{0, +\frac{2}{\sqrt{14}}, +\frac{4}{\sqrt{14}}, +\frac{6}{\sqrt{14}}\right\}}{\arg\max}\ p_{R_2|S_2}(r_2 \mid s_2). \tag{19}$$

For all α, there are the six approximate decision boundaries as follows;

$$\begin{aligned} r_2 &\simeq \pm|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 1 / \sqrt{14} \\ r_2 &\simeq \pm|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 3 / \sqrt{14} \\ r_2 &\simeq \pm|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 5 / \sqrt{14}. \end{aligned} \tag{20}$$

Such approximation errors are small and tolerable, resorting to the $68 - 95 - 99.7$ rule, for $\mathcal{N}\left(0, 1^2\right)$,

$$\begin{aligned} Q(3) &= \int_3^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \simeq 0.0015 \\ Q(2) &= \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \simeq 0.025 \\ Q(1) &= \int_1^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \simeq 0.16. \end{aligned} \tag{21}$$

Then, the decision region for $s_2 = 0$ is given by, for all α,

$$\begin{cases} r_2 < +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 1 / \sqrt{14} \\ r_2 > -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 1 / \sqrt{14} \end{cases} \tag{22}$$

and the decision region for $s_2 = +2 / \sqrt{14}$ is given by, for all α,

$$\begin{cases} +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 1 / \sqrt{14} < r_2 \\ < +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 3 / \sqrt{14}, \\ -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 3 / \sqrt{14} < r_2 \\ < -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 1 / \sqrt{14} \end{cases} \tag{23}$$

and the decision region for $s_2 = +4 / \sqrt{14}$ is given by, for all α,

$$\begin{cases} +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 3 / \sqrt{14} < r_2 \\ < +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 5 / \sqrt{14}, \\ -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 5 / \sqrt{14} < r_2 \\ < -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 3 / \sqrt{14} \end{cases} \tag{24}$$

and the decision region for $s_2 = +4 / \sqrt{14}$ is given by, for all α,

$$\begin{cases} r_2 > +|h_2|\sqrt{\alpha P} + |h_2|\sqrt{(1-\alpha)P} \cdot 5 / \sqrt{14} \\ r_2 < -|h_2|\sqrt{\alpha P} - |h_2|\sqrt{(1-\alpha)P} \cdot 5 / \sqrt{14}. \end{cases} \tag{25}$$

The probability of errors for the user-2 in BPSK/P4OOK NOMA, for all α,

$$P_e^{(2;\, P4OOK;\, BPSK/P4OOK\ NOMA;\, ML;\, practical)} \simeq 6 \cdot q_{h_2}^{(0;1)} + q_{h_2}^{(2;1)} - q_{h_2}^{(2;3)} + q_{h_2}^{(2;5)} - q_{h_2}^{(2;7)} + q_{h_2}^{(2;9)} - q_{h_2}^{(2;11)}. \tag{26}$$

Ⅳ. Results and Discussions

Assume that the channel gains are $\left|h_1\right| = 1.1$ and $\left|h_2\right| = 0.9$ and the total transmit signal power to one-sided power spectral density ratio is $P \,/\, N_0 = 15$. As shown in Fig. 1, for all α, the BPSK performance of BPSK/P4OOK NOMA is better than that in the standard BPSK/4PAM NOMA with the ideal perfect SIC for the user-1, i.e., even better than the BPSK performance itself. In addition, there is no perfect SIC to be implemented in the literature. Such outstanding BPSK performance of BPSK/P4OOK NOMA is the result of transforming effectively the inter user interference s_2 into the meaningful signal by P4OOK user-2. The numerical improvement example at the probability of errors $P_e = 10^{-3}$ is calculated as follows; we define the user-1 signal-to-noise ratio (SNR) as $\gamma_1 \triangleq \left|h_1\right|^2 \alpha P \,/\, N_0$. Then for the comparison, we use the notation $\gamma_1^{(standard\ NOMA)} \triangleq \left|h_1\right|^2 \alpha^{(standard\ NOMA)} P \,/\, N_0$ for the standard BPSK/4PAM NOMA with the ideal perfect SIC and $\gamma_1^{(proposed\ NOMA)} \triangleq \left|h_1\right|^2 \alpha^{(proposed\ NOMA)} P \,/\, N_0$

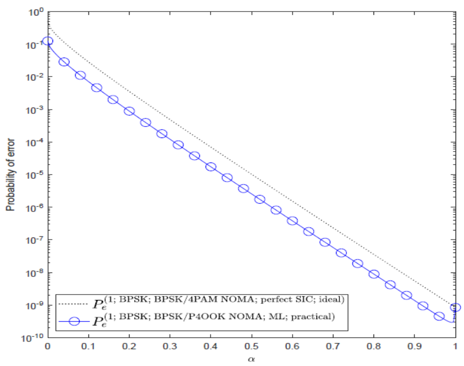

Fig. 1. Performance comparison of BPSK in BPSK/P4OOK NOMA to BPSK in BPSK/4PAM NOMA for the user-1, ($\left|h_1\right| = 1.1$).

for the proposed BPSK/P4OOK NOMA, respectively. The numerical improvement in SNR is given by

$$\left|h_1\right|^2 \alpha^{(standard\ NOMA)} P \,/\, N_0 - \left|h_1\right|^2 \alpha^{(proposed\ NOMA)} P \,/\, N_0. \tag{27}$$

Converting the improvement in dB,

$$\begin{aligned} &improvement\ (\mathrm{dB}) \\ &= 10\log_{10}\left|h_1\right|^2 \alpha^{(standard\ NOMA)} P \,/\, N_0 \\ &- 10\log_{10}\left|h_1\right|^2 \alpha^{(proposed\ NOMA)} P \,/\, N_0 \\ &= 10\log_{10}\left(\frac{\alpha^{(standard\ NOMA)}}{\alpha^{(proposed\ NOMA)}}\right). \end{aligned} \tag{28}$$

The power allocation factors are obtained numerically as

$$\begin{aligned} \alpha^{(standard\ NOMA)} &= 0.2607 \\ \alpha^{(proposed\ NOMA)} &= 0.1938 \end{aligned} \quad 29))$$

Then the improvement in dB is 1.2901 dB.

Now, we compare the user-2 performances. The performance to be compared to the proposed NOMA is the best ideal performance; there is no practical method to perform the perfect SIC for the user-2. Even in the standard NOMA, the SIC is not performed on the user-2. In fact, the NOMA capacity is achieved without the SIC for the user-2. However, in this paper, for the comparison purpose, the P4OOK performance of BPSK/P4OOK NOMA is compared to the 4PAM performance in the standard BPSK/4PAM NOMA with the ideal perfect SIC for the user-2. If the perfect SIC for the user-2 is assumed, then the received signal is given by,

$$y_2 = r_2 - \left|h_2\right|\sqrt{\alpha P} s_1 = \left|h_2\right|\sqrt{(1-\alpha)P} s_2 + n_2. \tag{30}$$

Then the performance is simply the probability of errors of the 4PAM modulation, for all α,

$$\begin{aligned} &P_e^{(2;\ 4PAM;\ BPSK/4PAM\ NOMA;\ perfect\ SIC;\ ideal)} \\ &= 6 \cdot \frac{1}{4} Q\left(\frac{\left|h_2\right|\sqrt{(1-\alpha)P} \,/\, \sqrt{5}}{\sqrt{N_0 \,/\, 2}}\right). \end{aligned} \tag{31}$$

As shown in Fig. 2, the P4OOK performance of BPSK/P4OOK NOMA is comparable to the 4PAM performance in the ideal perfect SIC BPSK/4PAM NOMA.

In addition, in order to investigate the proposed scheme for more various channel environments, we consider the stronger channel gain for the user-1, i.e.,

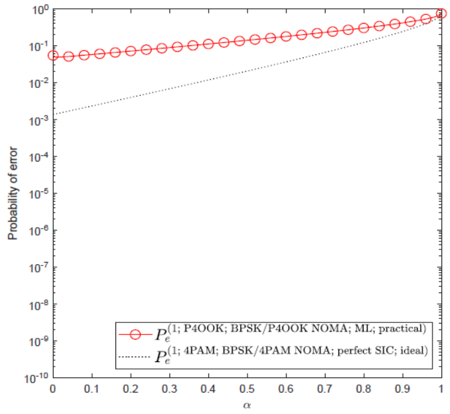

Fig. 2. Performance comparison of P4OOK in BPSK/P4OOK NOMA to 4PAM in BPSK/4PAM NOMA for the user-2, ($|h_2| = 0.9$).

$|h_1| = 2.0$ and the worse channel gain for the user-2, i.e., $|h_2| = 0.7$. As shown in Fig. 3 and 4, the performances of the proposed scheme show the similar patterns as in Fig. 1 and 2 with the higher/ lower probabilities of errors.

The last comment on the performance of both users is that with the existing modulation schemes, even if we increase the power enormously, the performances for both users in the standard NOMA never improve in the vicinity of $\alpha = 0.5$, (the proof is found in [2]). However, the performances for both users in the proposed NOMA improve linearly, i.e., as the allocated power to each user increases.

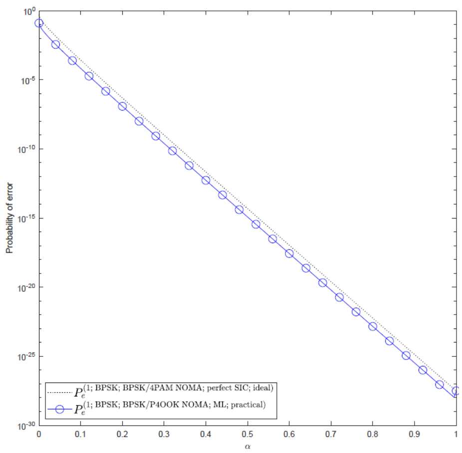

Fig. 3. Performance comparison of BPSK in BPSK/P4OOK NOMA to BPSK in BPSK/4PAM NOMA for the user-1, ($|h_1| = 2.0$).

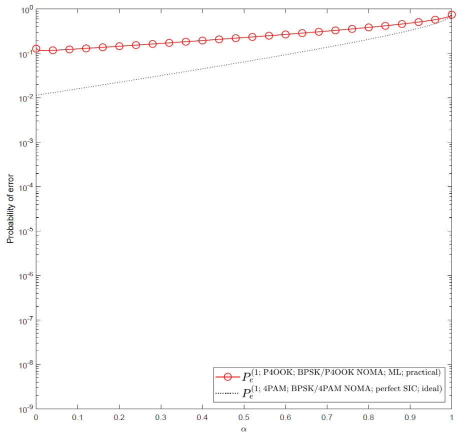

Fig. 4. Performance comparison of P4OOK in BPSK/P4OOK NOMA to 4PAM in BPSK/4PAM NOMA for the user-2, ($|h_2| = 0.7$).

V. Conclusion

This paper expanded POOK into P4OOK. It was shown that POOK can be extended in multilevel and still preserves the outstanding features over the standard modulation schemes, such as 4PAM. Consequently, NOMA with the help of P4OOK could be considered in multilevel for 5G and beyond mobile networks.

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