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How can Perfect SIC NOMA Be Implemented?

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ABSTRACT

Currently, the perfect successive interference cancellation (SIC) implementation has not been reported in the literature of non-orthogonal multiple access (NOMA). However, NOMA capacity is derived with the assumption of the perfect SIC. We try to find how to achieve the perfect SIC NOMA capacity and performance, under the practical modulation, such as the binary phase shift keying (BPSK) modulation. One possible way to achieve the near perfect SIC capacity/performance NOMA, proposed in this paper, is the dynamic superposition coding (DSC), which is a combination of the normal superposition coding (NSC) and the symmetric superposition coding (SSC), based on the power allocation factor.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, maximum likelihood, binary phase shift keying, superposition coding.

Ⅰ. Introduction

Non-orthogonal multiple access (NOMA) has opened the door to the possibility of increasing system capacity and reducing latency, so that it has been considered as a promising candidate for fifth generation (5G) mobile networks[1-5]. Such advantages are based on the perfect successive interference cancellation (SIC). However, the perfect SIC implementation has not been reported in the literature of NOMA. There are two main factors for the non-perfect SIC, such as channel state information (CSI) errors and non-perfect decoding of the inter user interference. In order to simplify the analysis, we consider only the latter factor, and the perfect CSI is assumed to be a priori. In this paper, one possible way to achieve the near perfect SIC is presented. The paper is organized as follows. Section II defines the system and channel model. In Section III, we suggest one candidate to achieve the near perfect SIC. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \leq \alpha \leq 1$, and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The superimposed signal is given by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \qquad (1)$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P} s_1 + \left(|h_1|\sqrt{(1-\alpha)P} s_2 + n_1\right) \\ r_2 &= |h_2|\sqrt{(1-\alpha)P} s_2 + \left(|h_2|\sqrt{\alpha P} s_1 + n_2\right) \end{aligned} \qquad (2)$$

where n_1 and $n_2 \sim \mathcal{N}\left(0, N_0/2\right) = \mathcal{N}\left(0, \sigma^2\right)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signals are given by

$$\begin{aligned} y_1 &= |h_1|\sqrt{\alpha P} s_1 + n_1 \\ y_2 &= r_2 . \end{aligned} \qquad (3)$$

We consider the binary phase shift keying (BPSK) modulation, with $s_1, s_2 \in \{+1, -1\}$. Let the

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information bits for the user-1 and the user-2 be $b_1, b_2 \in \{0,1\}$. Then in normal superposition coding (NSC), the modulation signal mapping is normal as

$$\begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases} \begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1 \end{cases}. \tag{4}$$

However, in symmetric superposition coding (SSC), the modulation signal mapping is changed as

$$\begin{cases} s_1(b_1 = 0, b_2 = 0) = +1 \\ s_1(b_1 = 1, b_2 = 0) = -1 \end{cases} \begin{cases} s_1(b_1 = 0, b_2 = 1) = -1 \\ s_1(b_1 = 1, b_2 = 1) = +1 \end{cases}$$
$$\begin{cases} s_2(b_2 = 0) = +1 \\ s_2(b_2 = 1) = -1. \end{cases} \tag{5}$$

Ⅲ. DSC NOMA Capacity/Performance

In the standard NOMA, NSC is used. NSC NOMA gets away from the perfect SIC NOMA for the power allocation factor interval $0.2 \le \alpha \le 0.8$ [6,7]. Such degraded capacity/performance can be partially recovered with SSC NOMA for $0.2 \le \alpha \le 0.8$ [8,9]. However, such gain pays the price of degraded capacity/performance for $0.8 \le \alpha \le 1$ [8,9].

Therefore, one possible way to achieve the near perfect SIC capacity/performance NOMA, proposed in this paper, is dynamic superposition coding (DSC), which is a combination of NSC and SSC, based on the power allocation factor. This DSC concept is inspired by shaping gain from the capacity of equiprobable *M*-ary constellations[10] to the capacity of the ideal Gaussian modulation[11]. So, we also call DSC as superposition shaping.

Now we present how to combine two superposition coding schemes to achieve the near perfect SIC NOMA. To simplify the analysis, we use the fact that SIC is performed not on the user-2 but only on the user-1 in the standard NOMA. Therefore, we consider only the capacity/performance of the user-1 to achieve the near perfect SIC. First, the capacity/performance of the perfect SIC NOMA for the user-1 are summarized as follows; The capacity of the perfect SIC NOMA is given by [10],

$$\begin{aligned} C_1^{(b;\, pecfect\ SIC)} &= -\int_{-\infty}^{\infty} p_{Y_1}(y_1) \log_2 p_{Y_1}(y_1)\, dy_1 \\ &\quad - 0.5 \log_2 \left(2\pi e \sigma^2\right) \end{aligned} \tag{6}$$

where

$$p_{Y_1}(y_1) = 0.5\left(\boldsymbol{\mathcal{N}}\left(|h_1|\sqrt{\alpha P}, \sigma^2\right) + \boldsymbol{\mathcal{N}}\left(-|h_1|\sqrt{\alpha P}, \sigma^2\right)\right). \tag{7}$$

and the performance is simply the BPSK modulation performance, for all α,

$$P_e^{(1;\, perfect\ SIC)} = q_{h_1}^{(0;1)} \tag{8}$$

where for the simplification, we define the notation as

$$q_{h_1}^{(I;A)} = Q\left(|h_1|\sqrt{P}\left(I\sqrt{(1-\alpha)} + A\sqrt{\alpha}\right) / \sigma\right). \tag{9}$$

where $Q(x) = \int_x^{\infty} e^{-\frac{z^2}{2}} / \sqrt{2\pi}\, dz$.

Second, the capacity/performance of the non-SIC NSC NOMA for the user-1 are summarized as follows; the capacity is represented as [7]

$$\begin{aligned} C_1^{(b;\, non-SIC;\, NSC)} &= -\int_{-\infty}^{\infty} p_{R_1}(r_1) \log_2 p_{R_1}(r_1)\, dr_1 \\ &\quad + \int_{-\infty}^{\infty} p_{N_4}(n_4) \log_2 p_{N_4}(n_4)\, dn_4 \end{aligned} \tag{10}$$

where

$$\begin{aligned} & p_{R_1}(r_1) \\ &= \frac{1}{4}\sum_{i=0}^{1}\sum_{j=0}^{1} \boldsymbol{\mathcal{N}}\left((-1)^i |h_1|\sqrt{\alpha P} + (-1)^{i+j}|h_1|\sqrt{(1-\alpha)P}, \sigma^2\right) \end{aligned} \tag{11}$$

and $p_{N_4}(n_4) = \frac{1}{2}\sum_{i=0}^{1} \boldsymbol{\mathcal{N}}\left((-1)^i |h_1|\sqrt{(1-\alpha)P}, \sigma^2\right)$.

And the performance is calculated as [6], for $\alpha > 0.5$,

$$P_e^{(1;\, non-SIC\ ML;\, NSC)} = \frac{1}{2} q_{h_1}^{(-1;1)} + \frac{1}{2} q_{h_1}^{(1;1)} \tag{12}$$

and for $\alpha < 0.5$,

$$\begin{aligned} & P_e^{(1;\, non-SIC\ ML;\, NSC)} \\ &\simeq q_{h_1}^{(0;1)} + \frac{1}{2} q_{h_1}^{(1;-1)} - \frac{1}{2} q_{h_1}^{(1;1)} - \frac{1}{2} q_{h_1}^{(2;-1)} + \frac{1}{2} q_{h_1}^{(2;1)}. \end{aligned} \tag{13}$$

Third, the capacity/performance of the non-SIC SSC NOMA for the user-1 are summarized as follows; The capacity is represented as [9]

$$
\begin{aligned}
C_1^{(b;\, non-SIC;\, SSC)} &= -\int_{-\infty}^{\infty} p_{R_1}(r_1)\log_2 p_{R_1}(r_1)dr_1 \\
&+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1 \mid b_1=0)p_{B_1}(b_1=0)\log_2 p_{R_1|B_1}(r_1 \mid b_1=0)\,dr_1 \\
&+\int_{-\infty}^{\infty} p_{R_1|B_1}(r_1 \mid b_1=1)p_{B_1}(b_1=1)\log_2 p_{R_1|B_1}(r_1 \mid b_1=1)\,dr_1
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
&p_{R_1|B_1}(r_1 \mid b_1=0) \\
&= \frac{1}{2}\sum_{i=0}^{1} \mathcal{N}\left(\left(-1\right)^i \left|h_1\right|\sqrt{\alpha P} + \left(-1\right)^i \left|h_1\right|\sqrt{(1-\alpha)P}, \sigma^2\right)
\end{aligned} \tag{15}
$$

and

$$
\begin{aligned}
&p_{R_1|B_1}(r_1 \mid b_1=1) \\
&= \frac{1}{2}\sum_{i=0}^{1} \mathcal{N}\left(\left(-1\right)^i \left|h_1\right|\sqrt{\alpha P} + \left(-1\right)^{i+1} \left|h_1\right|\sqrt{(1-\alpha)P}, \sigma^2\right).
\end{aligned} \tag{16}
$$

And the performance is calculated as [8], for $\alpha > 0.5$,

$$
P_e^{(1;\, non-SIC\ ML;\, SSC)} \simeq q_{h_1}^{(0;1)} - \frac{1}{2}q_{h_1}^{(2;1)} + \frac{1}{2}q_{h_1}^{(2;-1)} \tag{17}
$$

and for $\alpha < 0.5$,

$$
P_e^{(1;\, non-SIC\ ML;\, SSC)} \simeq q_{h_1}^{(1;0)} - \frac{1}{2}q_{h_1}^{(1;2)} + \frac{1}{2}q_{h_1}^{(-1;2)}. \tag{18}
$$

Now we put together all the pieces into one plot

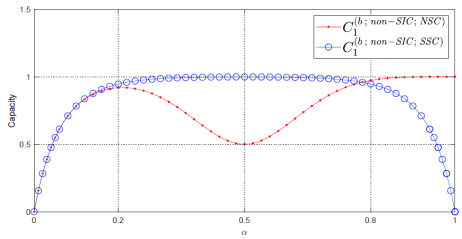

Fig. 1. Capacities of errors with non-SIC NSC/SSC ML for the user-1.

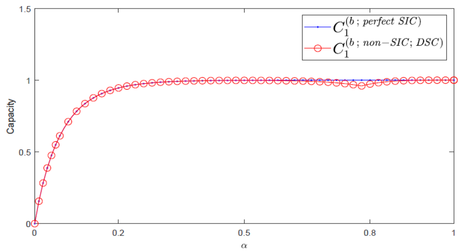

Fig. 2. Capacities of errors with perfect SIC/non-SIC ML DSC for the user-1.

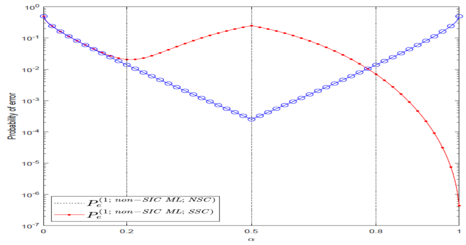

Fig. 3. Probabilities of errors with non-SIC NSC/SSC ML for the user-1.

in Fig. 1 and 3. Then the way looks unveiled; DSC is obtained as

$$
\begin{aligned}
&C_1^{(b;\, non-SIC;\, DSC)} = \max_{\alpha}\left\{C_1^{(b;\, non-SIC;\, NSC)}, C_1^{(b;\, non-SIC;\, SSC)}\right\} \\
&P_e^{(1;\, non-SIC\ ML;\, DSC)} \\
&= \min_{\alpha}\left\{P_e^{(1;\, non-SIC\ ML;\, NSC)}, P_e^{(1;\, non-SIC\ ML;\, SSC)}\right\}.
\end{aligned} \tag{19}
$$

Ⅳ. Results and Discussions

As we mention in Section I, we assume the perfect CSI is available at the receiver. Therefore, our analyses do not include the impact of the CSI errors. Assume that the channel gain is $\left|h_1\right| = 1.1$. The total transmit signal power to one-sided power spectral density ratio is $P / N_0 = 10$. In a point of view of capacity, DSC NOMA loses a little from the perfect SIC NOMA, as shown in Fig. 1 and 2. One question about this paper is whether or not the proposed scheme in this paper can be applied to the case of more than two users. Currently, we don't have the analytical results, even for the existing schemes with more than two users. First, we try to analyze the existing schemes for the arbitrary number of users, and then we can consider the proposed scheme for any number of users. Another question on this proposed scheme is about modeling the channel gains as the random variables (RVs), instead of the specific values, because the proposed scheme should be compared in various channel environments. Such problems could be meaningful, if the difficulty of the analysis is overcome. Next, in Fig. 3, the NSC and SSC NOMA performances are shown. Then by taking the minimum performance between them, we obtain the performance of the proposed scheme in Fig. 4, which is compared to the perfect SIC NOMA performance, i.e., the goal of this paper. We observe that the proposed scheme performance achieves the perfect SIC NOMA performance, for the power allocation factor less than 50 %. However, we also observe that there is a large performance gap between the perfect SIC NOMA and the proposed scheme, for the power allocation factor greater than 50 %. Such gap is currently not explained clearly. We need to research more for this gap. In addition, in order to investigate the proposed scheme for more various channel environments, we consider the stronger channel gain, i.e., $\left|h_1\right| = 1.9$. As shown in Fig. 5 and 6, the capacity of the proposed scheme approaches that of the perfect SIC more closely and the gap between them cannot be distinguishable. And the

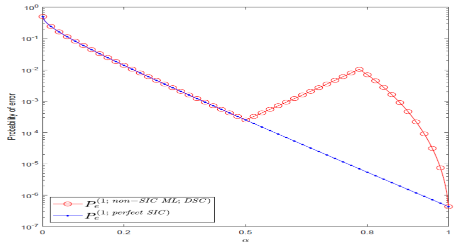

Fig. 4. Probabilities of errors with perfect SIC/non-SIC ML DSC for the user-1.

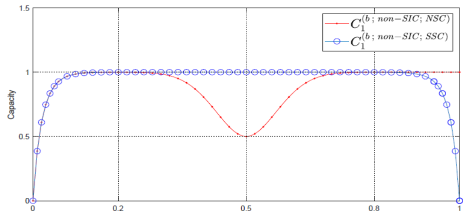

Fig. 5. Capacities of errors with non-SIC NSC/SSC ML for the user-1.

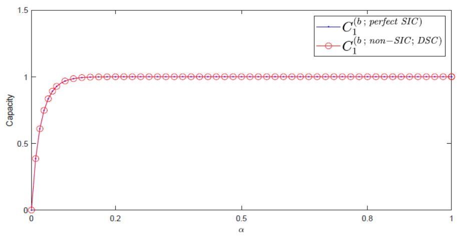

Fig. 6. Capacities of errors with perfect SIC/non-SIC ML DSC for the user-1.

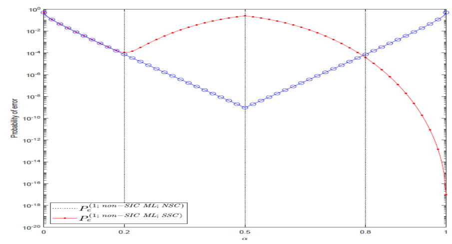

Fig. 7. Probabilities of errors with non-SIC NSC/SSC ML for the user-1.

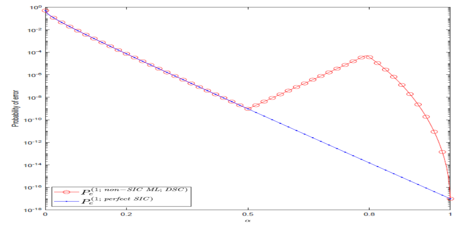

Fig. 8. Probabilities of errors with perfect SIC/non-SIC ML DSC for the user-1.

performances of the proposed scheme for $\left|h_1\right| = 1.9$ in Fig. 7 and 8 show the similar patterns as in Fig. 3 and 4 with the lower probabilities of errors. Lastly, we should give the intuitive perspective on the proposed scheme; both NSC and SSC NOMA capacities/performances are very close to those of the perfect SIC for $0 \le \alpha \le 0.2$. First, SSC NOMA performs better than NSC NOMA for $0.2 \le \alpha \le 0.8$. And NSC NOMA performance is better than SSC NOMA performance for $0.8 \le \alpha \le 1$. Then the proposed scheme takes the better performance NOMA according to the power allocation factor. Therefore, DSC NOMA, the proposed scheme, can be better than both NSC and SSC NOMA.

V. Conclusion

We considered achieving the perfect SIC NOMA. It was shown that the near perfect SIC NOMA could be achieved by the superposition shaping of DSC NOMA. In the future work, we will continue to find the way to achieve the perfect SIC NOMA.

References

[1] Y. Saito, Y. Kishiyama, A. Benjebbour, T. Nakamura, A. Li, and K. Higuchi, "Non-orthogonal multiple access (NOMA) for cellular future radio access," in *Proc. IEEE 77th VTC Spring*, pp. 1-5, 2013.

[2] Z. Ding, P. Fan, and H. V. Poor, "Impact of user pairing on 5G nonorthogonal multiple-access downlink transmissions," *IEEE Trans. Veh. Technol.*, vol. 65, no. 8, pp. 6010-6023, Aug. 2016.

[3] S. R. Islam, J. M. Kim, and K. S. Kwak, "On non-orthogonal multiple access (NOMA) in 5G systems," *J. KICS*, vol. 40, no. 12, pp. 2549-2558, Dec. 2015.

[4] M. H. Lee, V. C. M. Leung, and S. Y. Shin, "Dynamic bandwidth allocation of NOMA and OMA for 5G," *J. KICS*, vol. 42, no. 12, pp. 2383-2390, Dec. 2017.

[5] M. B. Uddin, M. F. Kader, A. Islam, and S. Y. Shin, "Power optimization of NOMA for multi-cell networks," *J. KICS*, vol. 43, no. 7, pp. 1182-1190, Jul. 2018.

[6] K. Chung, "Performance analysis on non-SIC ML receiver for NOMA strong channel user," *J. KICS*, vol. 44, no. 3, pp. 505-508, Mar. 2019.

[7] K. Chung, "Non-SIC NOMA: Gaussian mixture channel capacity under BPSK and QPSK modulations," *J. KICS*, vol. 44, no. 4, pp. 666-669, Apr. 2019.

[8] K. Chung, "Performance analysis of NOMA with symmetric superposition coding," *J. IKEEE*, vol. 23, no. 1, pp. 314-317, Mar. 2019.

[9] K. Chung, "SSC NOMA capacity: zero capacity bit to symbol mapping over Gaussian mixture channel," *J. KICS*, vol. 44, no. 6, pp. 1076-1079, Jun. 2019.

[10] G. D. Forney Jr. and G. Ungerboeck, "Modulation and coding for linear Gaussian channels," *IEEE Trans. Inf. Theory*, vol. 44, pp. 2384-2415, Oct. 1998.

[11] C. E. Shannon, "A mathematical theory of communication," *The Bell Syst. Technical J.*, vol. 27, no. 3, pp. 379-423, Oct. 1948.