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Orthogonal NOMA (Part II): Achieving All Users Perfect SIC Performance at Price Below 3 dB

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ABSTRACT

The simple mathematical equation, the conditional mutual information with Gaussian modulations, gives birth to non-orthogonal multiple access (NOMA). The equation requires the perfect successive interference cancellation (SIC) for one of two users. However, the existing modulation schemes do not guarantee the perfect SIC performance, for the entire operating range of power allocation, even though it is possible for the part of the whole range. Recently, new modulation techniques, such as polar on-off keying (POOK), make the equation hold for all power allocation factor [6]. This paper considers all two users perfect SIC performance. The equation does not guarantee the perfect SIC performance for one of two users. Therefore we pay the price for all users perfect SIC performance. It is shown that the price is less than an additional 3 dB power. In result, the trade-off between performance and power could be considered, with the plenty power of the base station.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, channel capacity, binary phase shift keying.

Ⅰ. Introduction

5G and beyond mobile networks have been commercialized in Korea, on April 3, 2019 for the first time in the world. However, the standardization is still in progress. One of 5G mobile radio access techniques is non-orthogonal multiple access (NOMA)[1-5]. In NOMA, one of two users is guaranteed to communicate at the perfect successive interference cancellation (SIC) performance. Recently, such performance has been achieved by the orthogonal NOMA (O NOMA) with polar on-off keying (POOK) over binary phase shift keying (BPSK)[6]. This paper considers NOMA with all two users perfect SIC performance. Such performance implies the orthogonal multiple access (OMA), such as almost all current communication systems. If the orthogonality is achieved in NOMA, we could double the system capacity of OMA with the same channel resources. We consider in this paper the NOMA with the two users; the analysis of three or more users in NOMA is discussed in Section V. The paper is organized as follows. Section II defines the system and channel model. In Section III, a brief review of POOK is summarized. In Section IV, achieving all users perfect SIC Performance is presented. In Section V, the results are presented and discussed. The paper is concluded in Section VI.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, ($0\% \le \alpha \le 100\%$), and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The expectation notation $\mathbb{E}[u]$ is defined as

$$\mathbb{E}[u] = \int_{-\infty}^{\infty} u p_U(u)\, du \tag{1}$$

where $p_U(u)$ is the probability density function (PDF). The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{2}$$

Before the successive interference cancellation

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(SIC) is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} r_1 &= |h_1|\sqrt{\alpha P}s_1 + \left(|h_1|\sqrt{(1-\alpha)P}s_2 + n_1\right) \\ r_2 &= |h_2|\sqrt{(1-\alpha)P}s_2 + \left(|h_2|\sqrt{\alpha P}s_1 + n_2\right) \end{aligned} \quad (3)$$

where n_1 and $n_2 \sim \mathcal{N}(0, N_0/2)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed,

$$y_1 = r_1 - |h_1|\sqrt{(1-\alpha)P}s_2 = |h_1|\sqrt{\alpha P}s_1 + n_1. \quad (4)$$

Ⅲ. Brief Review of POOK

We, now, briefly review POOK [6]. On-off keying (OOK) is the simplest modulation technique. The carrier is sent or not. Assume the binary phase shift keying (BPSK) modulation for the user-1, with $s_1 \in \{+1, -1\}$. Then POOK, with $s_2 \in \{+\sqrt{2}, 0, -\sqrt{2}\}$, is the inter user interference s_1 dependent OOK. The power is normalized as

$$\mathbb{E}\left[|s_2|^2\right] = \frac{1}{4}\left(+\sqrt{2}\right)^2 + \frac{1}{2}\left(\sqrt{0}\right)^2 + \frac{1}{4}\left(-\sqrt{2}\right)^2 = 1. \quad (5)$$

Compare the standard OOK, $s_{OOK} \in \{+\sqrt{2}, 0\}$, with

$$\mathbb{E}\left[|s_{OOK}|^2\right] = \frac{1}{2}\left(\sqrt{2}\right)^2 + \frac{1}{2}(0)^2 = 1. \quad (6)$$

If there exists the inter user interference s_1, polar OOK gets away from the interference in the direction from the origin to the interference. Therefore we give polarity to OOK, with the information input bits for the user-1 and the user-2 being $b_1, b_2 \in \{0,1\}$, as

$$\begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases}$$

$$\begin{cases} s_2(b_2 = 0 \mid b_1 = 0) = +\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 0) = 0 \end{cases} \quad \begin{cases} s_2(b_2 = 0 \mid b_1 = 1) = -\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 1) = 0. \end{cases} \quad (7)$$

Ⅳ. Achieving All Users Perfect SIC Performance

The main equation in NOMA,

$$\log\left(\frac{P_1 + P_2 + N_0}{N_0}\right) = \log\left(\frac{P_1 + N_0}{N_0}\right) + \log\left(\frac{P_1 + P_2 + N_0}{P_1 + N_0}\right) \quad (8)$$

does not guarantee the perfect SIC capacity for one of two users. If we want all users perfect SIC capacity, the equation should be changed into

$$\begin{aligned} &\log\left(\frac{P_1 + N_0}{N_0}\right) + \log\left(\frac{P_2 + N_0}{N_0}\right) \\ &= \log\left(\frac{P_1 + P_2 + N_0}{N_0}\right) + \left(\log\left(\frac{P_2 + N_0}{N_0}\right) - \log\left(\frac{P_1 + P_2 + N_0}{P_1 + N_0}\right)\right). \end{aligned} \quad (9)$$

Note that

$$\left(\log\left(\frac{P_2 + N_0}{N_0}\right) - \log\left(\frac{P_1 + P_2 + N_0}{P_1 + N_0}\right)\right) \geq 0 \quad (10)$$

where the equality hold for $P_2 = 0$. Therefore, in order for NOMA to achieve all users perfect SIC capacity, we need more nonnegative capacity in the inequality (10) . Such capacity can be interpreted practically as less than an additional 3 dB power. Now, we see that; for the fair comparison, we assume the BPSK modulations for both users in the standard NOMA, i.e., the BPSK/BPSK NOMA,

$$s_1, s_2 \in \{+1, -1\}. \quad (11)$$

Then, if the perfect SIC is assumed, then the performance of the user-1 is simply the probability of errors of the BPSK modulation, for all α,

$$P_e^{(1;\, BPSK;\, BPSK/BPSK\ NOMA;\, perfect\ SIC;\, ideal)} = Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0/2}}\right). \quad (12)$$

And also, if the perfect SIC for the user-2 is assumed, then the received signal is given by,

$$y_2 = r_2 - |h_2|\sqrt{\alpha P}s_1 = |h_2|\sqrt{(1-\alpha)P}s_2 + n_2. \quad (13)$$

Then the performance is also simply the probability of errors of the BPSK modulation, for all α,

$$P_e^{(2;\, BPSK;\, BPSK/BPSK\ NOMA;\, perfect\ SIC;\, ideal)} = Q\left(\frac{|h_2|\sqrt{(1-\alpha)P}}{\sqrt{N_0/2}}\right). \quad (14)$$

Now, we summarize the performance results in [6]; also, in the orthogonal NOMA, i.e., the BPSK/POOK

NOMA, the BPSK modulation is assumed for the user-1,

$$s_1 \in \{+1, -1\}. \quad (15)$$

Then the probability of error for the user-1 is calculated as, for all α,

$$\begin{aligned} & P_e^{(1;\, BPSK;\, BPSK/POOK\ NOMA;\, ML;\, practical)} \\ &= \frac{1}{2} Q\left(\frac{|h_1|\sqrt{\alpha P}}{\sqrt{N_0/2}} \right) \\ &+ \frac{1}{2} Q\left(\frac{|h_1|\sqrt{P}\left(\sqrt{2(1-\alpha)} + \sqrt{\alpha}\right)}{\sqrt{N_0/2}} \right). \end{aligned} \quad (16)$$

Note that for the same BPSK modulation, we have the different performances. And also note that

$$\begin{aligned} & P_e^{(1;\, BPSK;\, BPSK/POOK\ NOMA;\, ML;\, practical)} \leq \\ & P_e^{(1;\, BPSK;\, BPSK/BPSK\ NOMA;\, perfect\ SIC;\, ideal)} \end{aligned} \quad (17)$$

where the equality holds for $\alpha = 1$. With the equation (17) , the perfect SIC performance for the user-1 has been already achieved in the orthogonal NOMA. What is left is that the user-2 achieves the perfect SIC performance. The probability of errors for the user-2 in the orthogonal NOMA is presented in [6], for all α,

$$\begin{aligned} & P_e^{(2;\, POOK;\, BPSK/POOK\ NOMA;\, ML;\, practical)} \\ &\simeq Q\left(\frac{|h_2|\sqrt{(1-\alpha)P/2}}{\sqrt{N_0/2}} \right) \\ &- \frac{1}{2} Q\left(\frac{|h_2|\sqrt{P}\left(2\sqrt{\alpha} + 3\sqrt{(1-\alpha)/2}\right)}{\sqrt{N_0/2}} \right) \\ &+ \frac{1}{2} Q\left(\frac{|h_2|\sqrt{P}\left(2\sqrt{\alpha} + \sqrt{(1-\alpha)/2}\right)}{\sqrt{N_0/2}} \right). \end{aligned} \quad (18)$$

We approximate the above equation (18) further, for all α,

$$\begin{aligned} & P_e^{(2;\, POOK;\, BPSK/POOK\ NOMA;\, ML;\, practical)}\left((1-\alpha)P\right) \\ &\simeq Q\left(\frac{|h_2|\sqrt{(1-\alpha)P/2}}{\sqrt{N_0/2}} \right) \end{aligned} \quad (19)$$

where the calculation error is small and tolerable, resorting to the $68-95-99.7$ rule, for $\mathcal{N}\left(0,1^2\right)$,

$$\begin{aligned} Q(3) &= \int_3^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.0015 \\ Q(2) &= \int_2^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.025 \\ Q(1) &= \int_1^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz \simeq 0.16. \end{aligned} \quad (20)$$

It is time to use the additional power for the user-2 as, for all α,

$$\begin{aligned} & P_e^{(2;\, POOK;\, BPSK/POOK\ NOMA;\, ML;\, practical)}\left(2(1-\alpha)P\right) \\ &\simeq Q\left(\frac{|h_2|\sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right). \end{aligned} \quad (21)$$

Finally, we achieve the perfect SIC performance for the user-2, too, for all α,

$$\begin{aligned} & P_e^{(2;\, POOK;\, BPSK/POOK\ NOMA;\, ML;\, practical)}\left(2(1-\alpha)P\right) \simeq \\ & P_e^{(2;\, BPSK;\, BPSK/BPSK\ NOMA;\, perfect\ SIC;\, ideal)} \\ &= Q\left(\frac{|h_2|\sqrt{(1-\alpha)P}}{\sqrt{N_0/2}} \right). \end{aligned} \quad (22)$$

The additional power is used only for the user-2. Therefore, the additional power is less than 3 dB, because

$$\alpha P + 2(1-\alpha)P = 2P - \alpha P \leq 2P \quad (23)$$

where the equality holds for $\alpha = 0$. In this case, the additional power is the maximum 3dB and all power of $2P$ is allocated to the user-2. It is reasonable, because the performance of OOK, including POOK, is worse than that of BPSK by 3 dB. Except the worst case $\alpha = 0$, the additional power is less than 3 dB. As α increases, the additional power decreases because the BPSK performance in the BPSK/POOK NOMA is better than the BPSK performance in the BPSK/BPSK NOMA.

Ⅴ. Results and Discussions

Assume that the channel gains are $|h_1| = 1.1$ and $|h_2| = 0.9$. The total transmit signal power to one-sided power spectral density ratio is $P/N_0 = 15$, ($11.76\ \text{dB} = 10\log_{10}(15)$). The probability of errors of the perfect SIC NOMA in the equation (12) is compared to that of O NOMA in the equation (16) , for the user-1, in Fig. 1, with different power allocations, $0 \leq \alpha \leq 1$. As shown in Fig. 1, for all α, the performance of O NOMA is better than that of the perfect SIC NOMA for the user-1. We also compare the probability of errors of the perfect

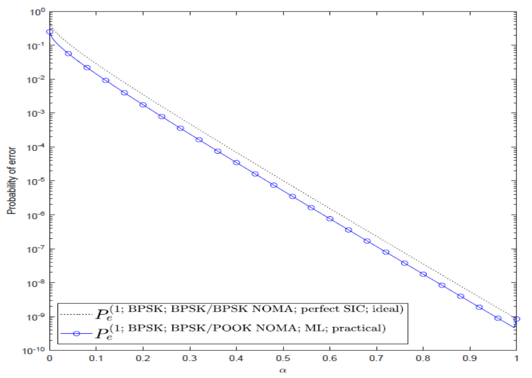

Fig. 1. Probabilities of errors with NOMA and O NOMA for the user-1.

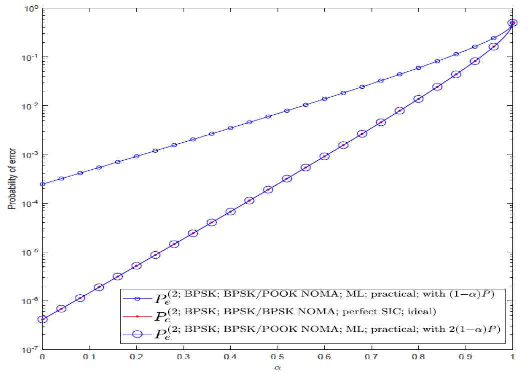

Fig. 2. Probabilities of errors with NOMA and O NOMA for the user-2.

SIC NOMA in the equation (14) to those of O NOMA, for the user-2 in the equation (19) and (21) , in Fig. 2. As shown in Fig. 2, for the user-2, the probability of errors of O NOMA with $2(1-\alpha)P$ is the same as that of the perfect SIC NOMA.

Lastly, the authors in this paper should mention the current status of the polarity modulations; first, POOK has been invented for the two users of the BPSK/POOK NOMA. Therefore, in the future researches, it would be meaningful to extend the polarity modulations into NOMA with the three or more users; the key idea of the polarity modulations is to mitigate the serious effect of the superposition. The more the users are served, the more severe the effect is.

Ⅵ. Conclusion

This paper proposed NOMA with all two users perfect SIC performance. First, in order for NOMA to achieve such performance, it was shown that the additional nonnegative capacity is required. Then we also presented such practical performance with an additional power less than 3 dB. In result, the trade-off between the performance and the power could be considered with the plenty power of the base station.

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